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PHILOSOPHICAL TRANSACTIONS.

I. *On the Circulation of Air observed in* KUNDT'S *Tubes, and on some Allied Acoustical Problems.*

By Lord RAYLEIGH, *D.C.L.*, *F.R.S.*

Received October 23,—Read November 15, 1883.

EXPERIMENTERS in Acoustics have discovered more than one set of phenomena apparently depending for their explanation upon the existence of regular currents of air, resulting from vibratory motion, of which theory has as yet rendered no account. This is not, perhaps, a matter for surprise, when we consider that such currents, involving as they do *circulation* of the fluid, could not arise in the absence of friction, however great the extent of vibration. And even when we are prepared to include in our investigations the influence of friction, by which the motion of fluid in the neighbourhood of solid bodies may be greatly modified, we have no chance of reaching an explanation, if, as is usual, we limit ourselves to the supposition of infinitely small motion and neglect the squares and higher powers of the mathematical symbols by which it is expressed.

In the present paper three problems of this kind are considered, two of which are illustrative of phenomena observed by FARADAY.* In these problems the fluid may be treated as incompressible. The more important of them relates to the currents generated over a vibrating plate, arranged as in CHLADNI'S experiments. It was discovered by SAVART that very fine powder does not collect itself at the nodal lines, as does sand in the production of CHLADNI'S figures, but gathers itself into a cloud which, after hovering for a time, settles itself over the places of maximum vibration. This was traced by FARADAY to the action of currents of air, rising from the plate at

* "On a Peculiar Class of Acoustical Figures; and on certain Forms assumed by groups of particles upon Vibrating Elastic Surfaces," Phil. Trans., 1831, p. 299.

the places of maximum vibration, and falling back to it at the nodes. In a vacuum the phenomena observed by SAVART do not take place, all kinds of powder collecting at the nodes. In the investigation of this, as of the other problems, the motion is supposed to take place in two dimensions.

It is probable that the colour phenomena observed by SEDLEY TAYLOR* on liquid films under the action of sonorous vibrations are to be referred to the operation of the aerial vortices here investigated. In a memoir on the colours of the soap-bubble,† BREWSTER has described the peculiar arrangements of colour accompanied by whirling motions, caused by the impact of a gentle current of air. In Mr. TAYLOR'S experiments the film probably divides itself into vibrating sections, associated with which will be aerial vortices reacting laterally upon the film.

The third problem relates to the air currents observed by DVORAK in a KUNDT'S tube, to which is apparently due the formation of the dust figures. In this case we are obliged to take into account the compressibility of the fluid.

[My best thanks are due to Mr. W. M. HICKS, who has been good enough to examine the mathematical work of the paper. The results are thus put forward with greater confidence than I could otherwise have felt.]

§ 1. In the usual notation the equations of motion in two dimensions are

$$\left.\begin{aligned}\frac{1}{\rho}\frac{dp}{dx}&=-\frac{du}{dt}+\nu\,\nabla^2u-u\frac{du}{dx}-v\frac{du}{dy}\\ \frac{1}{\rho}\frac{dp}{dy}&=-\frac{dv}{dt}+\nu\,\nabla^2v-u\frac{dv}{dx}-v\frac{dv}{dy}\end{aligned}\right\}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

and since the fluid is incompressible,

$$\frac{du}{dx}+\frac{dv}{dy}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

In virtue of (2) we may write

$$u=\frac{d\psi}{dy},\quad v=-\frac{d\psi}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Eliminating p between equations (1), we get

$$\nu\,\nabla^2\left(\frac{du}{dy}-\frac{dv}{dx}\right)-\frac{d}{dt}\left(\frac{du}{dy}-\frac{dv}{dx}\right)=\frac{d}{dy}\left(u\frac{du}{dx}+v\frac{du}{dy}\right)-\frac{d}{dx}\left(u\frac{dv}{dx}+v\frac{dv}{dy}\right).$$

Now

$$u\frac{du}{dx}+v\frac{du}{dy}=\tfrac{1}{2}\frac{d(u^2+v^2)}{dx}+v\left(\frac{du}{dy}-\frac{dv}{dx}\right)$$

$$u\frac{dv}{dx}+v\frac{dv}{dy}=\tfrac{1}{2}\frac{d(u^2+v^2)}{dy}-u\left(\frac{du}{dy}-\frac{dv}{dx}\right),$$

* Proc. Roy. Soc., 1878.

† Edinburgh Transactions, 1866–67.

and

$$\frac{du}{dy}-\frac{dv}{dx}=\nabla^2\psi,$$

so that

$$\nabla^4\psi-\frac{1}{\nu}\frac{d}{dt}\nabla^2\psi=\frac{u}{\nu}\frac{d\nabla^2\psi}{dx}+\frac{v}{\nu}\frac{d\nabla^2\psi}{dy} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

For the first approximation we neglect the right-hand member of (4), as being of the second order in the velocities, and take simply

$$\nabla^2\left(\nabla^2-\frac{1}{\nu}\frac{d}{dt}\right)\psi=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

The solution of (5) may be written*

$$\psi=\psi_1+\psi_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6),$$

where

$$\nabla^2\psi_1=0, \quad \left(\nabla^2-\frac{1}{\nu}\frac{d}{dt}\right)\psi_2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

We will now introduce the suppositions that the motion is periodic with respect to x, and also (to a first approximation) with respect to t. We thus assume that ψ_1 and ψ_2 are proportional to $\cos kx$, and also to e^{int}. The wave-length (λ) along x is $2\pi/k$, and the period τ is $2\pi/n$. The equations (7) now become

$$\left(\frac{d^2}{dy^2}-k^2\right)\psi_1=0, \quad \left(\frac{d^2}{dy^2}-k^2-\frac{in}{\nu}\right)\psi_2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8),$$

by which ψ_1 and ψ_2 are to be determined as functions of y. If we write

$$k'^2=k^2+\frac{in}{\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9),$$

we have as the most general solutions of (8)

$$\psi_1=\mathrm{A}e^{-ky}+\mathrm{B}e^{+ky} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10),$$

$$\psi_2=\mathrm{C}e^{-k'y}+\mathrm{D}e^{+k'y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

With respect to the value of k', we see from (9) that it is complex. If we write

$$k^2=\mathrm{P}^2\cos 2\alpha, \quad \frac{n}{\nu}=\mathrm{P}^2\sin 2\alpha,$$

then

$$k'=\mathrm{P}\cos\alpha+i\,\mathrm{P}\sin\alpha.$$

* STOKES "On Pendulums," Camb. Phil. Trans., vol. ix., 1850.

In all the applications that we shall have occasion to make, an approximate value of k' is admissible. On account of the smallness of ν, n/ν is very large in comparison with k^2, that is to say, the thickness of the stratum through which the tangential motion can be propagated in time τ is very small relatively to the wave-length λ. We may therefore neglect k^4 in the equation

$$P^4=k^4+\frac{n^2}{\nu^2},$$

and take simply

$$P^2=n/\nu.$$

Again

$$(\sin\alpha-\cos\alpha)^2=1-\sin 2\alpha=\tfrac{1}{2}\frac{k^4\nu^2}{n^2},$$

so that the difference between $\cos\alpha$ and $\sin\alpha$ may be neglected. We will therefore write

$$k'=\beta(1+i) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12),$$

where

$$\beta=\sqrt{\frac{n}{2\nu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13).$$

We must now distinguish the cases which we have to investigate. In the first we suppose that a wave motion is in progress in a vessel whose horizontal bottom occupies a fixed plane $y=0$. We may conceive the fluid to be water vibrating in stationary waves under the action of gravity, the question being to examine the influence of the bottom upon the motion. If there are no other solids in the neighbourhood of the bottom, we may put $D=0$, y being measured upwards, and β being taken positive.

The conditions to be satisfied at $y=0$ are that u and v should there vanish. Thus

$$A+B+C=0, \quad -kA+kB-k'C=0,$$

so that

$$\psi=C\left\{-\cosh ky+\frac{k'}{k}\sinh ky+e^{-k'y}\right\},$$

and

$$u=C\{-k\sinh ky+k'\cosh ky-k'e^{-k'y}\}.$$

At a short distance from the bottom, $u=k'C$. If we denote by u_0 the maximum value of u near the bottom, we have

$$k'C=u_0\, e^{int}\cos kx,$$

and then

$$\psi=u_0\, e^{int}\cos kx\left\{-\frac{\cosh ky}{k'}+\frac{\sinh ky}{k}+\frac{e^{-k'y}}{k'}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14),$$

$$u=u_0\, e^{int}\cos kx\left\{-\frac{k}{k'}\sinh ky+\cosh ky-e^{-k'y}\right\}\quad . \quad . \quad . \quad . \quad . \quad (15),$$

$$v=u_0\, e^{int}\sin kx\left\{-\frac{k}{k'}\cosh ky+\sinh ky+\frac{k}{k'}e^{-k'y}\right\}\quad . \quad . \quad . \quad . \quad (16).$$

These are the symbolical values. If we throw away the imaginary parts, we have as the solution in real quantities by (12),

$$\psi=u_0\cos kx\left\{-\frac{\cosh ky}{\beta\sqrt{2}}\cos(nt-\tfrac{1}{4}\pi)+\frac{\sinh ky}{k}\cos nt+\frac{e^{-\beta y}}{\beta\sqrt{2}}\cos(nt-\tfrac{1}{4}\pi-\beta y)\right\}\quad . \quad (17),$$

$$u=u_0\cos kx\left\{-\frac{k\sinh ky}{\beta\sqrt{2}}\cos(nt-\tfrac{1}{4}\pi)+\cosh ky\cos nt-e^{-\beta y}\cos(nt-\beta y)\right\}\quad . \quad . \quad (18),$$

$$v=u_0\sin kx\left\{-\frac{k\cosh ky}{\beta\sqrt{2}}\cos(nt-\tfrac{1}{4}\pi)+\sinh ky\cos nt+\frac{k\,e^{-\beta y}}{\beta\sqrt{2}}\cos(nt-\tfrac{1}{4}\pi-\beta y)\right\}\quad (19).$$

This is the solution to a first approximation. At a very small distance from the bottom the terms in $e^{-\beta y}$ become insensible.

Although the values of u and v in (18) and (19) are strictly periodic, it is proper to notice that the same property does not attach to the motions thereby defined of the particles of the fluid. In our notation u is not the velocity of any particular particle of the fluid, but of the particle, whichever it may be, that *at the moment under consideration* occupies the point x, y. If $x+\xi$, $y+\eta$ be the actual position at time t of the particle whose mean position during several vibrations is x, y, then the real velocities of the particle at time t are not u, v, but

$$u+\frac{du}{dx}\xi+\frac{du}{dy}\eta,\quad v+\frac{dv}{dx}\xi+\frac{dv}{dy}\eta\,;$$

and thus the mean velocity parallel to x is not necessarily zero, but is equal to the mean value of

$$\frac{du}{dx}\xi+\frac{du}{dy}\eta,$$

in which again

$$\xi=\int u\,dt,\quad \eta=\int v\,dt.$$

From the general form of u, viz., $\cos kx\,\mathrm{F}(y,t)$, it follows readily that $\int\frac{du}{dx}\xi\,dt=0$. For the second term we must calculate from the actual values as given in (18), (19). Thus

$$\eta=\frac{u_0 \sin kx}{n}\left\{-\frac{k \cosh ky}{\beta\sqrt{2}}\sin(nt-\tfrac{1}{4}\pi)+\sinh ky \sin nt+\frac{k e^{-\beta y}}{\beta\sqrt{2}}\sin(nt-\tfrac{1}{4}\pi-\beta y)\right\},$$

$$\frac{du}{dy}=u_0 \cos kx\left\{-\frac{k^2 \cosh ky}{\beta\sqrt{2}}\cos(nt-\tfrac{1}{4}\pi)+k \sinh ky \cos nt+\sqrt{2}.\beta e^{-\beta y}\cos(nt+\tfrac{1}{4}\pi-\beta y)\right\},$$

of which the two first terms may be neglected relatively to the third (containing the large factor β). The product of η and $\frac{du}{dy}$ will consist of two parts, the first independent of t, and the second harmonic functions of $2nt$. It is with the first only that we are here concerned. The mean value of the velocity parallel to x is thus

$$\frac{u_0^2 \sin 2kx\, e^{-\beta y}}{4n}\left\{k \cosh ky \cos \beta y+\sqrt{2}.\beta \sinh ky \quad \sin(\beta y-\tfrac{1}{4}\pi)-k e^{-\beta y}\right\}.$$

On account of the factor $e^{-\beta y}$, this quantity is insensible except when ky is extremely small. We may therefore write it

$$\frac{u_0^2 \sin 2kx\, e^{-\beta y}}{4V}\left\{\cos \beta y+\beta y(\sin \beta y-\cos \beta y)-e^{-\beta y}\right\} \quad . \quad . \quad . \quad . \quad (20),$$

V (equal to k/n) being the velocity of propagation of waves corresponding to k and n.

The only approximation employed in the derivation of (15) and (16) is the neglect of the right hand member of (4), and the corresponding real values of u and v could if necessary be readily exhibited without the use of a merely approximate value of k'. To proceed further we must calculate the value of

$$\frac{u}{\nu}\frac{d\nabla^2\psi}{dx}+\frac{v}{\nu}\frac{d\nabla^2\psi}{dy} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

in (4), for which it will be sufficient to take the values given by the first approximation. Thus

$$\nabla^2\psi=\nabla^2\psi_2=\frac{1}{\nu}\frac{d\psi_2}{dt},$$

and by (17)

$$\frac{d\psi_2}{dt}=-\frac{nu_0 \cos kx\, e^{-\beta y}}{\beta\sqrt{2}}\sin(nt-\tfrac{1}{4}\pi-\beta y),$$

from which we find as the value of (21),

$$\frac{nku_0^2 \sin 2kx\, e^{-\beta y}}{4\nu^2\beta\sqrt{2}}\left\{\left(\frac{k}{\beta\sqrt{2}}-\frac{\beta\sqrt{2}}{k}\right)\sinh ky \sin \beta y-\sqrt{2}\cosh ky \cos \beta y+\sqrt{2}\, e^{-\beta y}\right\}$$

$$+ \text{ terms in } 2nt.$$

On account of the factor $e^{-\beta y}$ this quantity is sensible only when y is very small. We may write it with sufficient approximation

$$\frac{nku_0^2 \sin 2kx\, e^{-\beta y}}{4\nu^2\beta}\left\{-\beta y \sin \beta y - \cos \beta y + e^{-\beta y}\right\} \quad . \quad . \quad . \quad . \quad . \quad (22).$$

The terms in $2nt$, corresponding to motions of half the original period, are not required for our purpose, which is to investigate the non-periodic motion of the second order. The equation with which we have to proceed is found by equating (22) to $\nabla^4\psi$. The solution will consist of two parts, one resulting from the direct integration of (22) and involving the factor $e^{-\beta y}$, the second a complementary function with arbitrary coefficients satisfying $\nabla^4\psi=0$. In the calculation of the first part we may identify ∇^4 with d^4/dy^4, on account of the smallness of k relatively to β. In this way our equation becomes

$$\frac{d^4\psi}{d(\beta y)^4}=\frac{nku_0^2 \sin 2kx\, e^{-\beta y}}{4\nu^2\beta^5}\left\{-\beta y \sin \beta y - \cos \beta y + e^{-\beta y}\right\} \quad . \quad . \quad . \quad . \quad (23),$$

of which the solution is

$$\psi=\frac{nku_0^2 \sin 2kx\, e^{-\beta y}}{4\nu^2\beta^5}\left\{\tfrac{3}{4} \cos \beta y + \tfrac{1}{2} \sin \beta y + \tfrac{1}{4}\beta y \sin \beta y + \tfrac{1}{16} e^{-\beta y}\right\} \quad . \quad . \quad (24).$$

The complementary function, being proportional to $\sin 2kx$, may be written

$$\frac{nku_0^2 \sin 2kx}{4\nu^2\beta^5}\{(A+By)e^{-2ky}+(A'+B'y)e^{+2ky}\}.$$

If the fluid be uninterrupted by a free surface, or otherwise, within distances for which ky is sensible, we must suppose $(A'+B'y)=0$, so that by (13) the complementary function may be written

$$\frac{u_0^2 \sin 2kx}{\beta V}(A+By)e^{-2ky}.$$

The condition that v (equal to $-d\psi/dx$) must vanish when $y=0$, gives $A=-\tfrac{13}{16}$. For the velocity parallel to x we have

$$u=\frac{u_0^2 \sin 2kx}{V}\Big[e^{-\beta y}\{-\sin \beta y - \tfrac{1}{4} \cos \beta y + \tfrac{1}{4}\beta y \cos \beta y - \tfrac{1}{4}\beta y \sin \beta y - \tfrac{1}{8}e^{-\beta y}\}$$
$$+\beta^{-1}e^{-2ky}\{B-2k(A+By)\}\Big].$$

In order that u should vanish when $y=0$, we must have

$$B=2kA+\tfrac{3}{8}\beta=\tfrac{3}{8}\beta-\tfrac{13}{8}k=\tfrac{3}{8}\beta,$$

approximately. Thus

$$u=\frac{u_0^2\sin 2kx}{V}\left[e^{-\beta y}\{-\sin\beta y-\tfrac{1}{4}\cos\beta y+\tfrac{1}{4}\beta y\cos\beta y-\tfrac{1}{4}\beta y\sin\beta y-\tfrac{1}{8}e^{-\beta y}\}\right.$$
$$\left.+\tfrac{3}{8}e^{-2ky}\{1-2ky\}\right] \quad . \quad (25),$$

and

$$v=-\frac{2ku_0^2\cos 2kx}{\beta V}\left[e^{-\beta y}\{\tfrac{3}{4}\cos\beta y+\tfrac{1}{2}\sin\beta y+\tfrac{1}{4}\beta y\sin\beta y+\tfrac{1}{16}e^{-\beta y}\}\right.$$
$$\left.+e^{-2ky}\{-\tfrac{13}{16}+\tfrac{3}{8}\beta y\}\right] \quad . \quad (26).$$

To obtain the mean velocity parallel to x of a particle, we must add to (25), the terms previously investigated and expressed by (20). If we call the total u', we have

$$u'=\frac{u_0^2\sin 2kx}{V}\left[e^{-\beta y}\{-\sin\beta y-\tfrac{3}{8}e^{-\beta y}\}+\tfrac{3}{8}e^{-2ky}\{1-2ky\}\right] \quad . \quad . \quad . \quad (27).$$

At a short distance from the bottom $e^{-\beta y}$ becomes insensible, and we have simply

$$u'=\tfrac{3}{8}\frac{u_0^2\sin 2kx}{V}e^{-2ky}(1-2ky). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28),$$

$$v'=-\frac{2ku_0^2\cos 2kx}{\beta V}e^{-2ky}(-\tfrac{13}{16}+\tfrac{3}{8}\beta y) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29).$$

The steady motion expressed by (28) and (29) is of a very simple character. It consists of a series of vortices periodic with respect to x in a distance $\frac{1}{2}\lambda$. For a given x the horizontal motion is of one sign near the bottom, and of the opposite sign at a distance from it, the place of transition being at $y=(2k)^{-1}=\lambda/4\pi$. The horizontal motion of the first order near the bottom being by (18) $u=u_0\cos kx\cos nt$, we see that it is a maximum when $kx=0,\ \pi,\ 2\pi, \ldots$ If we call these places loops, and the places of minimum velocity nodes, (29) shows that v' is negative and a maximum at the loops, positive and a maximum at the nodes. The fluid therefore rises from the bottom over the nodes and falls back again over the loops, the horizontal motion near the bottom being thus directed towards the nodes and from the loops. The maximum horizontal motion is simply $\frac{3}{8}u_0^2/V$, and is *independent of the value of* ν. We cannot, therefore, avoid considering this motion by supposing the coefficient of viscosity to be very small, the maintenance of the vortices becoming easier in the same proportion as the forces tending to produce the vortical motion diminish.

To ascertain the character of the motion quite close to the bottom, we must include the terms in $e^{-\beta y}$. When y is extremely small

$$u'=u_0^2V^{-1}\sin 2kx\{-\tfrac{1}{4}\beta y+\ldots\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30),$$

so that the motion is here in the opposite direction to that which prevails when $e^{-\beta y}$ can be neglected.

A few corresponding values of βy and of $-(\sin \beta y + \frac{3}{8}e^{-\beta y})e^{-\beta y} + \frac{3}{8}$ are annexed, in order to show the distribution of velocities within the thin frictional layer.

βy.		βy.	
$\frac{\pi}{16}$	$-\cdot038$	$\frac{3\pi}{8}$	$+\cdot055$
$\frac{\pi}{8}$	$-\cdot054$	$\frac{\pi}{2}$	$+\cdot151$
$\frac{3\pi}{16}$	$-\cdot049$	π	$+\cdot374$
$\frac{\pi}{4}$	$-\cdot025$	$\frac{3\pi}{2}$	$+\cdot384$

It appears that ($\sin 2kx$ being positive) the velocity is negative from the plate outwards until βy somewhat exceeds $\frac{1}{4}\pi$, after which it is positive, until reversed by the factor $(1-2ky)$. The greatest negative velocity in the layer is about $\frac{1}{7}$ of that which is found at a little distance outside the layer.

FARADAY found that fine sand, scattered over the bottom, tended to collect at the loops. This is in agreement with what the present calculation would lead us to expect, provided that we can suppose that the sand is controlled by the layer at the bottom whose motion is negative. The exceeding thinness of the layer, however, presents itself as a difficulty. The subject requires further experimental investigation; but in the meantime the following data may be worth notice, though in some respects, *e.g.*, the shallowness of the liquid in relation to the wave-length, the circumstances differed materially from those assumed in the theoretical investigation.

The liquid was water ($\nu = \cdot014$ C.G.S.), and the period of vibration was $\frac{1}{15}$, so that $n = 2\pi \times 15$. The thickness of the layer

$$= \frac{\pi}{4}\sqrt{\frac{2\nu}{n}} = 0135 \text{ centim.}$$

Measurements of the diameters of the particles of sand gave about $\cdot02$ centim., so that the grains would be almost wholly immersed in the negative layer, even if isolated. It seems therefore that the observed motion to the loops gives rise in this case to no difficulty. But it is possible that the behaviour of the sand is materially influenced by the vertical motion of the vessel by which in these experiments the liquid vibrations are maintained.*

§ 2. In the problem to which we now proceed the motion will be supposed to have its origin in the assumed motion of a flexible plate situated when in equilibrium at $y=0$. Thus for a first approximation we take $u=0$, $v=v_0 \sin kx\, e^{int}$, when $y=0$, and the question is to investigate the resulting motion of the fluid in contact with the plate.

* See a paper "On the Crispations of Fluid resting upon a Vibrating Support," Phil. Mag., July, 1883

The solution to a first approximation is readily obtained. As in (10), (11), we have

$$\psi=\psi_1+\psi_2=e^{int}\cos kx(\mathrm{A}e^{-ky}+\mathrm{C}e^{-k'y}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31),$$

in which we may take as before

$$k'=\sqrt{\frac{n}{2\nu}}\cdot(1+i)=\beta(1+i) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32).$$

By the condition at $y=0$,

$$\mathrm{A}=-\frac{k'}{k}\mathrm{C}, \qquad \mathrm{C}=\frac{v_0}{k'-k},$$

so that

$$\psi=\frac{v_0e^{int}\cos kx}{k-k'}\left\{-\frac{k'}{k}e^{-ky}+e^{-k'y}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33),$$

$$u=\frac{v_0e^{int}\cos kx}{k-k'}\left\{k'e^{-ky}-k'e^{-k'y}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34).$$

In passing to real quantities it will be convenient to write

$$\frac{v_0}{k-k'}=\mathrm{H}e^{i\epsilon} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35).$$

Thus throwing away the imaginary parts of (33), (34), we get

$$\psi=\cos kx\left\{-\frac{\beta\sqrt{2}}{k}e^{-ky}\cos(nt+\epsilon+\tfrac{1}{4}\pi)+e^{-\beta y}\cos(nt+\epsilon-\beta y)\right\} \quad . \quad . \quad . \quad (36),$$

$$u=\sqrt{2}.\beta\,\mathrm{H}\cos kx\left\{e^{-ky}\cos(nt+\epsilon+\tfrac{1}{4}\pi)-e^{-\beta y}\cos(nt+\epsilon+\tfrac{1}{4}\pi-\beta y)\right\} \quad . \quad (37),$$

$$v=\mathrm{H}\sin kx\left\{-\beta\sqrt{2}\,e^{-ky}\cos(nt+\epsilon+\tfrac{1}{4}\pi)+ke^{-\beta y}\cos(nt+\epsilon-\beta y)\right\} \quad . \quad . \quad (38).$$

From (32), (35), the approximate value of H is $-v_0/\beta\sqrt{2}$, and that of ϵ is $-\frac{1}{4}\pi$. More exact values will however be required later. We find

$$\mathrm{H}=-\frac{v_0}{\sqrt{\{(\beta-k)^2+\beta^2\}}}=-\frac{v_0}{\beta\sqrt{2}}\left(1+\frac{k}{2\beta}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (39),$$

$$\cos\epsilon=\frac{\beta-k}{\sqrt{\{(\beta-k)^2+\beta^2\}}}=\frac{1}{\sqrt{2}}\left(1-\frac{k}{2\beta}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40).$$

The values of u and v above expressed give $u=0$, $v=v_0\sin kx\cos nt$, when $y=0$. This is sufficient for a first approximation, but in proceeding further we must remember

that these prescribed velocities apply in strictness not to $y=0$, but to $y=\frac{v_0}{n}\sin kx \sin nt$. Substituting the latter value of y in the expressions (37), and (38), we find

$$u=\sqrt{2}.\beta \mathrm{H}\cos kx\left\{-ky\cos(nt+\epsilon+\tfrac{1}{4}\pi)+\sqrt{2}.\beta y\cos(nt+\epsilon+\tfrac{1}{2}\pi)\right\}$$

$$=\frac{\beta^2 v_0 \mathrm{H}}{n}\sin 2kx\sin nt\left\{-\frac{k}{\beta\sqrt{2}}\cos(nt+\epsilon+\tfrac{1}{4}\pi)+\cos(nt+\epsilon+\tfrac{1}{2}\pi)\right\}$$

$$=\frac{\beta^2 v_0 \mathrm{H}}{2n}\sin 2kx\left\{\frac{k}{\beta\sqrt{2}}\sin(\epsilon+\tfrac{1}{4}\pi)-\sin(\epsilon+\tfrac{1}{2}\pi)\right\}+\text{terms in } 2nt.$$

The first term within the bracket is of the *second* order in k/β relatively to the latter term, and may be omitted. Thus

$$u=-\frac{\beta^2 v_0 \mathrm{H}}{2n}\sin 2kx\cos\epsilon.$$

The terms in $2nt$ we need not further examine. From (39), (40), $\mathrm{H}\cos\epsilon=-v_0/2\beta$, very approximately, so that we may write

$$u=\frac{\beta v_0^2}{4n}\sin 2kx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41).$$

To the same degree of approximation, $v=v_0\sin kx\cos nt$, simply.

We have next, as in the first problem, to consider the complete equation

$$\nabla^4\psi=\frac{u}{\nu^2}\frac{d^2\psi_2}{dx\,dt}+\frac{v}{\nu^2}\frac{d^2\psi_2}{dy\,dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

in the right hand member of which we use the approximate values given by (36), (37), (38). Thus

$$\frac{d\psi_2}{dt}=-n\mathrm{H}\cos kx\, e^{-\beta y}\sin(nt+\epsilon-\beta y),$$

and (42) becomes

$$\nabla^4\psi=\frac{nk\beta\mathrm{H}^2\sin 2kx e^{-\beta y}}{4\nu^2}\left\{e^{-ky}\left(\frac{2\beta}{k}\sin\beta y-\sin\beta y-\cos\beta y\right)+2e^{-\beta y}\right\} . \quad . \quad (43).$$

It will be found presently that the term divided by k disappears from the final result, and thus we have to pursue the approximation further than might at first appear necessary. We may however neglect terms of order k^2/β^2, in comparison with the principal term. Thus ∇^4 may be identified with $\frac{d^4}{dy^4}$, and the equation becomes

$$\frac{d^4\psi}{d(\beta y)^4}=\frac{nk\mathrm{H}^2\sin 2kx\, e^{-\beta y}}{4\nu^2\beta^3}\left\{\left(\frac{2\beta}{k}-1\right)\sin\beta y-\cos\beta y-2\beta y\sin\beta y+2e^{-\beta y}\right\}\quad . \quad (44),$$

whence

$$\psi=\frac{nk\mathrm{H}^2\sin 2kx\, e^{-\beta y}}{4\nu^2\beta^3}\left\{\left(-\frac{\beta}{2k}+\tfrac{5}{4}\right)\sin\beta y+\tfrac{5}{4}\cos\beta y+\tfrac{1}{2}\beta y\sin\beta y+\tfrac{1}{8}e^{-\beta y}\right\}\quad . \quad (45).$$

And

$$u=\frac{d\psi}{dy}=\frac{nk\mathrm{H}^2\sin 2kx\, e^{-\beta y}}{4\nu^2\beta^2}\left\{\left(\frac{\beta}{2k}-2\right)\sin\beta y-\frac{\beta}{2k}\cos\beta y-\tfrac{1}{2}\beta y\sin\beta y+\tfrac{1}{2}\beta y\cos\beta y-\tfrac{1}{4}e^{-\beta y}\right\}(46).$$

To obtain the value of u at the surface of the plate it will be sufficient to put $y=0$ in (46). Thus

$$u=\frac{nk\mathrm{H}^2\sin 2kx}{4\nu^2\beta^2}\left\{-\frac{\beta}{2k}-\tfrac{1}{4}\right\}\quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (47).$$

By (32), (39)

$$\frac{nk\mathrm{H}^2}{4\nu^2\beta^2}=\frac{kv_0^2}{2n}\left(1+\frac{k}{\beta}\right)=\frac{v_0^2}{2\mathrm{V}}\left(1+\frac{k}{\beta}\right),$$

if as before we put V for k/n. Thus in (47)

$$u=\frac{v_0^2}{4\mathrm{V}}\left(-\frac{\beta}{k}-\tfrac{3}{2}\right)\sin 2kx\quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (48).$$

To obtain the complete value of u at the surface of the plate, corresponding to (37), (46), we have to add to (48) that given in (41). The term of lowest order disappears, and we are left simply with

$$u=-\frac{3v_0^2}{8\mathrm{V}}\sin 2kx.\quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (49).$$

In like manner we find for the complete value of v at the surface of the plate corresponding to (38), (45),

$$v=v_0\sin kx\cos nt-\frac{11v_0^2k\cos 2kx}{8\beta\mathrm{V}}\quad . \; . \; . \; . \; . \; . \; . \; . \quad (50).$$

The values of u and v expressed in (49) and the second part of (50) must be cancelled by a suitable choice of the complementary function, satisfying $\nabla^4\psi=0$, so that to the second order of approximation the fluid in contact with the plate may have no relative motion.

The complementary function is

$$\psi=(\mathrm{A}+\mathrm{B}y)e^{-2ky}\sin 2kx,$$

whence

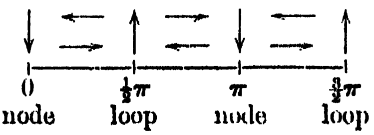
$$u=\{B-2k(A+By)\}e^{-2ky}\sin 2kx,$$
$$v=-2k(A+By)e^{-2ky}\cos 2kx.$$

Determining the constants as indicated above we get

$$u=\frac{3v_0^2}{8V}(1-2ky)e^{-2ky}\sin 2kx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51),$$

$$v=-\frac{kv_0^2}{8\beta V}(11+6\beta y)e^{-2ky}\cos 2kx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52).$$

The velocities given by (51), (52) are the only part of the motion of the second order which is sensible beyond a very small distance from the vibrating plate. The nodes of the plate (where sand would collect) are at the points given by $kx=0, \pi, 2\pi\ldots$, and the loops at the points $kx=\frac{1}{2}\pi, \frac{3}{2}\pi\ldots$ At the former points v is negative, and at the latter positive. For $kx=\frac{1}{4}\pi$, u is positive, and for $kx=\frac{3}{4}\pi$, u is negative.



The magnitude of the vortical motion is independent of the coefficient of friction.

The complete value of u to the second order of approximation (except the terms in $2nt$) is obtained by adding together (37), (46), and (51), and it will contain the term divided by k in (46), whose appearance, however, is misleading. The objectionable term will be got rid of, if we express the mean velocity of a particle, instead of as in (46), the mean velocity *at a point*. For this purpose we are to add to (46), (51), the mean value of

$$\xi\frac{du}{dx}+\eta\frac{du}{dy},$$

as calculated from the first approximation where

$$\xi=\int udt, \quad \eta=\int vdt.$$

As in the former problem the mean value of $\xi\frac{du}{dx}$ is zero.

Multiplying together $\frac{du}{dy}$, and $\int vdt$ as found from (37), (38), and rejecting the terms in $2nt$, we get with omission of k^2,

$$-\frac{k\beta^2 H^2 e^{-\beta y}}{n}\frac{\sin 2kx}{}\left\{\frac{\beta}{2k}(1-ky)(\sin\beta y-\cos\beta y)+\tfrac{1}{2}e^{-\beta y}\right\} \quad . \quad . \quad . \quad . \quad (53),$$

in which we may write

$$-\frac{k\beta^2 H^2}{n}=-\frac{k\beta^4 H^2}{n\beta^2}=-\frac{nkH^2}{4\nu^2\beta^2}.$$

Combining (53), (46), and (51), we get finally

$$u'=\frac{nkH^2 e^{-\beta y}\sin 2kx}{4\nu^2\beta^2}\left\{-2\sin\beta y-\tfrac{3}{4}e^{-\beta y}\right\}+\frac{3v_0^2}{8V}(1-2ky)e^{-2ky}\sin 2kx$$

$$=\frac{v_0^2\sin 2kx}{2V}\left\{-e^{-\beta y}(2\sin\beta y+\tfrac{3}{4}e^{-\beta y})+\tfrac{3}{4}(1-2ky)e^{-2ky}\right\} \quad . \quad . \quad . \quad . \quad . \quad (54),$$

which expresses the mean particle velocity.

When βy is very small, (54) gives

$$u'=\frac{v_0^2\sin 2kx}{2V}(-\tfrac{1}{2}\beta y+\ldots) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55).$$

from which it appears that quite close to the plate the mean velocity is in the opposite direction to that which is found outside the frictional layer.

§ 3. In the third problem, relating to KUNDT's tubes, the fluid must be treated as compressible, as the motion is supposed to be approximately in one dimension, parallel (say) to x. The solution to a first approximation is merely an adaptation to two dimensions of the corresponding solution for a tube of revolution by KIRCHHOFF,* simplified by the neglect of the terms relating to the development and conduction of heat. It is probable that the solution to the second order would be practicable also for a tube of revolution, but for the sake of simplicity I have adhered to the case of two dimensions. The most important point in which the two problems are likely to differ can be investigated very simply, without a complete solution.

If we suppose $p=a^2\rho$, and write σ for $\log\rho-\log\rho_0$, the fundamental equations are

$$a^2\frac{d\sigma}{dx}=-\frac{du}{dt}-u\frac{du}{dx}-v\frac{du}{dy}+\nu\nabla^2 u+\nu'\frac{d}{dx}\left(\frac{du}{dx}+\frac{dv}{dy}\right) \quad . \quad . \quad . \quad . \quad . \quad (56),$$

with a corresponding equation for v, and the equation of continuity,

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{d\sigma}{dt}+u\frac{d\sigma}{dx}+v\frac{d\sigma}{dy}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57).$$

* POGG. Ann., t. cxxxiv., 1868.

Whatever may be the actual values of u and v, we may write

$$u=\frac{d\phi}{dx}+\frac{d\psi}{dy},\quad v=\frac{d\phi}{dy}-\frac{d\psi}{dx} \quad . \quad . \quad . \quad . \quad (58),$$

in which

$$\nabla^2\phi=\frac{du}{dx}+\frac{dv}{dy},\quad \nabla^2\psi=\frac{du}{dy}-\frac{dv}{dx} \quad . \quad . \quad . \quad . \quad (59).$$

From (56), (57),

$$\left(a^2+\nu'\frac{d}{dt}\right)\frac{d\sigma}{dx}=-\frac{du}{dt}+\nu\,\nabla^2 u-u\frac{du}{dx}-v\frac{du}{dy}-\nu'\frac{d}{dx}\left(u\frac{d\sigma}{dx}+v\frac{d\sigma}{dy}\right) \quad . \quad . \quad . \quad (60),$$

$$\left(a^2+\nu'\frac{d}{dt}\right)\frac{d\sigma}{dy}=-\frac{dv}{dt}+\nu\,\nabla^2 v-u\frac{dv}{dx}-v\frac{dv}{dy}-\nu'\frac{d}{dy}\left(u\frac{d\sigma}{dx}+v\frac{d\sigma}{dy}\right) \quad . \quad . \quad . \quad (61).$$

Again from (60), (61),

$$\left(a^2+\nu\frac{d}{dt}+\nu'\frac{d}{dt}\right)\nabla^2\sigma-\frac{d^2\sigma}{dt^2}=\frac{d}{dt}\left(u\frac{d\sigma}{dx}+v\frac{d\sigma}{dy}\right)-(\nu+\nu')\nabla^2\left(u\frac{d\sigma}{dx}+v\frac{d\sigma}{dy}\right)$$
$$-\frac{d}{dx}\left(u\frac{du}{dx}+v\frac{du}{dy}\right)-\frac{d}{dy}\left(u\frac{dv}{dx}+v\frac{dv}{dy}\right). \quad (62).$$

For the first approximation the terms of the second order in u, v, and σ are to be omitted. If we assume that as functions of t, all the periodic quantities are proportional to e^{int}, and write q for $a^2+in\nu+in\nu'$, (62) becomes

$$q\nabla^2\sigma+n^2\sigma=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (63).$$

Now by (57), (59),

$$\nabla^2\phi=-in\sigma=i\frac{q}{n}\nabla^2\sigma,$$

so that

$$\phi=i\frac{q}{n}\sigma,*$$

and

$$u=\frac{iq}{n}\frac{d\sigma}{dx}+\frac{d\psi}{dy},\quad v=\frac{iq}{n}\frac{d\sigma}{dy}-\frac{d\psi}{dx} \quad . \quad . \quad . \quad . \quad (64).$$

Substituting in (60), (61), with omission of terms of the second order, we get in view of (63),

$$(\nu\nabla^2-in)\frac{d\psi}{dy}=0,\qquad (\nu\nabla^2-in)\frac{d\psi}{dx}=0,$$

* It is unnecessary to add a complementary function ϕ', satisfying $\nabla^2\phi'=0$, as the motion corresponding thereto may be regarded as covered by ψ.

whence

$$(\nu\nabla^2 - in)\psi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65).$$

If we eliminate σ directly from the fundamental equations (56), we get

$$\left(\nu\nabla^4 - \frac{d}{dt}\nabla^2\right)\psi = \frac{d}{dy}\left(u\frac{du}{dx} + v\frac{du}{dy}\right) - \frac{d}{dx}\left(u\frac{dv}{dx} + v\frac{dv}{dy}\right) = \frac{d}{dy}(v\nabla^2\psi) + \frac{d}{dx}(u\nabla^2\psi)$$

$$= \left(\frac{du}{dx} + \frac{dv}{dy}\right)\nabla^2\psi + u\frac{d\nabla^2\psi}{dx} + v\frac{d\nabla^2\psi}{dy} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66).$$

If we now assume that as functions of x the quantities σ, ψ, &c., are proportional to e^{ikx}, equations (63), (65) may be written

$$\left(\frac{d^2}{dy^2} - k''^2\right)\sigma = 0, \qquad \text{where } k''^2 = k^2 - \frac{n^2}{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad (67),$$

$$\left(\frac{d^2}{dy^2} - k'^2\right)\psi = 0, \qquad \text{where } k'^2 = k^2 + \frac{in}{\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (68).$$

If the origin for y be in the middle between the two parallel boundaries, σ must be an even function of y, and ψ must be an odd function. Thus we may write

$$\sigma = \mathrm{A} \cosh k''y . e^{int} e^{ikx}, \qquad \psi = \mathrm{B} \sinh k'y . e^{int} e^{ikx} \quad . \quad . \quad . \quad . \quad (69),$$

$$\left.\begin{aligned} u &= \left(-\frac{kq}{n}\mathrm{A} \cosh k''y + k'\mathrm{B} \sinh k'y\right)e^{int} e^{ikx} \\ v &= \left(\frac{ik''q}{n}\mathrm{A} \sinh k''y - ik\mathrm{B} \sinh k'y\right)e^{int} e^{ikx} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (70).$$

If the fixed walls are situated at $y = \pm y_1$, u and v must vanish for these values of y. Eliminating from (70) the ratio of A to B, we get as the equation for determining k,

$$k^2 \tanh k'y_1 = k'k'' \tanh k''y_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71),$$

in which k', k'' are given as functions of k by (67), (68). We now introduce further approximations dependent upon the assumption that the direct influence of friction extends through a layer whose thickness is a small fraction only of y_1. On this supposition k' is large, and k'' is small, so that we may put $\tanh k'y_1 = \pm 1$, $\tanh k''y_1 = \pm k''y_1$. Equation (71) then becomes

$$k^2 = k'k''^2 y_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72),$$

or if we introduce the values of k', k'' from (67), (68),

$$k^2=\left(k^2-\frac{n^2}{q}\right) y_1\sqrt{\left(k^2+\frac{in}{\nu}\right)}.$$

Since in/ν is great, $k^2=\frac{n^2}{q}=\frac{n^2}{a^2}$ approximately.

Thus

$$k^2=\frac{n^2}{q}+\frac{k^2}{y_1\sqrt{\left(k^2+\frac{in}{\nu}\right)}}=\frac{n^2}{a^2}\left\{1+\frac{1}{y_1\sqrt{\frac{in}{\nu}}}\right\}$$

and

$$k=\pm\frac{n}{a}\left\{1+\frac{1-i}{2y_1\sqrt{\frac{2n}{\nu}}}\right\} \qquad (73).$$

If we write $k=k_1+ik_2$,

$$k_1=\pm\frac{n}{a}\left\{1+\frac{\sqrt{\frac{\nu}{2n}}}{2y_1}\right\}, \quad k_2=\mp\frac{n}{a}\,\frac{\sqrt{\frac{\nu}{2n}}}{2y_1} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (74),$$

which agrees with the result given in § 347 (11) of my book on the Theory of Sound.

In taking approximate forms for (70), we must distinguish which half of the symmetrical motion we contemplate. If we choose that for which y is *negative*, we replace $\cosh k'y$ and $\sinh k'y$ by $\frac{1}{2}e^{-k'y}$. For $\cosh k''y$ we may write unity, and for $\sinh k''y$ simply $k''y$. If we change the arbitrary multiplier so that the maximum value of u is unity, we have

$$\left.\begin{aligned} u&=(-1+e^{-k'(y+y_1)})e^{ikx}e^{int} \\ v&=\frac{ik}{k'}\left(\frac{y}{y_1}+e^{-k'(y+y_1)}\right)e^{ikx}e^{int} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (75),$$

in which, of course, u and v vanish when $y=-y_1$.

If in (75) we change k into $-k$, and then take the mean, we obtain

$$\left.\begin{aligned} u&=(-1+e^{-k'(y+y_1)})\cos kx\, e^{int} \\ v&=-\frac{k}{k'}\left(\frac{y}{y_1}+e^{-k'(y+y_1)}\right)\sin kx\, e^{int} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (76).$$

Although k is not absolutely a real quantity, we may consider it to be so with sufficient approximation for our purpose. If we write as before

$$k'=\sqrt{\left(\frac{n}{2\nu}\right)}.(1+i)=\beta(1+i),$$

we get from (76) in terms of real quantities

$$\left.\begin{aligned} u &= \cos kx[-\cos nt + e^{-\beta(y+y_1)}\cos\{nt-\beta(y+y_1)\}] \\ v &= -\frac{k}{\beta\sqrt{2}}\sin kx\left[\frac{y}{y_1}\cos(nt-\tfrac{1}{4}\pi)+e^{-\beta(y+y_1)}\cos\{nt-\tfrac{1}{4}\pi-\beta(y+y_1)\}\right] \end{aligned}\right\} \quad (77).$$

It will shorten the expressions with which we have to deal if we measure y from the wall (on the negative side) instead of as hitherto from the plane of symmetry, for which purpose we must write y for $y+y_1$. Thus

$$\left.\begin{aligned} u &= \cos kx\{-\cos nt + e^{-\beta y}\cos(nt-\beta y)\} \\ v &= \frac{k\sin kx}{\beta\sqrt{2}}\left\{\frac{y_1-y}{y_1}\cos(nt-\tfrac{1}{4}\pi)-e^{-\beta y}\cos(nt-\tfrac{1}{4}\pi-\beta y)\right\} \end{aligned}\right\} \quad . \; . \; . \; . \quad (78).$$

From (78) approximately

$$\nabla^2\psi = \beta\sqrt{2}.\cos kx\, e^{-\beta y}\sin(nt-\tfrac{1}{4}\pi-\beta y) \quad . \; . \; . \; . \; . \; . \quad (79),$$

$$\frac{du}{dx}+\frac{dv}{dy} = k\sin kx\cos nt \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (80),$$

$$u\frac{d\nabla^2\psi}{dx}+v\frac{d\nabla^2\psi}{dy} = \tfrac{1}{2}k\beta\sin 2kx\, e^{-\beta y}(-\cos\beta y+e^{-\beta y}) + \text{terms in } 2nt \quad . \; . \quad (81),$$

$$\left(\frac{du}{dx}+\frac{dv}{dy}\right)\nabla^2\psi = -\tfrac{1}{4}k\beta\sin 2kx\, e^{-\beta y}(\sin\beta y+\cos\beta y)+\text{terms in } 2nt \quad . \; . \quad (82).$$

As in former problems the periodic terms in $2nt$ will be omitted. For the non-periodic part of ψ of the second order, we have from (66)

$$\nabla^4\psi = -\frac{k\beta}{4\nu}\sin 2kx\, e^{-\beta y}\{\sin\beta y+3\cos\beta y-2e^{-\beta y}\} \quad . \; . \; . \; . \quad (83).$$

In this we identify ∇^4 with $\dfrac{d^4}{dy^4}$, so that

$$\psi = \frac{k\sin 2kx\, e^{-\beta y}}{16\,\nu\beta^3}\{\sin\beta y+3\cos\beta y+\tfrac{1}{2}e^{-\beta y}\} \quad . \; . \; . \; . \; . \; . \quad (84),$$

to which must be added a complementary function, satisfying $\nabla^4\psi=0$, of the form

$$\psi = \frac{\sin 2kx}{16\,\nu\beta^3}\{\mathrm{A}\sinh 2k(y_1-y)+\mathrm{B}(y_1-y)\cosh 2k(y_1-y)\} \quad . \; . \; . \quad (85),$$

or as we may take it approximately, if y_1 be small compared with the wave-length λ,

$$\psi=\frac{k\sin 2kx}{16\,\nu\beta^3}\{A'(y_1-y)+B'(y_1-y)^3\}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (86).$$

The value of σ to a second approximation would have to be investigated by means of (62). It will be composed of two parts, the first independent of t, the second a harmonic function of $2nt$. In calculating the part of $d\phi/dx$ independent of t from

$$\nabla^2\phi=-\frac{d\sigma}{dt}-u\frac{d\sigma}{dx}-v\frac{d\sigma}{dy},$$

we shall obtain nothing from $d\sigma/dt$. In the remaining terms on the right-hand side it will be sufficient to employ the values of u, v, σ of the first approximation. From

$$\frac{d\sigma}{dt}=-\frac{du}{dx}-\frac{dv}{dy},$$

in conjunction with (80), we get

$$\sigma=-\frac{u_0}{a}\sin kx\sin nt,$$

whence

$$\frac{d^2\phi}{d(\beta y)^2}=\frac{ku_0^2}{2a\beta^2}\cos^2 kx\, e^{-\beta y}\sin\beta y.$$

It is easily seen from this that the part of u resulting from $d\phi/dx$ is of order $k^2\beta^2$ in comparison with the part (87) resulting from ψ, and may be omitted.

Accordingly by (84), with introduction of the value of β and (in order to restore homogeneity) of u_0^2

$$u=-\frac{u_0^2\sin 2kx\, e^{-\beta y}}{8a}\{4\sin\beta y+2\cos\beta y+e^{-\beta y}\}\quad . \quad . \quad . \quad . \quad . \quad . \qquad (87),$$

$$v=-\frac{2ku_0^2\cos 2kx\, e^{-\beta y}}{8\beta a}\{\sin\beta y+3\cos\beta y+\tfrac{1}{2}e^{-\beta y}\}\quad . \quad . \quad . \quad . \quad . \qquad (88);$$

and from (86)

$$u=-\frac{u_0^2\sin 2kx}{8\beta a}\{A'+3B'(y_1-y)^2\}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (89),$$

$$v=-\frac{2ku_0^2\cos 2kx}{8\beta a}\{A'(y_1-y)+B'(y_1-y)^3\}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (90).$$

When $y=0$, the complete values of u and v, as given by the four last equations, must vanish. Determining in this way the arbitrary constants A′ and B′, we get as the complete values at any point,

$$u=-\frac{u_0^2 \sin 2kx}{8a}\left\{e^{-\beta y}(4 \sin \beta y+2 \cos \beta y+e^{-\beta y})+\tfrac{3}{2}-\tfrac{9}{2}\frac{(y_1-y)^2}{y_1^2}\right\} \quad . \quad . \quad (91),$$

$$v=-\frac{2ku_0^2 \cos 2kx}{8\beta a}\left\{e^{-\beta y}(\sin \beta y+3 \cos \beta y+\tfrac{1}{2}e^{-\beta y})+\tfrac{3}{2}\beta(y_1-y)-\tfrac{3}{2}\beta\frac{(y_1-y)^3}{y_1^2}\right\} \quad (92).$$

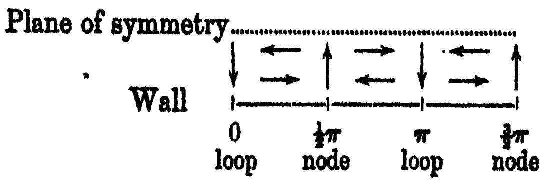
Outside the thin film of air immediately influenced by the friction we may put $e^{-\beta y}=0$, and then

$$u=-\frac{3u_0^2 \sin 2kx}{16a}\left\{1-3\frac{(y_1-y)^2}{y_1^2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (93),$$

$$v=-\frac{3u_0^2 2k \cos 2kx}{16a}\left\{y_1-y-\frac{(y_1-y)^3}{y_1^2}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (94).$$

From (93) we see that u changes sign as we pass from the boundary $y=0$ to the plane of symmetry $y=y_1$, the critical value of y being $y_1(1-\sqrt{\tfrac{1}{3}})$, or $\cdot 423\ y_1$.

The principal motion being $u=-u_0 \cos kx \cos nt$, the loops correspond to $kx=0$, π, $2\pi, \ldots$, and the nodes correspond to $\tfrac{1}{2}\pi$, $\tfrac{3}{2}\pi, \ldots$ Thus v is positive at the nodes and negative at the loops, vanishing of course in either case both at the wall $y=0$, and at the plane of symmetry $y=y_1$.



To obtain the mean velocities of the *particles* parallel to x, we must make an addition to u, as in the former problems.

In the present case the mean value of

$$\frac{du}{dx}\xi+\frac{du}{dy}\eta=-\frac{u_0^2 \sin 2kx\, e^{-\beta y}}{4a}\left\{e^{-\beta y}-\cos \beta y\right\},$$

so that

$$u'=-\frac{u_0^2 \sin 2kx}{8a}\left\{e^{-\beta y}(4 \sin \beta y+3e^{-\beta y})+\tfrac{3}{2}-\tfrac{9}{2}\frac{(y_1-y)^2}{y_1^2}\right\} \quad . \quad . \quad . \quad (95).$$

When βy is small,

$$u'=-\frac{u_0^2 \sin 2kx}{8a}\left\{-2\beta y+\ldots\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96).$$

Inside the frictional layer the motion is in the same direction as just beyond it.

We have seen that the width of the direct current along the wall is $\cdot 423\ y_1$, and that of the return current (measured up to the plane of symmetry) is $\cdot 577\ y_1$, so that the direct current is distinctly narrower than the return current. This will be still more the case in a tube of circular section. The point under consideration depends only upon a complementary function analogous to (86), and is so simple that it may be worth while to investigate it.

The equation for ψ is

$$\left(\frac{d^2}{dr^2}-\frac{1}{r}\frac{d}{dr}-4k^2\right)^2\psi=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97),$$

but if we suppose that the radius of the tube is small in comparison with λ, k^2 may be omitted. The general solution is

$$\psi=\{A+Br^2+B'r^2\log r+Cr^4\}\sin 2kx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98),$$

so that

$$u=\frac{1}{r}\frac{d\psi}{dr}=\{2B+B'(2\log r+1)+4Cr^2\}\sin 2kx,$$

whence $B'=0$, by the condition at $r=0$. Again

$$v=-\frac{1}{r}\frac{d\psi}{dx}=-2k\{Ar^{-1}+Br+Cr^3\}\cos 2kx,$$

whence $A=0$.

We may take therefore

$$\left.\begin{aligned}u&=\{2B+4Cr^2\}\sin 2kx\\ v&=-2k\{Br+Cr^3\}\cos 2kx\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99).$$

If $v=0$, when $r=R$, $B+CR^2=0$, and

$$u=2C(2r^2-R^2)\sin 2kx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1\ \ .$$

Thus u vanishes, when

$$r=\frac{R}{\sqrt{2}}=\cdot 707\ R, \qquad R-r=\cdot 293\ R.$$

The direct current is thus limited to an annulus of thickness $\cdot 293$ R, the return current occupying the whole interior, and having therefore a diameter of

$$2\times\cdot 707\ R=1\cdot 414\ R.$$

II. *On the Solubility of Salts in Water at High Temperatures.*

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[PLATES 1, 2.]

THE experiments of which an account is given in the following paper originated in a desire to investigate further the remarkable anomalies which have been observed in the solubility of sulphate of sodium in water. It appears to have been first discovered by GAY LUSSAC (Ann. Chim. Phys., xi., 313) that whilst the solubility of this salt in water increases rapidly with rise of temperature above zero, it attains a maximum at 33° or thereabouts, and above that temperature diminishes till the boiling point (about 103°) of the saturated solution is reached.

On referring, however, to the curve of solubility for this salt traced by GAY LUSSAC, or to the numbers published many years afterwards by LÖWEL (Ann. Chim. Phys. [3], xlix., 32), it will be seen that the rate of decrease of solubility, though at first rapid, soon slackens, and the descending curve becomes nearly parallel with the axis of the abscissæ representing temperatures. It appeared probable that if the solubility of sulphate of sodium could be traced through a range of higher temperatures, the solubility would be found once more to increase, and the curve resume an upward direction.

It was not possible, however, to undertake the investigation of an isolated case of this kind without extending the inquiry to the phenomena of solution in general, and we have thus been led to consider several questions, which were not immediately involved in the subject of our first experiments. One important point which our experiments illustrate is the relation, hitherto assumed rather than determined, between fusibility and solubility.

In our earlier experiments the ordinary crystallised sulphate of sodium containing ten molecules of water of crystallisation was enclosed together with a suitable quantity of water in glass tubes, either bent in the middle to an angle of 130°–140°, or straight and divided midway by a strainer of fine platinum wire gauze.

In either case the materials were placed at one end of the tube, which was held in an inclined position within an air bath, with double walls, and provided with a

thermometer at each end, and a thermostat. The tube and its contents were then exposed to the desired temperature for a considerable time, usually about four-and-a-half to five hours, so as to ensure complete saturation. By then gradually tilting the air bath, which was slung upon suitable supports, the solution was caused to drain away from the undissolved residue of the salt, and the whole was then allowed to cool. Finally the tube was cut open, the end containing the solution was weighed, and after removing the solution to a tared dish, was reweighed. The solution was then evaporated to dryness with due precautions, and the weight of the dry residue determined. From the data so obtained the proportion of anhydrous salt to water in the solution was calculated.

Throughout these operations two tubes were invariably employed in each experiment, and two determinations thus made simultaneously. The mean of the results was taken. Preliminary experiments were also made which indicated that by a proper disposition of the four BUNSEN burners used as the source of heat, and the employment of the mercurial gas regulator, the temperature of the air bath could be maintained within a range of $\pm$ 2° for many hours. We also satisfied ourselves of the efficiency of the platinum gauze strainers, by frequently examining the decanted solution whilst still warm and liquid. It may also be added that the salts operated upon were pure and were almost always specially prepared in the form of agglomerated masses of small crystals, with the object alike of exposing a greater surface to the action of the solvent, and of allowing the solution to flow away freely when the tube was reversed.

The employment of bent glass tubes was soon abandoned, owing to the difficulty of keeping both ends of the tube at precisely the same temperature, also in consideration of the greater risk of bursting to which they were liable. The employment of tubes of glass under any form is, in the case of sulphate of sodium, undesirable, as we find that this salt has the power of attacking glass at high temperatures in an unexpected degree. Consequently, although we succeeded in satisfying ourselves that the solubility of sodium sulphate in water at temperatures above the normal boiling point of the solution does increase in the manner we anticipated, the numerical results of the successive experiments were not sufficiently concordant to show clearly the form of the continuation of GAY LUSSAC'S curve. It therefore became necessary to employ tubes of metal, and of a somewhat different construction.

In arranging the experiments, which were afterwards extended to many other metallic salts, the following considerations had to be taken into account:—

1. It is necessary to allow the solution to drain away completely from the undissolved residue of salt before allowing the tube to cool; otherwise a part of the dissolved salt may be deposited, and the solution collected for analysis would be weaker than it should be. Provision is made against this source of error in the metal tube we have used in the later experiments.

2. If the two extremities of the tube are not kept at absolutely equal temperatures,

distillation of water from one part to another will take place. A paraffin bath appeared to offer greater probability of uniformity than an air bath, and was consequently employed in most of the best experiments.*

3. Further, it is obvious that the part of the tube not occupied by salt and solution must be filled with vapour of water, which, on cooling, will condense to the liquid state, and mix partly with the undissolved residue and partly with the solidified solution which has been drained away. Unfortunately, too, this circumstance is complicated by the fact that the tension of such water vapour is not that which would be given by pure water, but is the smaller tension given by the saline solution contained within the tube.

By ascertaining the volume of this vapour, approximate correction of the results can be effected with the aid of the tables of vapour tension of salt solutions, published by WÜLLNER (POGGEND. Ann., ciii., 529; cx., 564).

In any case the error on this account could not be great, but we nevertheless thought it advisable to make a few experiments with the object of testing directly its probable magnitude.

A number of bent glass tubes containing crystals of sodium sulphate, $Na_2SO_4.10H_2O$, in one limb, were sealed up and heated in a paraffin bath to temperatures ranging from 115° to 150°, the experiment being conducted in all respects as if a solubility were to be determined, except that the solution was not decanted. They were then allowed to cool, cut in two at the bend, and the end remote from the salt was at once closed by a stopper and weighed. It was then dried and re-weighed. Subsequently its capacity was ascertained. The following were the results:—

Experiment.	Weight of Water.		Difference.
No.	Calculated.	Found.	
	grm.	grm.	grm.
1	·0103	·0153	+·0050
2	·0102	·0116	+·0014
3	·0141	·0129	−·0012
4	·0067	·0061	−·0006
5	·0151	·0137	−·0014
6	·0106	·0173	+·0067

As the quantity of solution dealt with was never less than 4 grms., and was generally about 8 grms., it is obvious that the greatest error to be expected on account of occasional slight distillation from inequality of temperature, or on account of the water vapour always present, is inappreciable. When the metal tube is used the error is still less, because its greater mass and superior conducting power are favourable to the maintenance of a constant temperature.

Attempts were made to employ a metal tube provided with valves of various kinds,

* The bath was in fact carefully tested, and the constancy of its temperature ascertained.

arranged so that after draining the solution from the residual crystals it was shut off in a portion of the tube of known capacity, further entrance or escape of vapour being prevented, the risk of an impoverished solution continuing to drain into it during cooling being also done away with. But though we were successful in getting such a tube constructed, the difficulty of working it and of keeping it in working order, led us to abandon it in favour of a simpler form.

4. The influence of pressure.—Mr. SORBY'S experiments (Proc. Roy. Soc., xii., 538) show that pressure exerts an influence upon the solubility of salts in water, but the effect due to the pressure existing at high temperatures in our tubes is too small to affect materially these estimations of solubility. In our experiments, with only two or three exceptions, the pressure of the vapour in the tube could never have exceeded some 10 or 12 atmospheres.

5. The choice of salts at our disposal is more limited than might be expected.—In some cases the solubility becomes so great at temperatures above 100°, that quantitative experiments were found to be impracticable. In others, as may be supposed, water at high temperatures decomposes the salts with formation of precipitates, or other signs of chemical change.

In one case, namely, chloride of barium, the crystals of the salt fall to a powder, from which it was found impossible to withdraw the solution.

6. We intended originally to have made a much larger number of determinations at temperatures above 200°, but it was not found practicable, partly in consequence of the difficulty of maintaining constant so high a temperature in the air bath (and paraffin could not be used), partly because the pressure at 250° and above becomes so considerable (amounting probably to about 30 atmospheres), that the lead washer in the joint of the metal tube was forced out, no matter how tightly the parts were screwed together.

The melting point of the most fusible salt tried (silver nitrate, m.p. 217°), is easily reached, but its solubility at temperatures far below this was so excessive that further determinations became impossible.

The apparatus finally adopted is represented (half-size) in Plate 1, figs. 1 and 2. The tube is made of gun metal, electroplated with silver all over. The two parts screw together at C. Each of the faces, *b*, *b*, *b'*, *b'*, has two circular grooves cut in it, an electroplated lead washer fits between these two faces, and when the proper amount of pressure is applied in screwing up, the lead is forced into these grooves. Plate 1, fig. 1 B, shows the end of B. It is hexagonal for convenience in screwing up. *a*, *a*, is a disk or screen of silver having a semicircular opening, *e*, cut through it. When the tube is closed, *a* does not quite touch *f*. By means of the handle *h*, the tube may be turned round its longitudinal axis in its support during an experiment, so that liquid may be free to flow from A to B through *e*, or not, as may be desired. In using the tube the materials are placed in A, a disk of platinum gauze is placed above at about *g*, and the two parts are screwed together very firmly. The tube is then placed

in an adjustable cradle (Plate 1, fig. 2), which consists of a stout beam of wood, B, supported by a horizontal rod of iron which passes through *o*, and serves as a pivot on which the beam can turn. Attached to the upper side of this beam is an index, which moves against the face of a semicircle of wood S, fixed rigidly to the rod on which B turns. The index can be pinned to the semicircle at the holes *h, h, h, h*, so that the beam can be inclined to the horizontal at any desired angle. The cradle of sheet copper, C, is supported by the wires *w, w*, and in this the tube lies.

The tube being in position in the cradle, with the end B slightly elevated, and the opening *e* in the screen downwards, the whole is lowered into a paraffin bath by lowering the rod which supports the beam, and which is itself fixed by an ordinary clamp to the upright of a large retort stand. After heating at a steady temperature for four-and-a-half or five hours, the end A of the tube is raised, and B depressed, very gradually, keeping the whole tube below the surface of the paraffin. When time has been allowed for the solution to drain away, the tube is turned half round its long axis by the handle *h*, so that the screen is interposed between the solution and the residual salt, and so liquid from the latter is prevented from continuing to drain into the former. The tube is then placed in a position more nearly horizontal, but still with the end A raised somewhat higher than B, which now contains the solution.

The cradle, with the tube, is then lifted from the bath, and allowed to cool in the air. When cold it is opened, the disk is removed, and a stopper placed in the mouth of B. After cleansing the outside of the tube by washing in benzoline, the tube and solution are weighed. Then the solution, which is usually in a solid or semi-solid state, having been removed to a tared dish for analysis, the dry tube is finally reweighed.

The capacity of B to the mouth was 16·4 cub. centims.

The results given below with sodium sulphate show that the process yields very satisfactory results. It necessitates, however, a good deal of trouble, and each experiment occupies more than a day.

In order that our results might be readily compared with the determinations made by other experimenters at lower temperatures, we have added many of these latter to the tables. The numbers are taken from the several original sources, and have been recalculated to show the weight of salt in 100 parts of water when not so given in the memoir referred to. In the curves which accompany the tables the results of other experimenters are put in dotted lines. We have also added at the head of each table the melting point of the salt. For these we have adopted the values obtained by Professor T. CARNELLEY (Jour. Chem. Soc., 1876, i., 489; 1878, i., 273), though in one or two cases, where data were not to be found, a rough determination of the melting point has been made by ourselves.

SOLUBILITY of sodium sulphate.—Melting point 860° C.

Observer.	Temperature.		Anhydrous salt Na_2SO_4 in 100 parts of water.	
GAY LUSSAC (Ann. Chim. Phys., xi., 312)	° 0	..	5·02	
	11·6	..	10·12	
	13·3	..	11·74	
	17·9	..	16·73	
	25·0	..	28·10	
	28·7	..	37·35	
	30·7	..	43·05	
	31·8	..	47·37	
	32·7	..	50·65	
LÖWEL (Ann. Chim. Phys. [3], xlix., 42)	33	..	50·76	
	34	..	55·00	
	40·1	..	48·78	
	45·0	..	47·81	
	50·4	..	46·82	
	59·7	..	45·42	
	70·6	..	44·35	
	84·4	..	42·96	
T. and S.	100·0	..	42·41	
GAY LUSSAC	103·1	..	42·65	
T. and S.	120	*a*. 41·9 *b*. 42·0	Mean 41·95	In the metal tube.
	140	*a*. 41·9 *b*. 42·1	" 42·00	
	160	*a*. 42·8 *b*. 43·0	" 42·90	
	180	*a*. 44·2 *b*. 44·3	" 44·25	
	230	..	" 46·40	

In this table and curve (Plate 1), the results of LÖWEL between 33° and 34° are substituted for those of GAY LUSSAC, as being probably more correct. At 34°, or a fraction above, crystals of the ordinary salt with $10H_2O$ melt, and, if the experiment be conducted with due care, without separation of anhydrous salt. Such a liquid is a solution of 78·8 parts of Na_2SO_4 in 100 parts of water, and falls naturally into its place at the highest point of the curve.

The solubility of sodium sulphate at 100° was determined in several ways, in order to ascertain whether the mode of operating had any influence on the result.

(*a*.) By heating up crystals of the decahydrated salt, without addition of water, in a bath of constant temperature.

(*b*.) By dropping crystals of the same salt into water, heated and maintained at 100°.

(*c*.) By adding the anhydrous salt to water at 100°.

The result was the same in each case.

Reference to the curve will show that, as we anticipated, the solubility of sodium sulphate does increase again when the temperature is carried high enough.

The peculiarities of sodium sulphate in regard to its solubility have always formed an interesting problem, but we venture to think its interest is enhanced by this

discovery, because it seems impossible, by appeal to the commonly received theories of solution, to find a satisfactory explanation of all the facts of the case. If we admit that sodium sulphate, placed in contact with water at temperatures below 34°, dissolves in virtue of its power of entering into union with water to form liquid hydrates, the diminished solubility above that temperature must be due to dissociation of these hydrates, and production of the anhydrous salt, which is apparently much less soluble. What then is the cause of the much greater solubility of the ordinary crystals in which the salt is already united with a large quantity of water, and how can we explain the fact that the anhydrous salt increases in solubility in accordance with the common rule when the temperature is raised? The explanation appears to be found in the difference of fusibility of the two compounds, $Na_2SO_4 10H_2O$ melting at 34°, and Na_2SO_4, which melts at 860°.

There is nothing new in the idea that readiness to melt by heat is associated with disposition to dissolve by contact with liquid solvents, for even so far back as 1819 we find GAY LUSSAC quoting with approval a still older explanation given by LAVOISIER ('Traité Elem. de Chimie.,' ii., 39)* of the action of heat in causing increase of solubility. But we are not aware that it has been definitely brought to the test of experiment before.

Supposing a substance heated with a solvent to the melting point of the former, three cases might present themselves:—

(*a.*) The liquids might be miscible in all proportions.

This is true of sulphate of sodium at 34°. The melted salt, $Na_2SO_4 10H_2O$, may be mixed with an indefinitely small quantity of water, or in other words is infinitely soluble.

We have also ascertained that it is nearly true of benzoic acid, which melts at 120°. This compound is stated to be soluble in about 600 parts of water at 0°, in 200 parts at 18°, in 25 parts at 100°. By sealing it up with water in a glass tube and heating to a few degrees beyond the melting point, intermixture occurs in all proportions, and the liquid so obtained on cooling to 120°, or about 1° lower, becomes turbid from deposition of oily drops, which, however, immediately crystallise.

(*b.*) The solvent might become saturated and the excess of undissolved substance remain over in a liquid state.

(*c.*) Or both might become saturated, the one with the other, forming two distinct liquids.

This occurs in the case of phenol (carbolic acid) and certain of the fatty series of acids, besides other well-known substances.

Supposing either *b* or *c* to occur, the two liquids become miscible when the temperature is raised, as may easily be shown in either of the cases referred to.

But the connexion between fusibility and solubility, though proved, does not wholly explain the nature of the initial stage in the process of solution of a solid. It does,

* In the reprint of LAVOISIER's works, vol. i., p. 305.

however, strongly support a kinetic theory of solution based on the mechanical theory of heat. The solution of a solid in a liquid would accordingly be analogous to the sublimation of such a solid into a gas, and proceeds from the intermixture of molecules detached from the solid with those of the surrounding liquid.

Such a process is promoted by rise of temperature, partly because the molecules of the still solid substance make longer excursions from their normal centre, partly because they are subjected to more violent encounter with the moving molecules of liquid. Such a view does not necessarily involve the assumption of an "attraction" between the molecules of the solvent and those of the solvend (compare DOSSIOS, Jahresb., 1867, 92; and NICOL, Phil. Mag., Feb., 1883).

Indeed, it is difficult to disconnect "attraction" from the idea of combination resulting from such attraction. In some of the cases we are considering, as for instance the solution of anhydrous sulphate of sodium in water at 100°, nothing like combination between the water and salt seems to occur.

We have satisfied ourselves by direct experiment that anhydrous sulphate of sodium at the temperature of 100° dissolves in water at the same temperature without any sign of previous combination, and the solution so prepared contains exactly the same amount of solid as the solution made by gradually heating up a solution prepared at a lower temperature. But when anhydrous sulphate of sodium is introduced into water below 34°, all the phenomena of combination are manifested, and the salt sets into a solid crystalline mass previous to dissolving.

Whilst, therefore, we still think the act of hydration a factor in a great many cases of solution, it appears that it must be abandoned as a hypothesis of general applicability.

We now proceed to describe the results we have obtained with other salts.

CALCIUM sulphate.—Melting point a red heat.

SULLIVAN (Rep. Brit. Assoc., 1859, 292) states that he has proved this salt to be insoluble in water at 150°, but we can find no detail of any experiments of his. COUSTÉ (Ann. des Mines [5], v., 140–144) describes experiments upon the solubility of calcium sulphate at high temperatures, but they appear to have been all made with sea-water, and there are many objections that might be raised to his mode of operating.

In our experiments pure precipitated calcium sulphate, which had been most thoroughly washed, was used and distilled water. Glass is rapidly attacked by the solution, and the determinations were therefore made in the silvered metal tube.

Observer.	Temperature.		Parts of $CaSO_4$ in 100 of water.
Poggiale (Ann. Chim. Phys. [3], viii., 469)	0°	..	·205
	20	..	·241
	35	..	·254
	70	..	·244
	100, &c.	..	·217, &c.
T. and S.	140	Exp. 1. ·080	Mean ·078
		„ 2. ·076	
		„ 3. ·080	
	160–165	„ 1. ·056	„ ·056
		„ 2. ·056	
	175–185	„ 1. ·024	„ ·027
	178–183	„ 2. ·030	
	240	..	·018
	250	..	·018

(See Plate 2.)

This curve is interesting as having considerable resemblance to that of sodium sulphate. Experiments at 250° were repeated, but without any indications that the solubility was about to increase, and this was the highest temperature at which we have been able to work. Calcium sulphate is much less fusible than sodium sulphate, and we could not therefore expect that a change would be observable in the direction of the curve, unless the temperature were carried much higher than we found it possible to go.

Potassium sulphate.—Melting-point ?.

Temperature.	Parts of K_2SO_4 in 100 parts of water.	
	Gay Lussac.	T. and S.
0°	8·3	..
12·7	10·5	..
16	..	9·76
20	..	10·30
28	..	12·59
36	..	13·28
39	..	14·21
49	16·9	..
59	..	17·39
63·9	19·2	..
98	..	23·91
101·5	26	..
120	..	26·5
143	..	28·8
170	..	32·9

(See Plate 1.)

These results give a curve which is nearly a straight line. Our figures are uniformly somewhat lower than those of Gay Lussac. Curves constructed with the

two sets of figures are nearly parallel. Hence we think it possible that the salt used by GAY LUSSAC may have been slightly acid. On the other hand, our own results at temperatures above 120° may be somewhat too low.

COPPER sulphate.

Temperature.	Parts of $CuSO_4$ in 100 parts of water.
120°	90·1
135	85·5
140	84·4
157	82·0
188	74·5

These results are of no value as showing the solubility of the salt, for at temperatures above 120° chemical action ensues, with production of a green basic sulphate. And even at 120° there is reason to suppose that decomposition has commenced, inasmuch as 90·1 is below the solubility that would be inferred from a consideration of the solubilities (see POGGIALE, Ann. Chim. Phys. [3], viii., 467) determined at lower temperatures.

SODIUM chloride.—Melting point 772°.

Temperature.	Parts of salt in 100 parts of water.	
118°	39·8	exp. made in the metal tube.
140	42·1	
160	43·6	
180	44·9	

(See Plate 2).

Reference to the curve shows that the solubility increases at temperatures above 100° faster than below.

POTASSIUM chloride.—Melting point 734°.

Temperature.	Parts of salt in 100 parts of water.	
125°	59·6	in metal tube.
133	69·3	in glass tube.
144	70·8	
175	75·2	
180	77·5	in metal tube.

POTASSIUM bromide.—Melting point 699°.

Temperature.	Parts of salt in 100 parts of water.
140°	120·9 in metal tube.
181	145·6

POTASSIUM iodide.—Melting point 634°.

Temperature.	Parts of salt in 100 parts of water.
124°	233·9 } in glass tube.
133	249·3
144	264·6
175	310·4

(See Plate 2.)

These results, when expressed graphically, correspond in each case very nearly to a straight line. And when the four preceding salts are compared together they serve to illustrate very well the relation of solubility to fusibility. In Plate 2, iodide of potassium, the most easily fusible, is shown to be not only the most soluble at common temperatures, but its solubility increases at a more rapid rate than either of the others, which follow in succession.

POTASSIUM nitrate.—Melting point 339°.

Two determinations of the solubility of this salt were done at 125°. At this temperature 100 parts of water dissolved—

Exp. I. 495·9 parts of salt, KNO_3.
Exp. II. 492·7 „ „ „

From the great solubility of the salt at higher temperatures, and the peculiar viscosity of the solution, in consequence of which it was difficult to separate it from the solid, no determinations of any value could be obtained in the experiments made at 180° and thereabouts.

SILVER nitrate.—Melting point 217°.

Notwithstanding the extreme solubility of this compound, two pairs of concordant determinations were made.

Temperature.	Parts of salt dissolved in 100 parts of water.
125°	1622·5 } in glass tube.
133	1941·4

(See Plate 1.)

Beyond this temperature it was useless to attempt quantitative experiments, as we already have nearly 20 parts of salt dissolved in 1 part of water. It may be observed that this corresponds to rather more than two molecules of silver nitrate, $AgNO_3$, to one molecule of water. This, therefore, is an example of a solution in which it is difficult to conceive the existence of liquid hydrates. It must rather be regarded as melted silver nitrate mixed with a small quantity of water.

POTASSIUM chlorate.—Melting point 359°.

Temperature.	Parts of salt in 100 parts of water.	
120°	73·7	in glass tube.
136	98·9	in glass tube.
160	148·0	in glass tube.
190	183·0	in metal tube.

(See Plate 2.)

BARIUM chlorate, $Ba(ClO_3)_2$.—Melting point of the anhydrous salt about 400°.

Temperature.	Parts of $Ba(ClO_3)_2$ dissolved by 100 parts of water.	
116°	195·5	in glass tube.
135	287·4	in glass tube.
146	365·6	in glass tube.
180	522·6	in metal tube.

(See Plate 2.)

This salt gave some trouble, partly on account of its considerable solubility, partly from the viscosity of the solution. No oxygen was found in any of the tubes after heating, but two tubes were split in a somewhat singular manner by the solution in process of solidification during cooling.

POTASSIUM dichromate, $K_2Cr_2O_7$.—Melting point about 400°.

Temperature.	Parts of salt dissolved by 100 parts of water.	
117°	128·3	in glass tube.
129	153·8	in glass tube.
148	200·6	in glass tube.
180	262 7	in glass tube.

(See Plate 2.)

BARIUM acetate, $Ba(C_2H_3O_2)_2$.—Melting point of the anhydrous salt about 450°.

Temperature.	Anhydrous salt dissolved by 100 parts of water.	
22°	48·5	
40	76·5	
60	79·0	
110	79·3	in glass tube.
130	85·6	in glass tube.
136	91·9	in glass tube.
180	141·6	in metal tube.

(See Plate 2.)

This salt was examined chiefly because it had been represented as an example of solubility diminishing with rise of temperature. When these results are plotted out the curve does suggest a change of this kind, and it is possible that acetate of barium may resemble sulphate of sodium and sulphate of calcium in parting with its water of crystallisation when the solution is heated. It is known to yield crystals containing one molecule of water of crystallisation when deposited from a warm solution, or three molecules of water when crystallised by cooling a weaker solution. It is probable that the peculiarities of its solubility are connected with the fusibility of these hydrates respectively.

CALCIUM HYDRATE.

This compound is known to be less soluble in hot water than in cold water. We have made some experiments at temperatures above 100°, but as the solution seemed to attack the metal, and the quantity of liquid we could operate upon is but small, the results are of no quantitative value.

SUMMARY.

Altogether we have examined sixteen salts, whereof three, namely, barium chloride, copper sulphate, and calcium hydrate, gave no results at high temperatures for reasons already given. Barium acetate presents anomalies which cannot be explained without further inquiry. Sodium sulphate and calcium sulphate are salts which certainly exist in solution in two forms, that is, in chemical combination with water and in the anhydrous state.

The remaining ten salts, with one exception, barium chlorate, form crystals which, when deposited from solution at any ordinary temperatures, contain no water of crystallisation.

It is almost fair to infer, though of course it is not certain, that these salts do not combine with water when the temperature is raised, and therefore when in solution at high temperatures exist there in the anhydrous state.

Concerning these ten salts the following remark may be made. If we write them down in the order of their melting points, beginning with the most fusible, we do not indeed find that the figures expressing their solubilities follow the same order. Though it is true that one example of this may be seen in the iodide, bromide, and chloride of potassium, such a relation could hardly be expected to be general amongst the rest, differing as they do in composition and in other properties. But if their solubilities are compared at temperatures of 100° and above, we find that the *rate of increase of solubility follows the order of the melting points.* In other words the ratio of the solubility at, say, 180°, to the solubility at 100°, is greatest in the most easily fusible, whilst the rest follow in regular succession. This is shown in the following table.

Formula of salt.	Melting point.	Parts of salt* dissolved by 100 parts of water at 0°	100°	180°	180°	Ratio of VI. to IV.	Ratio of VI. to V.	Ratio of V. to IV.
I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
$AgNO_3$	217°	121·9	830	1825	?	..	..	2·20
KNO_3	339	13·3	265	565	?	..	..	2·13
$KClO_3$	359	3·3	56·5	88·5	190	3·36	2·14	1·56
$K_2Cr_2O_7$	400	4·9	102	156	285	2·79	1·82	1·52
KI	634	130	204	243	327	1·60	1·34	1·19
KBr	699	53·4	102	118	143	1·40	1·21	1·15
KCl.	734	29·2	56·5	66	78	1·38	1·18	1.16
NaCl	772	35·5	36·6	40·3	44·9	1·23	1·11	1·10
Na_2SO_4	860	hydrates	hydrates	42·0	44·2	..	1·05	..
K_2SO_4 †	?	8·3	25	28	34	1·36	1·21	1·12

The only salt which does not fall strictly into order is potassium sulphate; but concerning this we are in doubt as to the melting point, and since it gave a good deal of trouble the determinations of solubility at the higher temperatures may not be quite exact. It is difficult to believe that the relation we have indicated is merely accidental.

* The solubility, when not the result of a direct experiment, is taken from the curve by interpolation.

† The melting point of potassium sulphate is doubtful. It is probably higher than that of sodium sulphate. According to Carnelley (*loc. cit.*) the potassium salts generally melt at temperatures above the melting points of the corresponding sodium salts. Thus:

$NaNO_3$, m.p. 316°, KNO_3, m.p. 339°.
$NaClO_3$, m.p. 302°, $KClO_3$, m.p. 359°.
NaI, m.p. 628°, KI, m.p. 634°.
Na_2CO_3, m.p. 814°, K_2CO_3, m.p. 834°.

On the other hand—

KCl, m.p. 734°, NaCl, m.p. 772°.

III. *The Influence of Pressure on the Temperature of Volatilization of Solids.*

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1. THE experiments to be described in this paper were undertaken in order to ascertain whether solids have definite volatilizing* points under different pressures, as liquids have definite boiling-points, and whether these pressures are identical with their vapour-tensions at those temperatures.

It has been long known that arsenic, which volatilizes without melting under atmospheric pressure, melts when the pressure is raised; and some years ago CARNELLEY proved that ice, mercuric chloride, and camphor do not melt below certain pressures peculiar to each substance; but above these pressures they melt when heated. He proposed the term "critical pressure" to denote that pressure below which a solid cannot melt. Preliminary experiments appeared to show that the solid might be raised in temperature above its ordinary melting-point without melting; but it has since been experimentally proved that this is not the case.

In January, 1881, shortly after the publication of CARNELLEY's experiments, one of us read a paper before the Chemical Society of the Owens College, in which it was pointed out that, theoretically, at pressures below the triple point of JAS. THOMSON, water should be unable to exist as such. It was at that time experimentally undecided whether ice could be heated above 0° C. or not; and the annexed diagram (fig. 1) was designed to show the relations of solid, liquid, and gas, to temperature and pressure.

A somewhat similar diagram was subsequently published in 'Nature' by PETTERSSEN, (June 23rd, 1881), which, however, did not show JAS. THOMSON's ice-steam line, and in which the triple-point was placed below, instead of above 0°. This was pointed out in a letter to 'Nature' (July 14th, 1881).

2. Although it appeared extremely probable that the temperature of a solid corresponding to a definite vapour-tension is identical with the temperature at which it

* By the term "volatilizing" we wish to imply the condition of a solid analogous to that of a liquid when it is said to be "boiling"; and not the mere passing off into vapour analogous to *evaporating* in the case of a liquid; in other words, the "volatilizing point" of a solid at a given pressure is the maximum temperature to which the solid can attain under that pressure.

volatilizes under the same pressure, yet it has never been satisfactorily proved. It is true that Mr. J. B. HANNAY states that in experiments performed to disprove the possibility of raising the temperature of ice above 0°, for which a Florence flask was used, the bulb being placed in a freezing mixture, while the neck, in which a thermometer surrounded with ice was suspended, was heated by a BUNSEN's flame, the temperature of the bulb or condenser was nearly identical with that of the ice. From our experiments we know that the temperatures of the freezing mixture and of the condenser are never identical, and Mr. HANNAY failed to describe any arrangement by which the internal temperature of the condenser could be found.

Fig. 1.

The experiments of PETTERSSEN are more conclusive. By connecting the apparatus with a manometer he found that when the block of ice surrounding the thermometer was at any given temperature, the pressure in the manometer roughly corresponded to the vapour-tension of ice at the same temperature; but as he published the results of only two experiments, and as the errors are comparatively large, the question could not be regarded as settled.

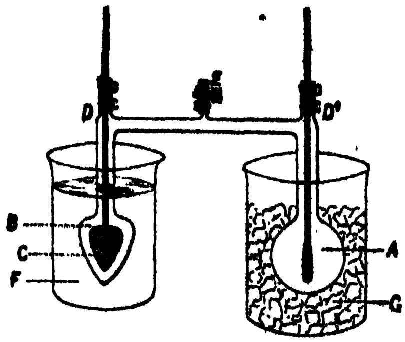
3. We resolved first to study the behaviour of ice at low pressures. For this purpose an apparatus was constructed which we afterwards found to have some resemblance to that described by BOUTLEROW (I. Russ. Phys. Chem. Soc., 1881, i., 316). The annexed figure (fig. 2) shows its form.

It was found that the leakage of air into the apparatus through the indiarubber connexions was extremely minute, and that an almost perfect vacuum could be maintained for several days.

4. The plan of operation was as follows:—Both bulbs were filled with water through the opening at E, and the water was boiled down until reduced to the volume of the bulb B. While the water was boiling, the screw-clip at E was closed, and the flames then removed. After cooling, the water was transferred to B, which was placed in a freezing mixture. When the water was frozen the bulb was gently

warmed, and the water which had melted from its interior surface transferred to A. The bulb A was then surrounded by a freezing mixture, and by gently warming B, all ice in contact with the sides was removed. The bulb B was now surrounded with boiling water, and the temperatures registered by the thermometers in A and B were carefully observed, while the temperature of the freezing mixture was altered from time to time.

Fig. 2.



A. Bulb, named condenser, placed in freezing-mixture.
B. Pear-shaped bulb, containing a block of ice, C, frozen round thermometer.
D, D'. Narrower tubes fused on to the wider tubes, through which the thermometers passed, secured by wired indiarubber connexions.
E. Exit tube for steam, closed by indiarubber tube and screw-clip.
F. Hot bath of water or paraffin.
G. Freezing mixture of hydrochloric acid and ice.

During freezing the phenomenon of supersaturation was nearly always observed. The temperature fell occasionally as low as $-11°$, while the water was still liquid. A sudden formation of ice then occurred, and the temperature rose to 0°. After remaining constant for some time while ice was being formed on the sides of the bulb, we were surprised to find the temperature fall below 0° (on one occasion as low as $-10°$), before the whole of the water was frozen. This we afterwards ascertained to be owing to the thermometer transferring heat to the ice with which it was in contact, and which was cooled by the freezing mixture, for the temperature of the water had not fallen below 0°.

5. The pressure in the apparatus could be increased only by raising the temperature of the condenser or by admitting air. The former has the effect of increasing the total pressure of water-vapour, and also of the air already contained in the apparatus, and can be calculated from the tables of tension of vapour in contact with ice, given by REGNAULT. An attempt was made to measure the pressure of the air present by inverting the apparatus and transferring all the air to one bulb, and then reading the difference in level of the water in the two limbs; but as the apparatus itself is such a delicate air-thermometer, it was found impossible to obtain satisfactory results. The amount of air was therefore calculated by a method which will appear hereafter.

6. The first series of readings gave the following results :—

Number of readings.	Mean temperature of condenser.	Mean temperature of bulb.	Difference.	Difference calculated.
7	$-11^{\circ}\cdot1$	-8°	$3^{\circ}\cdot1$	$3^{\circ}\cdot2$
1	$-9\cdot2$	$-6\cdot4$	$2\cdot8$	$2\cdot8$
4	$-8\cdot05$	$-5\cdot6$	$2\cdot45$	$2\cdot5$
4	$-5\cdot5$	$-3\cdot3$	$2\cdot2$	$2\cdot3$
6	$-3\cdot2$	$-1\cdot3$	$1\cdot9$	$1\cdot9$

The second series gave similar numbers :-

Number of readings.	Mean temperature of condenser.	Mean temperature of bulb.	Difference.	Difference calculated.
4	$-17^{\circ}\cdot7$	$-9^{\circ}\cdot35$	$8^{\circ}\cdot35$	$8^{\circ}\cdot35$
1	$-16\cdot7$	$-8\cdot9$	$7\cdot8$	$7\cdot9$
3	$-15\cdot5$	$-8\cdot3$	$7\cdot2$	$7\cdot4$
4	$-14\cdot5$	$-7\cdot6$	$6\cdot9$	$6\cdot9$
3	$-13\cdot6$	$-6\cdot8$	$6\cdot8$	$6\cdot5$
1	$-12\cdot7$	$-6\cdot3$	$6\cdot4$	$6\cdot2$

At this point the bulb of the thermometer became slightly exposed, and it is seen that the temperature shown by this thermometer rises more rapidly :—

Number of readings.	Mean temperature of condenser.	Mean temperature of bulb.	Difference.	Difference calculated.
2	$-11^{\circ}\cdot45$	$-5^{\circ}\cdot45$	$6^{\circ}\cdot0$	$5^{\circ}\cdot7$
3	$-10\cdot6$	$-4\cdot5$	$6\cdot1$	$5\cdot4$
9	$-9\cdot0$	$-3\cdot2$	$5\cdot8$	$4\cdot9$
10	$-7\cdot3$	$-1\cdot5$	$5\cdot8$	$4\cdot4$

It is to be observed that in each case the thermometer in the ice-bulb shows a higher temperature than that in the condenser, and that as the temperature rises the difference decreases.

7. The question arises :—On what does this difference depend? The answer which naturally occurred to us was that the temperature of the ice depends on the pressure in the apparatus, which in its turn depends on the temperature of the condenser. Assuming the truth of this hypothesis, it was possible to calculate the pressure exerted by the air in the apparatus for any one temperature in the following way :— In the first series of experiments the tension, ascertained by REGNAULT's tables, corresponding to the temperature $-3^{\circ}\cdot2$ in the condenser, is 3·59 millims.; that corresponding to the temperature $-1^{\circ}\cdot3$ of the ice is 4·16 millims. Assuming the temperature of the ice wholly to depend on the pressure in the apparatus, the pressure would

be 4·16 millims., of which 3·59 millims. are due to the tension of vapour in the condenser, while the difference between 4·16 and 3·59=0·57 millim. must be due to air. Since this pressure of air is nearly constant throughout the experiment, the slight variation being due to difference in temperature of the apparatus, which can be approximately allowed for, it is possible to calculate from REGNAULT's table the variation in temperature of the ice caused by altering the temperature of the condenser. For instance, the lowest temperature of the condenser in series I. is $-11^\circ{\cdot}1$; this corresponds to a pressure of 1·905 millims.; the total pressure in the apparatus is therefore 1·905 + 0·570=2·475 millims., and the corresponding temperature is $-7^\circ{\cdot}9$; the temperature observed being $-8^\circ{\cdot}0$.

In the second series the pressure of the air found as above is 1·08 millims., the lowest temperature being taken as the basis of calculation. The agreement between the calculated and found differences is seen to be close until the bulb became exposed, when its temperature rose more rapidly.

8. As it was sufficiently proved by these experiments, which were confirmed by numerous others, that our hypothesis is fairly in agreement with experimental evidence, it was decided next to admit small quantities of air, and to ascertain what rise in temperature the ice underwent. But as a very small amount of air causes a great difference in temperature, we were unsuccessful in measuring the exact amount of each addition. In order to do this, it was necessary to have as nearly as possible a perfect vacuum in the apparatus. For this purpose a litre of distilled water was boiled down to about 200 cub. centims. and introduced, while almost at its boiling-point, into the apparatus. The thermometers were not inserted until steam was issuing rapidly from the three orifices. Before inserting them they were held for some time in the steam, so as to remove any adhering film of air. It was evident that the bulbs were nearly vacuous, for two columns of water enclosing vapour came completely together without showing any trace of a bubble of air. It is right here to observe that all attempts to produce a complete vacuum with the SPRENGEL'S pump which we then possessed were totally unsuccessful.

After the apparatus was in order the freezing mixture was changed, and the temperature of the ice fell to -17°; the temperature of the thermometer in the condenser, however, fell very slowly, owing possibly to absence of convection currents. The temperature of the ice rose, while that of the condenser fell, until at -13° they were equal, and remained constant for a considerable time. It thus appears that the temperature of volatilization of the ice is -13° when the pressure in the apparatus is equal to the vapour-tension of ice at that temperature. A minute quantity of air was then introduced; the temperature of the condenser was now $-12^\circ{\cdot}6$, while that of the ice rose at once to $-6^\circ{\cdot}7$, and then remained stationary. On a second addition of air, with the condenser at $-12^\circ{\cdot}9$, the temperature of the ice rose to $-1^\circ{\cdot}5$ Thus, with no air, the difference between the readings of the two thermometers was 0°; after the first addition, $5^\circ{\cdot}9$, and after the second addition of air, $11^\circ{\cdot}4$.

9. In Section 7 it was stated that the alteration of the temperature of the condenser influenced the pressure in the apparatus. Indeed, any alteration in the temperature of the apparatus must influence the pressure due to air; hence it follows that if the temperature of the bath is altered, the pressure in the apparatus must also be altered. When the temperature of the bath is kept constant the pressure due to air alters only with alteration in the temperature of the condenser, and its change is therefore very small. To ascertain the effect of changing the temperature of the bath, the following experiment was made:—

Temperature of bath.	Temperature of condenser.	Temperature of ice.	Pressure.	Difference found.	Difference calculated.
°	°	°	millim.	°	°
75	−11	−6·3	0·890	4·7	4·7
85	−10·8	−6·1	0·905	4·7	4·8
100	− 9·5	−5·0	0·930	4·5	4·4
110	− 9·1	−4·7	0·948	4·4	4·4
120	− 8·8	−4·3	0·962	4·5	4·35
140	− 8·2	−3·7	0·990	4·5	4·25

In this experiment the capacities of the condenser, of the bulb, and of the whole apparatus were known; the pressure (0·89 millim.) was calculated from the lowest reading, and the difference in pressure was calculated from the observed temperatures and volumes of the various parts of the apparatus.

The agreement between the observed and calculated results is sufficient to give probability to the above hypothesis.

10. These results, although fairly conclusive, are all deduced from the behaviour of one substance, ice, which on account of its low melting-point and vapour-tension offers considerable difficulties in manipulation. Acetic acid was chosen, and the following series of results were obtained, the method of experiment being precisely similar to that followed with ice. Acetic acid melts at 16°·4.

	Temperature of bath.	Temperature of condenser.	Temperature of ice.	Difference.
a. Preliminary	Cold	−14°·9	+ 6°·6	21°·5
	Cold	− 6·0	9·7	15·7
	100	− 4·9	9·9	14·8
	70	− 4·3	8·8	13·1
	70	− 2·7	10·9	13·6
	Cold	+ 1·0	13·6	12·6
b. Apparatus free from air . . .	75	+ 0·9	+ 0·6	+ 0·3
	75	12·2	11·9	0·3
c. x cub. centims. of air introduced (The absolute amount of air was not measured).	80	− 0·6	5·9	6·5
	80	+ 0·9	6·3	5·4
	80	2·3	6·9	4·6
	80	4·3	7·9	3·6
	80	12·9	15·0	2·1
d. 2x cub. centims. of air introduced	65	− 8·2	7·0	15·2
	65	− 6·4	7·4	13·8
	65	− 5·8	7·6	13·4
	65	− 4·9	7·9	12·8
	65	− 3·7	8·1	11·8
	80	+ 2·1	9·9	7·8
	69	3·1	10·9	7·8
	67	5·6	11·8	6·3
	80	8·9	14·9	6·0
e. 3x cub. centims. of air introduced	73	− 3·4	12·1	15·5

11. Benzene gave a good semi-transparent block of ice, but its vapour caused the indiarubber connexions to leak; moreover, its rate of volatilization was so rapid that it only partially solidified in the receiver, even at −15°. The consequence was that the temperature of the benzene ice was apparently much lower than that of the thermometer in the condenser. Although acetic acid vapour solidified at once on reaching the condenser, yet the rapid current of vapour evidently warmed the thermometer in the condenser, for its readings are apparently too high.

12. Naphthalene was next tried, the apparatus being in this case exhausted with the pump.

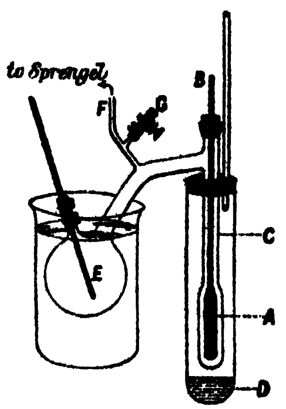
Temperature of bath.	Temperature of condenser.	Temperature of naphthalene.	Difference.
100°	−4°·9	73°·0	77°·9
”	−4·1	73·65	77·75
”	−3·6	74·15	77·75
”	−2·45	74·5	76·95
”	−1·6	74·75	76·35
”	−0·5	75·15	75·65
”	+0·4	75·6	75·20
”	+1·0	76·35	75·35
”	+2·3	76·8	74·50

The results with acetic acid and naphthalene generally agree with those obtained with ice; the differences at low temperatures being greater than at high temperatures. Owing to the low vapour-tension of naphthalene (9 millim.) at its melting-point (79·2), and to the want of data regarding its vapour-tension at lower temperatures, it was not thought worth while to make more extended experiments.

13. Camphor presents unusual facilities for the study of this question. Its melting-point is 175°; its vapour-tension at that temperature is 354 millims., and at 20° is 1 millim.; a great range is thus secured.

The apparatus, however, had to be modified in order to permit the tube containing the thermometer which supported a block of camphor to be jacketed with a vapour, and also to prevent stoppage of the passage by condensation of camphor vapour. It is represented in the accompanying figure (fig. 3).

Fig. 3.



A. Block of camphor round thermometer B, inserted in tube.
C, D. Jacketing tube containing aniline, the vapour of which could be made to surround C.
E. Condenser.
F. Tube connected with SPRENGEL's pump.
G. Indiarubber tube, closed by screw-clip, through which air could be admitted to alter pressure.

The thermometer was coated by dipping it repeatedly into melted camphor until a sufficiently thick layer had accumulated. The tube C was closed by an indiarubber cork through which the thermometer passed. The condenser E was placed in cold water during the experiment.

On establishing a fair vacuum, and boiling the aniline so as to jacket the tube containing the camphor with aniline-vapour at 184°·5, the temperature of the camphor rapidly rose, but no sublimation took place until the temperature had nearly reached its upper limit. The camphor then rapidly sublimed, and its temperature and the pressure indicated by a manometer connected with the air-pump were read. The pressure was then altered, and other readings taken. At very low pressures the camphor-vapour passed over into the condenser, which in these cases was cooled with a

freezing mixture; but at higher pressures condensation took place in the tube connecting the condenser with the heated tube.

Pressure.	Temperature of camphor.	Pressure.	Temperature of camphor.
millims.	°	millims.	°
1·7	41·2	92·8	136·3
7·2	48·9	105·0	140·3
15·4	92·4	109·4	141·7
27·2	101·0	155·1	147·0
35·0	109·4	197·6	154·3
46·0	116·7	218·5	157·9
66·3	127·4	240·7	160·1
88·6	134·2	297·8	168·0

14. When pressure was gradually increased to 370 millims. the camphor melted, and a drop hung from the end of the solid camphor coating the thermometer. By lowering the pressure to 358 millims. this drop solidified. The pressure of the solidification point was confirmed by a second experiment, but the pressure of melting seemed to vary.

15. The tension of camphor vapour in a barometer tube was next determined. When carried out with only ordinary precautions it was found impossible to exclude moisture, which rendered the results false. Correct results were obtained in the following manner. The upper end of the barometer-tube was drawn into a capillary, and connected with a SPRENGEL's pump; the whole tube was then jacketed, and surrounded with the vapour of boiling aniline, while a current of dry air was drawn through it. Some camphor was then introduced by the lower end, which was immediately dipped under hot mercury. The mercury was then pumped up the inclined tube; after it had reached a certain level, the camphor solidified, and adhered to the side of the tube. While the mercury rose further, bubbles of camphor vapour rushed up the tube, carrying with them all air and water vapour. After the mercury had entered the capillary portion of the tube the jacketing tube was slipped down, and the capillary tube was sealed through the mercury. We think it right to give details of the method of operation, as we found it a matter of extreme difficulty to expel all moisture and air. The temperatures given were obtained by jacketing the barometer-tube with the vapours of various pure liquids.

Temperature.	Pressure.	Temperature.	Pressure.
°	millims.	°	millims.
20·0	1·0	132·0	78·1
35·0	1·8	154·0	188·8
62·4	6·4	175·0*	354·0
78·4	9·5	184·5	431·0
100·0	22·6		

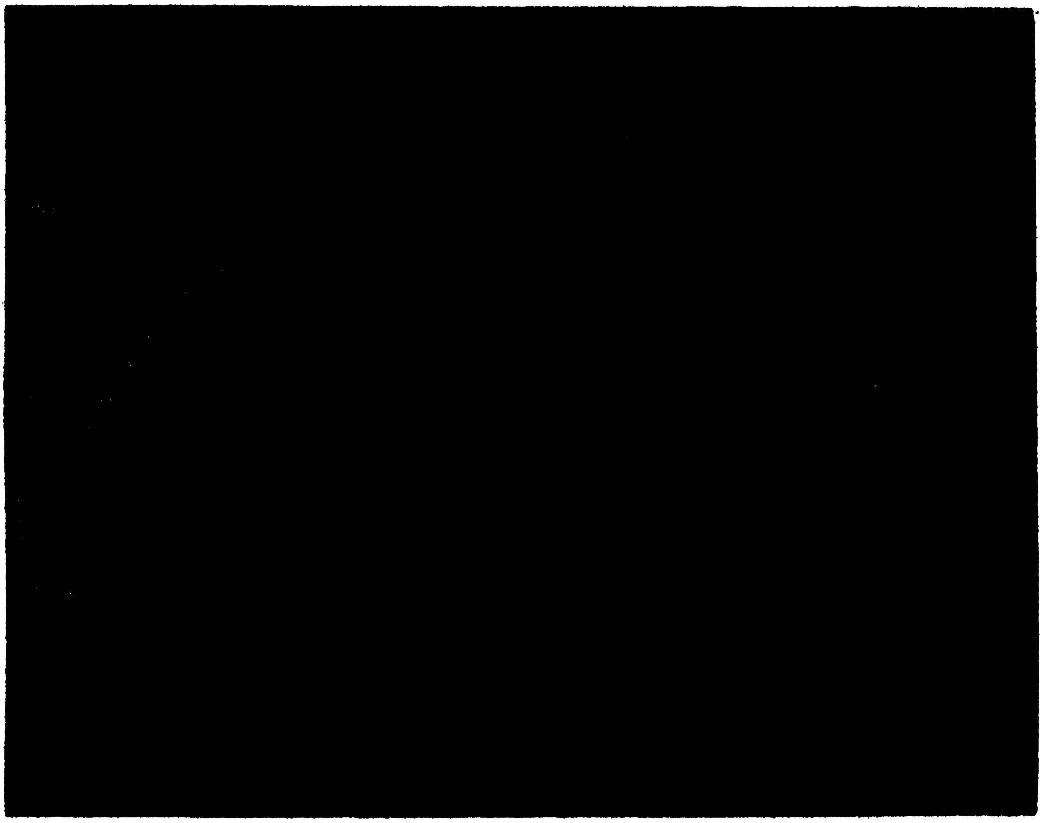
* Melting-point of camphor (GAY-LUSSAC, Ann. Chim. Phys., ix., 78).

The tension was observed three times when the camphor solidified in the barometer-tube. The readings differed only by 3·5 millims., and as a mean gave 350 millims.

The tension at which solidification took place in the bulb-apparatus was observed to be 358 millims. The mean of both determinations is 354 millims.

The annexed figure (fig. 4) shows the curve obtained from both sets of results; the pressures corresponding to temperatures of vaporization being indicated by a cross; the vapour-tensions by a dot surrounded by a circle.

Fig. 4.



16. It is thus proved that in the case of camphor the pressures corresponding to the temperatures of volatilization coincide with the vapour-tensions of solid camphor at these temperatures; and it appears that this assertion can also be made of ice.

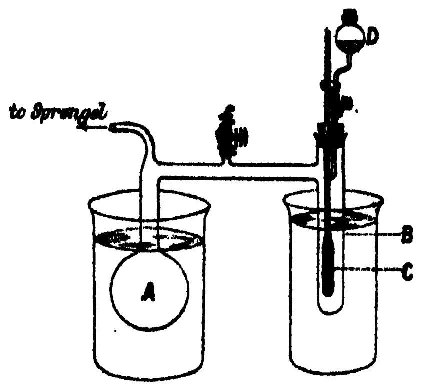
It may be advisable here to point out the difference between the evaporation of a liquid and of a solid. When the bottom of a vessel containing a liquid is heated, the whole of the liquid becomes hot owing chiefly to convection currents, and evaporation takes place only at its surface. When the temperature has reached the boiling point, either superheating or ebullition must take place. It would thus seem that the surface is not large enough to afford escape for the gaseous molecules, and in the former case the temperature of the liquid rises indefinitely, whereas in the latter the liquid

increases the extent of its surface by the formation of bubbles. In the case of a solid it is obvious that the surface is of limited extent, and it might therefore be expected that the solid should rise in temperature. Reasoning thus, the possibility of the existence of hot ice was maintained by CARNELLEY and other writers in a series of letters which appeared in 'Nature' during the years 1881 and 1882.

On the other hand, a liquid in the spheroidal state presents a free surface of evaporation in every direction, and yet, although exposed to the radiation of a white-hot surface, its temperature does not rise to the boiling-point (BALFOUR STEWART, 'Treatise on Heat,' 3rd edition, p. 124); and we find that when water is heated in a platinum basin by means of a blowpipe flame impinging on its surface, its temperature cannot be raised above 90°. In these cases the surface appears to be large enough to allow all vapour to escape with sufficient rapidity to prevent superheating.

If, then, the rate of evaporation at the surface of a solid is capable of indefinite increase, however much heat the solid receives, it follows that solids have definite temperatures of volatilization or volatilizing-points, corresponding to definite pressures, as liquids have definite boiling-points.

Fig. 5.



A. Condenser from which exit-tube leads to SPRENGEL's pump and manometer.

B. Tube in which thermometer C is suspended, placed in hot bath.

C. Thermometer, with bulb covered with cotton-wool.

D. Bulb containing liquid which could be admitted to apparatus by turning screw-clip, so as to trickle down the thermometer-stem, and moisten the cotton-wool.

E. Clipped indiarubber tube for admission of air.

17. It occurred to us that it would be advisable to ascertain if the boiling-points or maximum temperature of evaporation of liquids under conditions as nearly as possible identical with those to which the solids already mentioned were exposed are the same as the temperatures corresponding to their vapour-tensions. To enable this to be done, the apparatus was modified as shown in the figure (fig. 5).

By using a casing of cotton-wool round the thermometer bulb the liquid was

exposed to radiated heat in the same manner as the solids were. The vapour-tensions of water found in this way were identical with those of REGNAULT.

18. This method of determining vapour-tensions has great advantages compared with the usual method. For it is extremely difficult to obtain a number of constant temperatures when a long tube is heated, and the pressures depend on those temperatures; whereas by the method described, the temperature depends on the pressure, which can be varied at will by exhausting with the pump or by introducing air.

19. The experiments described have shown that solids have definite temperatures of volatilization, as liquids have definite boiling-points, depending on the pressure to which they are subjected, and that these temperatures are *sensibly* coincident with those of their vapour-tensions. That they cannot be *absolutely* identical is evident; for there must be a certain excess of pressure to produce a flow of vapour from the evaporating substance to the surrounding space, and consequently the evaporating substance must have a higher temperature corresponding to the higher pressure in its immediate neighbourhood. But by the ordinary method of measuring vapour-tensions, where the body emitting vapour is placed in the vacuous space above the mercury in a barometer tube, no flow is possible, and hence the level of the mercury is a true measure of the tension.

Our results, we venture to think, show that with solids as with liquids this difference, even when rapid evaporation is taking place, is an extremely minute one. They also show that the ice-steam line of JAS. THOMSON (see diagram, fig. 1, M, L) is the upper temperature limit of ice at pressures below the critical one.

IV. *Researches on Spectrum Photography in relation to New Methods of Quantitative Chemical Analysis.*—Part I.

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Communicated by Professor Stokes, *Sec. R.S.*

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[Plate 3.]

Contents.

Introduction.

In the year 1878 I submitted to the Council of the Chemical Society a series of photographs executed with roughly constructed and imperfect apparatus, which showed variations in the number, length, and strength of the lines exhibited by some of the elements in the ultra-violet region. These photographs proved that by alterations in the exposure of the photographic plates certain impurities became visible in metals otherwise apparently pure, without any changes being noticeable in the spectra such as might be introduced by over-exposure. Thus by increasing the period of exposure three-fold the lines of iron were plainly seen in a spectrum regarded as that of pure aluminium. Gold obtained in as pure a condition as possible by parting gave evidence of the presence of silver. Two specimens of indium were examined: they both yielded the flame spectrum and the reactions usual with this metal; one of them showed no strong lines in the ultra-violet which could not be attributed to tin, lead, or cadmium; the other contained tin and cadmium in such proportions that the lines of these metals were more prominent than those of indium. A third specimen, prepared by Professor Richter, of Freiberg, yielded no spectral lines which could be attributed to any foreign metal.

A striking fact was noticed which, as will be seen, is exemplified in the case of magnesium, namely, that it is not always what appears to be the strongest and longest lines which first make their appearance when an impurity is discernible; and it was considered that many observations were necessary to ascertain which are the most persistent lines in the spectra of the elements. It was proposed to extend these observations, but their importance at that time did not seem to be appreciated. Almost all varieties of spark spectra have since been investigated, with the object of applying spectrum photography to the purposes of chemical analysis. From time to time opportunities have occurred when the process which has been gradually developed and rendered practical has been advantageously put into operation for the solution of questions upon which it would have been difficult to arrive at a decision by other means. For instance, I employed photographs of spark spectra in an examination of the rare earth bases separated from the mineral rhabdophane, for testing the purity of certain cerium compounds, and for estimating the amount of beryllium contained therein (Journal of the Chemical Society, vol. xli., Transactions, 1882, p. 210). It may therefore be considered that the method is so far complete in detail that it may be with advantage described for publication, notwithstanding that it is manifestly capable of improvement.

At an early stage of the inquiry it was found necessary to devise some convenient method of examining solutions of salts, and ascertaining the best material from which to form electrodes.

On a method of photographing spark spectra of the elements with solutions of their compounds.

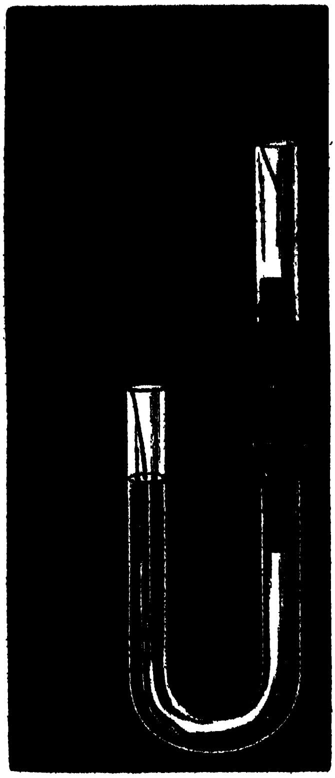
In the examination of substances for the purpose of chemical analysis it is undesirable to use metallic electrodes even when this is possible, owing to the fact that alloys are not generally of a homogeneous character, and the sparks pass from only one or two spots situated at the surface of the electrodes. A better means of judging of the composition of a mass, whatever be its dimensions, is afforded by solutions, because the amount of metal volatilised by the spark from mere points of the substance is exceedingly small—according to Messrs. LOCKYER and ROBERTS not more than 0·0001 gram. ('Studies in Spectrum Analysis,' LOCKYER, p. 226). Messrs. PARRY and TUCKER ('Engineering,' vol. xxvii., pp. 127, 429; and xxviii., p. 141) in the examination of iron and steel have made use of the charcoal points first used by BUNSEN, and by dispensing with a Leyden-jar have photographed the lines characteristic of metallic solutions in such a manner as to render them comparatively free from the air spectrum. This method does not appear satisfactory to me, because a prolonged exposure is necessary, and with the reduced density of the spark, fluted spectra of air make their appearance while the metallic lines are too short. These bands or flutings are composed of a large number of fine lines, the distance between

which increases with their increased refrangibility. They appear to be the spectrum of nitrogen of the first order. Messrs. PARRY and TUCKER's photographs are representations of the less refrangible rays only, such as are transmitted by glass lenses and prisms. For obtaining sparks from solutions, M. LECOCQ DE BOISBAUDRAN uses a concentrated solution contained in a glass tube, through the bottom of which passes a platinum wire which is fused into the glass. The wire is connected with the negative pole of an induction coil, the positive pole terminating in a wire placed conveniently above this. A well-known modification of this is the tube of MM. DELACHANEL and MERMET (SCHÜTZENBERGER's 'Traité de Chimie Générale,' vol. i., p. 284), but as the rays must necessarily pass through glass, it is not adapted to the examination of ultra-violet spectra. Any method is disadvantageous which necessitates the use of platinum electrodes, since the number of lines belonging to this element is embarrassing when they have to be eliminated from the spectra of other bodies. M. LECOCQ DE BOISBAUDRAN states that iridium points give no lines belonging to that metal, but this statement does not apply to photographed spectra of rays of high refrangibility obtained with a condensed spark, since about 180 lines have been counted in such a spectrum. Of all *metals* affording materials for electrodes gold appears to be the best; its spectrum is a weak one, containing comparatively few lines, it is an excellent conductor of electricity, and it is not attacked by solutions of metallic chlorides.

There is no very simple method of obtaining spectra free from air lines without resorting to either an inconveniently short or a too prolonged exposure, and if such existed it would be of doubtful advantage, since the number of the air lines in the ultra-violet region is not exceedingly large, and from their character they are easily recognisable. They are, in fact, a positive advantage, since by their well known appearance and positions they serve to fix the position of other lines. A simple and elegant apparatus devised by my colleague, Professor O'REILLY, consists of a metallic wick composed of a few twists of platinum wire, by which a BUNSEN flame is fed with the solution to be examined. Wicks of gold wire have been used by me, the metal projecting about one-eighth of an inch above the surface of the solution.

Of all elementary substances hitherto photographed, graphite yields the simplest spectrum ("Note on Certain Photographs of the Ultra-Violet Spectra of Elementary Bodies," Journal of the Chemical Society, vol. xli., p. 90). It shows lines and the edges of bands due to the air-spectrum to the number of sixty-six, of rays assignable to carbon it contains but twelve. Graphite being an excellent conductor of electricity, electrodes are made in the simplest possible manner by cutting plates of good Ceylon or Siberian graphite, tapering from one-fourth to one-eighth of a inch in breadth and three-eighths of an inch in length. Deep grooves or scratches are made in the sides of the lower electrode, which is inserted into the end of a small glass U tube containing the solution. The upper pole may be of metal or of graphite, either being fused into a glass tube or held in a screw clip. Platinum terminals from the coil are

connected with these. Capillary attraction keeps the lower electrode moist at its upper surface. Such electrodes have been constantly in use for producing sparks from which photographs of the spectra of saline solutions have been taken. Fresh electrodes have generally been made for each solution, but they may be used over again repeatedly provided they be well washed with hydrochloric acid and water. They wear away rather quickly, but their durability may be indicated by the fact that the same points have actually been employed continuously for a period of ten hours. Each of the two pieces of graphite should be sharpened so as to resemble a chisel, the edge of the one being exactly superposed above that of the other, while both are placed in a line with the slit; the movement of the spark is thus restricted to a direction backwards and forwards, but always in front of the slit and not to one side or the other.



Graphite electrodes (actual size).

The only two lines of a foreign element plainly visible in the spectrum of graphite are the first and third of the quadruple group in the magnesium spectrum, with wave-lengths 2794·4 and 2801·1. Faintly seen are the second and fourth lines, wave-lengths 2796·9 and 2789·6. This specimen of graphite came from Ceylon, and it was thought worth while to submit it to the ordinary method of analysis. Accordingly a considerable quantity was incinerated, and the ash was found to contain only magnesium and a trace of iron. After some hundreds of electrodes had been cut from

a lump of the material, it was found that the points showed more or less unmistakable indications of the presence of iron.

From this it appears that the same piece of the mineral may be of unequal purity.

Alterations in spectra caused by moistening the electrodes.

When electrodes of graphite are moistened with water, the only alteration remarkable in the spectrum is a lengthening of the short carbon lines, with wave-lengths 3590·0, 3583·5, 2836·8, 2836·0, 2511·6, 2508·7, and 2478·3, so that they extend from pole to pole. The other lines of carbon do not appear to be more than slightly lengthened, and in addition the line 4266·3 is much weakened. This change would receive a natural explanation by assuming the formation of some gaseous or at least volatile carbon compound. If it be due to carbon dioxide, a similar change should occur when the electrodes are surrounded by pure oxygen. In order to put this matter to the test of experiment, pure dry oxygen was passed for some time through a glass tube in which were fastened electrodes of graphite, the end of the tube being closed by a plate of quartz. The change seen in the spectrum under these circumstances is the following:—

Wave-lengths.	
4266·3	This line does not appear altered.
3919·5	This line remains short.
3875·9	This line remains short.
3590·7	This line remains short.
2836·8 2836·0	These lines are so lengthened as to be rendered continuous from pole to pole.
2746·6 2640·0	These lines are somewhat lengthened.
2511·6 2507·8 2478·3 2297·6	These lines are lengthened so as to be rendered continuous.

A map to the scale of wave-lengths is appended (Plate 3).

In addition to the above, the following lines are seen with difficulty, a pair with wave-lengths about 3168·0 and 3166·3 is much lengthened, and a very nebulous ray, extending from about 2995 to 2990 ($\lambda = 2993$) is rendered much larger and more distinct. It will be seen that the two spectra differ considerably; it does not, therefore, seem at all probable that the change is due to the cause suggested. Indeed, it is not, as may be learnt by a study of the action of the spark on metallic electrodes. When photographs are taken of dry electrodes of copper the lines in the spectrum with lesser wave-lengths than 2766·2 and extending to 2243·5 are all short lines. When the electrodes are moistened with a solution of a metallic chloride, with hydrochloric acid, or even with pure distilled water, these lines are greatly lengthened, some of them so much so as to become continuous. When platinum, iridium, and gold electrodes are partially immersed in water, the same lengthening of the short

lines takes place as in the case of copper and graphite. The lines of iridium yielded by dry electrodes extend no more than one-fourth the distance towards the opposite pole. When one pole is a point of iridium and the other of copper, the two series of short lines are seen, but those of iridium only are visible when that metal constitutes the negative pole, though the copper lines are weak, but still visible, at the positive pole.

When an iridium point, as the negative electrode, is partially immersed in water all the short lines are increased in length, so that they stretch four-fifths of the distance towards the copper; if the copper be negative and immersed in water the copper lines are lengthened, and but few of those belonging to iridium are discernible. The same lengthening of short lines takes place when gold electrodes are moistened. We have but few facts which serve to indicate the constitution of the electric spark and the circumstances under which it is altered in character. It may, however, be affirmed, with some degree of truth, that an increased intensity of the spark, such as is gained by placing a jar in circuit, which is usually considered to correspond to an increase of temperature, causes an increased length in the short lines, and also that the more volatile the metal the longer the lines. It was considered necessary to ascertain the effect of heating the electrodes, and accordingly two points of iridium wire were taken as suitable. When the spark passed at the ordinary temperature, a photograph which received five minutes' exposure showed, beside the large number of short lines, a continuous band of rays extending through the whole spectrum. On the same plate, and immediately beneath this, another photograph was taken while the negative electrode was heated to the most brilliant incandescence by the oxy-hydrogen blowpipe. The spark continued to pass during the whole period of five minutes, as was made evident by the scintillations visible at the cooler electrode. This latter was kept at a temperature sufficiently low as to show no signs of redness. The passage of the spark, but for the scintillations, was otherwise invisible and almost noiseless. On turning down the gas the spark appeared of a pale blue colour, the electrode being at a red-heat, and it now emitted a moderate crackling sound, similar to that caused when no jar is in circuit. On turning the gas out the spark grew brighter as the metal became colder, till finally the usual brilliancy and the sharp crackling noise were resumed. The second photograph, with the same exposure of five minutes, showed barely a trace of the iridium spectrum; some feeble rays emitted by the flame of the blow-pipe were visible, but the chief feature of the photograph was the beautiful group of nitrogen lines seen when the spark is passed between points without a jar in circuit. The wave-length of the line at the commencement of one of these groups was 3369, the commencement of the second group was a line with wave-length 3211·6, the spectrum terminating with a line with wave-length 3062·4.

At a low red-heat the spark passes with only a moderate crackling sound, and the spectra photographed under these conditions consist of the two spectra of air—namely, those of the first and second order superposed.

The same photographs are obtained when only a small jar is used and the electrodes are cold.

From these experiments it is evident that cooling the electrodes intensifies the spark, and consequently lengthens the short lines, just as heating them causes the reverse effect. The reason why heating the negative electrode acts so powerfully upon the spark is because the high temperature of the pole discharges the jar (GUTHRIE's 'Magnetism and Electricity,' p. 84).

In addition to the lengthening of the carbon lines, hydrochloric acid and solutions of metallic chlorides yield a weak continuous spectrum, and also a series of closely placed lines or flutings, further details concerning which are given on page 59. The continuous rays are most noticeable in spectra of solutions of chlorides of the alkali metals and of aluminium chloride; the flutings, however, appear most prominent in concentrated solutions of aluminium chloride and of zinc chloride.

Dry electrodes in air showed in some instances a total absence of the lines, with wave-lengths 4266·3, 3919·5, 3881·9, 3875·7, 3870·7, 3590·0, 3585·0, 3583·5, 2746·6, and 2640·0, while 2836·8 and 2836·0 were exceedingly faint. These lines are marked on the map with a star. No reason can be assigned at present for this difference in the spectrum, as the lines 4266·3, 3919·5, 3881·9, 3875·7, and 3870·7 are visible when the atmosphere surrounding the poles is carbon dioxide, and the photograph was perfect as regards the other part of the spectrum. A difference in the strength of the spark may occasion some similar alteration, but such a cause did not operate in this case.

On the spectra of solutions of binary compounds.

A complete series of photographs of metallic salts, chiefly chlorides, corresponding to the electrodes already enumerated and examined, was executed (Journal of the Chemical Society, vol. xli., p. 90). The reason for taking chlorides was that as a rule they are among the most soluble of salts. The solutions examined consisted of the chlorides of magnesium, zinc and cadmium, aluminium, thallium, iron, cobalt and nickel, arsenic, copper, strontium and tin. The cadmium, copper, and stannous chlorides were made by dissolving the electrodes used for the series of photographs of the elements. The zinc chloride was made from a very pure specimen of distilled zinc, the only impurity it betrayed being a trace of cadmium. The arsenic trichloride was obtained from a very carefully purified specimen of arsenious oxide. The ferric chloride was prepared from fine pianoforte wire. The thallous chloride was a particularly pure specimen made from the metal used as electrodes.

Some preparations of barium and strontium chlorides made with great care were found to contain calcium. They were purified by treating saturated solutions with an equal volume of strong hydrochloric acid, whereby the chlorides are precipitated in a crystalline form, and by filtration through a plug of asbestos the acid liquid is separated therefrom, the crystals are washed with strong acid twice, after which

they are dissolved in water, and if necessary treated again in the same manner. Aluminium chloride was prepared from an especially pure specimen of ammonia alum, the aluminium hydroxide was precipitated by ammonia and most thoroughly washed with hot water, it was then dissolved in hydrochloric acid. The cobalt chloride was a carefully prepared laboratory specimen. This series of solutions is very complete, inasmuch as spectra of almost every character are represented and may be referred to. For instance, there are comparatively few lines in the magnesium spectrum, but these are of a strongly marked character and they are closely grouped together. Exceedingly short and some straggling and long lines occur in the zinc spectrum, while in cadmium we see the position of two of the lines to be so near together that there is some difficulty in distinguishing them. In the iron and cobalt spectra there is a multitude of long and short lines crowded together in groups. In arsenic and antimony we have examples of metalloid spectra, with a considerable amount of diffused rays and lines distributed with tolerable regularity throughout the whole ultra-violet region.

By the juxtaposition of spectra on the same plate, the lines assignable to each metal were compared with those yielded by a solution of its chloride. With but two exceptions the two series of spectra are identical line for line; in the case of iron the number of lines reproduced is over 600, and in that of cobalt over 500. The group of five fine lines constituting the most refrangible group in the spectrum of magnesium were exactly reproduced by the solution of the chloride along with the other lines characteristic of the metal. The dual lines of cadmium were in like manner plainly seen. The one sole difference between the spectra of metallic electrodes and those from the salts of the metals was the greater degree of continuity of the lines shown by the salts. Thus all the lines discontinuous in the spectrum of iron were continuous in that afforded by a solution of ferric chloride. The short lines of the metals potassium and sodium, which are weak, appear as short lines in the spectra of their chlorides, likewise some of the shortest lines which are at the same time strong lines in the spectrum of aluminium are seen as short lines in the spectrum of a concentrated solution of its chloride. These lines form a triplet group with wave-lengths 3612·7, 3601·2, and 3584·5. This last line appears to be strongest in the aluminium spectrum. Whether the short lines appear or not depends upon the amount of metal present in the solution.

Regarding the two exceptional cases above-mentioned, they are referable to two distinct causes—the first to the extreme shortness of the lines, and the second to the presence of an impurity. Zinc is a metal with a number of lines in its spectrum so short that they can be described as merely dots. Their wave-lengths are the following: 2526·3, 2521·3, 2514·7, 2508·7, 2490·4, 2485·9, 2427·0, and 2418·8. All specimens of zinc yield these very short lines, but solutions made from them do not. As anhydrous zinc chloride contains only 65 per cent. of the metal, and as no solution of the chloride used in the spark apparatus contained more than 25 per cent. of this salt,

it is easy to see that the absence of these lines may be accounted for by the small quantity of metal in the spark.

When zinc electrodes are moistened these very short lines become somewhat lengthened.

When aluminium electrodes were employed to obtain a photograph of its spectrum (Journal of the Chemical Society, vol. xli., p. 90), a number of short lines were exhibited which did not appear in the spectra of solutions of the chloride. The wave-lengths of the aluminium lines as seen in solutions are the following: 3960·9, 3943·4, 3612·7, 3601·2, 3584·5, 3092·2, 3081·5, 3056·8 (faintly), 2815·6, 2659·8, 2651·7, 2631·0 (a group of five lines), and lastly 2567·5.

Nearly all the remaining short lines are caused by the presence of iron, a fact which may be easily ascertained by prolonging the exposure of the photographic plate. Commercial aluminium may contain as much as 2 per cent. of iron, and as the iron lines are strong, this quantity modifies the appearance of the spectrum to such a degree as to give it a considerable resemblance to that of iron, but there is the widest possible difference between the spectra of the two metals. J. L. SCHÖNN, who examined the ultra-violet spectra of several metals with Iceland spar prisms and a fluorescent eye-piece, has remarked on what he believes to be a similarity between the aluminium and iron spectra (WIEDEMANN'S Annalen, vol. ix., p. 483; vol. x., p. 143). It is possible that the likelihood of iron being contained in the aluminium had escaped his notice, or that he was not prepared to recognise the iron lines when they were somewhat altered in appearance by the metal being present in small quantities.

Mr. J. NORMAN LOCKYER, referring to the differences in the appearances of the spectrum of the same element under different conditions, has shown that by diminution of pressure under which the spark is taken, certain short lines disappear, while longer lines remain. Further, he states in reference to the spectra of chemical compounds: "It was found in all cases that the difference between the spectrum of the chloride and the spectrum of the metal was:—*That under the same spark conditions the short lines were obliterated, while the air lines remained unchanged in thickness*" (Phil. Trans., clxiii., p. 253, 1873).

It is obvious that this statement cannot be applied to solutions of chlorides examined in the manner described in the preceding pages, and the reasons for our different conclusions are the following. The short lines are not visible in the spectra of dry metallic chlorides, because the quantity of metal present in the spark is too small. Thus the spark does not pass from the chloride because it is a very bad conductor, or through it because it does not form a continuous and homogeneous covering to the metallic electrode, but only past or between the particles of the salt from the surface of which portions are volatilised. The conditions are quite different when metallic solutions are used; the salt is equally diffused throughout the liquid, which forms a continuous coating to the electrode through which the spark must pass, and

as the electrode is nearly immersed in water the intensity of the spark is increased. Hence, though the quantity of the element present in the solution might be insufficient to render the short lines visible with dry electrodes, the opposite effect introduced by the presence of water is sufficient to compensate for this.

In order to ascertain whether the elements oxygen and sulphur in combination could yield spark spectra, and whether insoluble compounds could be made to yield metallic lines to the spark, ferric oxide and ferrous sulphide were examined. The substances were finely powdered and mixed with glycerine to prevent them being dispersed by the spark too rapidly. No iron lines were detected on the photographic plate exposed to the action of the spark for the normal period of two minutes, though there was a good photograph of the graphite electrodes and the air spectrum.

To test the behaviour of insoluble but volatile substances, thallous chloride was treated in the same way, and the lines with wave-lengths 3778·4 and 3518·8 were rendered weakly. In no case did the non-metallic constituents cause any variations in the spectra. Hence we may conclude that:—*Insoluble and non-volatile compounds do not yield spark spectra when diffused in liquids.*

Several attempts to obtain a spectrum from selenium, selenic acid, and sodium selenate have proved unsuccessful.

On the spectra of ternary compounds.

In an examination of ternary compounds the salts examined were sulphates, nitrates, and phosphates of magnesium, cadmium, zinc, aluminium and iron. The bases of these salts were, as in the former case, prepared from the metallic electrodes, the aluminium and iron compounds being prepared from pure aluminium hydroxide and ferric oxide. The ferric oxide was obtained by heating pure ferrous sulphate with sodic sulphate, lixiviating with hot water, and washing many times therewith by decantation. The oxide was then dissolved in the requisite acids. The observations recorded in the case of metallic chlorides apply equally well to sulphates and nitrates, while the difficulty of obtaining a spectrum from ferric phosphate diffused in glycerine was equal to that in the case of the sulphide. When dissolved in hydrochloric acid the phosphate displayed the lines of iron only. A similar observation was made with cerous phosphate. Ammonium chloride was made into a saturated solution, and its spectrum photographed. There was no striking alteration in the spectrum of the graphite poles, but a careful examination showed that a group of lines which appear to be caused by the presence of nitrogen was greatly strengthened and made more prominent. These lines or flutings extend from about wave-lengths 3881·8 to 3829·0. The following facts point to their origin. They are never seen when metallic electrodes are used, wet or dry; they are therefore not air-bands. They are not seen when carbon electrodes are immersed in oxygen, and cannot therefore be due to carbon dioxide. They are always seen when solutions are used with carbon electrodes,

and are particularly strong when the carbons, wet or dry, are immersed in pure and dry nitrogen. When the negative pole is a metallic wire immersed in water, and the positive a point of carbon, the lines are strongly developed at the positive pole only. When the current is reversed the spectrum is not so strong. The bands not being traceable to carbon dioxide, to a hydrocarbon or to air, and being at the same time produced with great intensity when graphite is surrounded by pure and dry nitrogen, they must belong either to a modified spectrum of carbon or to a compound of carbon with nitrogen, such as cyanogen. In order to test the probability of these flutings belonging to the cyanogen spectrum, it was thought desirable to try the effect of cyanides in solution, using for the purpose metallic electrodes. A sample of perfectly pure cyanide of potassium was prepared by dissolving the commercial salt in alcohol and crystallising it therefrom. A saturated solution of the salt gave no sign of the bands when submitted to the spark passed between gold electrodes. A hot saturated solution of mercuric cyanide was similarly treated, the negative electrode being gold and the positive carbon, the gold being all but immersed in the solution. A strong mercury spectrum was obtained, but the flutings were absent except at that point where they might be expected under ordinary circumstances, namely, at the positive carbon electrode, and even here they appeared as faintly as if water and not a saline solution had been used. Cyanides therefore do not yield this spectrum.

It is remarkable that certain solutions which do not contain nitrogen in any form favour the formation of these bands, as for instance chlorides generally and zinc chloride as a saturated solution particularly. In order to ascertain whether the strength of the spectrum is dependent on the proportion of saline matter present, I examined a series of solutions containing varying proportions of calcium chloride, from $\frac{1}{100}$th to $\frac{1}{10000}$th, with the result that the strength of the bands was found to increase with the strength of the solutions.

There are two lines in this spectrum of graphite (Journal of the Chemical Society, vol. xli., p. 90) which apparently commence these flutings; their wave-lengths are 3875·7 and 3870·7, and yet two others which are absent from the spectrum of carbon when taken in oxygen with wave-lengths 3585·5 and 3584·0.

Professors Liveing and Dewar give the general appearance of the cyanogen spectrum as observed by them (Proc. Royal Soc., vol. xxxiv., p. 123), and the group of lines between K and L, extending from 3883 to about 3830, much resembles the first group mentioned above, while the second series near N, lying between 3580 and 3590, approximate closely to the second pair of lines to which I refer; nevertheless I cannot attribute these lines to cyanogen because they are not obtainable from cyanides.

That these lines are absent from the carbon spectrum when taken under certain conditions is not conclusive evidence that they are not carbon lines, because, as I have already shown, the lines 4266·3, 3919·5, 3881·9, 3875·7, 3870·7, &c., are occasionally absent, yet these unquestionably belong to the carbon spectrum.

Borates and silicates.—When soluble borates or boracic acid, soluble silicates, silicofluorides, or hydrofluosilicic acid are submitted to the spark, line spectra of the elements boron and silicon result, as I have already shown in a paper submitted at a recent date to the Royal Society. I then showed reason for suspecting that several lines attributed to carbon by Professors LIVEING and DEWAR (Proc. Roy. Soc., vol. xxxiii., p. 403) are in reality lines of silicon. In their communication, to which I have had to refer for a description of the cyanogen spectrum, I find (*loc. cit.*, vol. xxxiv., p. 123) a list of lines seen in the arc spectrum which they assign to carbon. I give for comparison with these the wave-lengths of the most prominent lines in the spectrum of silicon.

Approximate wave-lengths of carbon arc-lines (LIVEING and DEWAR).	Wave-lengths of silicon lines (HARTLEY).
2434·8	2435·5
2478·3	...
2506·6	2506·3
2514·1	2513·7
2515·8	2515·6
2518·8	2518·5
2523·9	2523·5
2528·1	2528·1
2881·1	2881·0

From this it seems probable that the only carbon line in the ultra-violet arc spectrum is the strongest of the series with wave-length 2478·3.

On spectra obtained from dilute solutions and on alterations caused by prolonged exposures.

The effect of prolonged exposure of the sensitive plate to the spark produced from weak solutions is a lengthening and strengthening of the metallic lines. It is reasonable to suppose that the density of the lines, or otherwise the intensity of chemical action, is a function of the concentration of the liquid and the period of exposure in all cases, but with very dilute solutions this certainly appears not to be exactly the case. Thus a solution containing six per cent. of calcium chloride exposed for half a minute, yields a spectrum with the calcium lines, showing a greater density than that produced by a solution containing one per cent. of the salt exposed for three minutes. Probably the conducting surface of the electrodes is not large enough to be kept constantly moistened, and so maintained in a condition capable of presenting a sufficient quantity of the solution to the spark. It has been observed in all cases that the effect of diluting solutions is to weaken the metallic lines; and, further, with one or two exceptions, to shorten even the longest and strongest lines until they finally disappear.

On the presence of impurities in certain spectra.

It has been noticed that when solutions are photographed with a Leyden-jar in circuit, and when these solutions are concentrated, two very fine and continuous lines of a strong character, which have been identified with the metal calcium, are occasionally seen. Their wave-lengths are 3967·6 and 3933, and they are designated H and K of the solar spectrum on M. CORNU's map. Sometimes a second pair of lines, with wave-lengths 3736·5 and 3705·5, are visible. They appear in strong solutions of cadmium chloride, ferric chloride, cobaltic chloride, and ferric nitrate. They are of very feeble intensity in strontium chloride and barium chloride, and then only the first pair are visible, attenuated and shortened.

They appear as short lines and rather feeble in photographs taken from electrodes of SIEMENS-MARTIN steel, and from electrodes of iridium. The most remarkable fact about these lines is, that they are not visible in the spectra of the alkaline chlorides, lithium, sodium and potassium, which were prepared from neutral solutions. Nor are they to be seen in the spectrum of aluminium chloride. They do not appear in photographs of the spectra of graphite points, either when dry or moistened with water, but they are seen as very faint and short lines when graphite points are moistened with hydrochloric acid. They are fairly well seen in solutions of ferric sulphate, prepared by dissolving the oxide in sulphuric acid. For some time the origin of these lines was a source of some perplexity, for although they had been ascertained to be calcium lines, yet they made their appearance on occasions when they were least expected, and when their presence could not be accounted for. Acids were always found to contain traces of calcium, and particularly hydrochloric acid; but the quantity was so minute as to be capable of detection only by photographing the spectrum. A quantity of pure hydrochloric acid was distilled in a previously very carefully cleaned glass retort and collected in a glass receiver. The operation was so conducted that by no possibility could any of the liquid have been carried over, except in the form of vapour. Still, in photographs of the spectrum of the distillate, the calcium lines were detected but little diminished in intensity. When platinum wire was substituted for graphite electrodes the lines were again present.

There is much evidence that the action of the acid upon the glass vessels dissolves out of the glass a small portion of calcium. The calcium lines are particularly strong, while those of the alkalies are just as weak. It is probable that the aluminium which does not appear would be less likely to enter into solution, and at the same time as the lines are not so strong as the calcium lines they would not be so easily rendered visible. Messrs. PARRY and TUCKER found that various samples of iron, after solution in hydrochloric acid and further chemical treatment, gave evidence of the presence of calcium. In this case the calcium probably was introduced by the acid, or at least a portion of it, and the strength of the lines was increased by the concentration of the solutions. It must, however, not be overlooked that both SIEMENS-MARTIN

steel and dry iridium electrodes yielded spectra with the calcium lines. In the case of the steel, either minute traces of slag are diffused through the metal or traces of calcium occur as one of its constituents. The iridium points were new, and it is just conceivable that traces of lime from the crucible in which the metal was fused were adhering to its outer surface. The specimens of graphite contained magnesium and iron, but no calcium. This conclusion was the result of spectrum observations, and likewise of a careful analysis of the ash made by the usual methods. Many of the photographs of solutions of metallic chlorides, as for instance the chlorides of copper and manganese, show no trace of the calcium lines. I have now no doubt that when the calcium salt is not derived from the action of acids on glass vessels, the spectra are contaminated by dust floating in the air, and that the calcium lines are to the ultra-violet region in this respect what the sodium lines are to the visible spectrum.

The alkali-metals yield but feeble ultra-violet spectra, otherwise the sodium lines would likewise be ever present.

The evidence that the calcium lines are due to dust is the following: two metallic electrodes, alloys of copper and silver, were filed up so as to present bright points; they were each opposed to a graphite electrode, and the spark was made to pass from the metal to the carbon; in the one case the calcium lines were either very faint or invisible, but in the second they were very strong. The interval of time between the taking of the photographs was not more than half a minute. The calcium lines were strongest at the metallic or negative pole.

The following facts have been established by the foregoing observations:—

When carbon or metallic electrodes are moistened the short lines are lengthened.

With very few exceptions the non-metallic constituents of a salt do not affect the spark spectra of solutions.

Insoluble and non-volatile compounds do not yield spark spectra.

The solution of a metallic chloride yields spectral lines identical in number and position with the principal lines of the metal itself.

Short lines become long lines, but otherwise their character is identical, whether the spectra are produced by metallic electrodes or solutions.

The effect of diluting solutions of metallic salts is first to weaken and attenuate the metallic lines, then with a more extensive dilution to shorten them, the length of the longest and strongest lines generally decreasing until they finally disappear.

Accidental differences in the passage of the spark or in the time of exposure of the photographic plate, when the normal period is from half a minute to five minutes, do not cause sensible variations in spectra obtained from the same substance.

V. *Measurements of the Wave-lengths of Lines of High Refrangibility in the Spectra of Elementary Substances.*

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Communicated by Professor STOKES, *LL.D., Sec. R.S.*

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[PLATES 4–6.]

INTRODUCTION.

IN the Philosophical Transactions, Vol. 170, pp. 257–274, 1879, one of us has described, in conjunction with Mr. A. K. HUNTINGTON, the first use of dry gelatine films, sensitised with silver bromide, for photographing ultra-violet spectra; and the application of the almost continuous spectrum emitted by the metals iron, nickel, and copper to the purpose of examining the ultra-violet absorption spectra of organic compounds. These researches, up to the present* time, have been prosecuted under considerable disadvantages, owing to the impossibility of describing accurately either absorption or emission spectra, by reason of the data for calculating wave-lengths being unfortunately insufficient. The object of this work is to give an exact description of the photographed spectra of some sixteen elementary substances, and to place on record the wave-lengths of so large a number of well-defined metallic lines, together with such other measurements of spectra, that subsequent workers may experience no difficulty in constructing interpolation curves capable of yielding fairly accurate numbers representing wave-lengths. The first measurements of rays of high refrangibility made by means of photography were the determinations of the wave-lengths of the lines of cadmium by M. MASCART ('Annales de l'École Normale,' vol. iv., 1867). He made use of a NOBERT's grating, a goniometer, and a photographic eye-piece. In addition to the splendid map of the ultra-violet portion of the solar spectrum given us by M. CORNU, we have wave-lengths most carefully calculated for a series of lines in the spectra of the metals cadmium, magnesium, aluminium, and zinc ('Annales de l'École

* Proc. Roy. Soc., xxxi., pp. 1–26. Journ. Chem. Soc., xxxix., pp. 57–60, and pp. 111–128; xxxvii., pp. 676–678, &c.

Normale,' vol. ix., 1880; and 'Archives des Sciences Physiques et Naturelles de Genéve,' (3), ii., pp. 119–126). Messrs. LIVEING and DEWAR, with an improved goniometer and a RUTHERFURD grating, have estimated the wave-lengths of the lines of carbon by a modification of MASCART's method (Proc. Roy. Society, vol. xxxii., 1882). As each line must be cussed independently of the rest of the spectrum and photographed on a different plate, and as the relative positions of the lines on the photographs are varied by very slight alterations in focus and by the removal and replacement of the plates, we have been led to apprehend that there are grave objections to this method of manipulation. The process, moreover, appears to be a lengthy one. In accordance with these views, which are the result of long experience, we have preferred to employ a method similar to that of CORNU.*

As much of each spectrum as possible is photographed on one plate, and together with this a series of ideal lines or plain reflections of the slit, each corresponding to a measured angular deviation, from which a scale of wave-lengths may be calculated.

The instruments.—For the production of spectra we have used one of Mr. RUTHERFURD's small diffraction gratings. This was mounted on a stand made six years ago by Mr. BROWNING, the telescope and collimator of which are fixed, and the grating movable. A tangent screw is used to give an angular motion to the grating, and measurements are made upon a divided arc of 9 inches radius. The position of the grating can be fixed at any required angle without the necessity of clamping. The original telescope and collimator fitted with glass lenses were removed from the stand, and replaced by a collimator and two lenses of 36 inches focus for the D lines. The material of the lenses was quartz, one of right- the other of left-handed rotation. Unless the lenses are approximately of the same thickness and correct one another, all fairly strong lines, whether produced by a prism or grating, are liable to be doubled, that is to say, accompanied by faint images resembling the "ghosts" that are seen when strong lines are viewed with a RUTHERFURD grating. The distance of the lenses from the grating was about 3 inches. Their considerable focal length gives an approximately flat field over a wide range of the spectra. Instead of a telescope, the photographic camera described in the Scientific Proceedings of the Royal Dublin Society, vol. iii. ("Description of the Instruments and Processes employed in Photographing Ultra-violet Spectra," 1881, W. N. HARTLEY) was altered and adjusted for use with the grating. The table upon which it was supported consisted of a large and massive slab of slate, immovably set upon solid stone foundations.

In taking photographs from metallic electrodes, it is of some importance that the spark be always in exactly the same position with regard to the slit, otherwise the relative positions of the lines are liable to variation; we have, therefore, always used an electrode of cadmium immovably fixed opposite to the slit, the other points of metal

* It may be mentioned that the best method for the determination of wave-lengths, and the precautions to be taken with regard to the accurate measuring of the positions of the lines, was the subject of several months' investigation by one of the authors, W. N. H.

being on the same stand below it, the arm by which they were held being capable of such motion that they could be renewed or replaced without disturbance of the fixed cadmium point. The image of the spark was projected on to the slit by a lens of 3 inches focus, which was also immoveably fixed. All lines photographed could be measured with reference to those in the spectrum of cadmium. The slit of the collimator, which was not more than $\frac{1}{1000}$th of an inch in width, was protected from dust by being covered with a thin plate of quartz. Photographs were taken of the first order, both to the right and to the left of the plain reflection of the slit. The average period of exposure was an hour to an hour and a half.

The spectrum of one order overlaps that of another, but this is of no consequence, because the lenses being uncorrected for chromatic aberration, and the spectra of different orders having different foci, only one image is visible on the developed plate.

The developer used was made with pyrogallic acid and potassium bromide.

Method of measuring the spectra.—When a series of photographs had been obtained, the distances between the lines of the various spectra were accurately measured by means of a microscope and a dividing engine with a screw 30 inches in length. The arrangement by which a forward motion only is given to the screw was thrown out of gear, and a divided wheel, 4 inches in diameter, with a handle attached, was placed at the opposite end of the screw. Each division on the wheel as it passed the pointer registered a longitudinal motion of the stage equal to $\frac{1}{5000}$th of an inch, and it was easy to read to $\frac{1}{10000}$th. The measurements were certainly accurate to $\frac{1}{2000}$th of an inch even when working on lines of different intensities. The microscope, which was placed on the stand of the machine, had a magnifying power of 25 diameters. Less than this is insufficient, and more is unsatisfactory, except in special cases.

A plate-glass stage was fitted to the carrier of the dividing engine, and by means of screw clamps the photographs were secured to this. The photographs must be so adjusted that a line passing from end to end of the spectrum and dividing it into two parts longitudinally lies parallel to the axis of the screw, otherwise the lines will not all occupy the same position with respect to the cross lines in the field of the microscope.

Again, it is necessary that the photographs be taken on patent plate-glass so as to present a perfectly flat surface, and the plates are more suitable if selected with regard to equality of thickness at each end. Such curvature as is ordinarily to be seen in flattened crown-glass would yield inaccurate measurements, and if the plates be not of the same thickness throughout, the two ends of the spectrum, when the plate was in position on the glass stage, would not be in the same horizontal plane, and so by the motion of the screw the lines of the spectrum would soon travel out of focus. Freedom from spherical aberration, which facilitates the measurement of the lines, is secured by using lenses of unusually long focus.

Three plates were taken for each spectrum, the first included all rays lying between the cadmium lines 6 and 12 ($\lambda=4676 \cdot 7$ and $3249 \cdot 5$), the second included Cd 11 and

Cd 18 (λ=3402·9 and 2572·2), and the third Cd 16 and Cd 26 (λ=2836·1 and 2145·7). Between these limits all the lines were accurately focussed. It will be seen that these plates overlap, so that on one series of photographs a certain number of lines may be measured twice over if necessary. The accurately measured portions of the spectra were 4 to 5 inches in length on each plate, so that the whole spectrum extended from 12 to 15 inches, and each inch was easily divisible into 10,000 parts. The measurements of the lines were made when the cross wires coincided with or equally divided the ends of the lines. This is necessary, as many lines are extremely short and cannot be measured at any other point. Lines continuous from pole to pole are comparatively few in number. "Ghosts" of very strong lines appear in the photographs of diffraction spectra; they are generally easy of recognition, but should it happen that by reason of a crowd of lines they are not easily distinguishable, they may be eliminated by comparing the diffraction with the prism spectrum.

No metallic line which is not common to both spectra has been measured. It is difficult to identify the lines rendered by a prism spectroscope when the original photographs only are examined, on account of the necessity of employing the microscope, which enables one to view only a small portion of the spectrum at one time. For the convenience of identifying the lines and registering their wave-lengths two sets of enlargements have been used, each containing about eight spectra, 36 inches in length. For the purpose of registering the wave-lengths of the air-lines and the very numerous lines of iron, enlargements 8 feet in length have been made.

Determination of Wave-Lengths.

Method of Working.—The determination of the wave-lengths of the lines in any photograph becomes very simple if we know the value of their linear positions on the plates in terms of the scale of the goniometer, and so be in a position to find their deviations. M. Cornu has described a method by which he determined the deviation of some of the lines in his photographs of the ultra-violet solar spectrum. After photographing a spectrum he moved his plate so as to obtain an impression of the image of the slit on the sensitised film on each side, and very close to the line he wished to measure. The points on the arc to which the images corresponded being known, the deviation of the line could be determined from them, since the images were sufficiently close together for linear distances between them to be taken as proportional to angular distances. We have followed the principle of this method. As stated in the introduction to this paper, after photographing a spectrum, the grating was moved so as to reflect on to the sensitised film a series of images of the slit, corresponding to equi-distant fixed points on the arc of the goniometer. In this way the spectrum was obtained, together with a number of images of the slit, disposed at regular intervals along its length, the images serving as fiducial lines, the deviations of which were known with all the accuracy afforded by the scale of the goniometer.

The wave-lengths corresponding with these deviations were calculated, and those of the spectral lines were determined from them by interpolation.

Calculation of the wave-lengths corresponding to the fiducial lines.—For the convenience of future reference the photographs of the three portions of the spectrum for which the plates were focussed, will be designated by the numbers 6–12, 11–18, and 17–26. These numbers have been assigned to the prominent cadmium lines by MASCART, and they serve to fix the limits of the less and more refrangible ends respectively of each portion of the spectrum.

The grating was placed so that to get the three portions of the spectrum into focus it was only necessary to move the plate-carrier, without shifting either the lens of the camera or the grating itself. The calculations of the wave-lengths corresponding to the deviations of the fiducial lines were made from the formula

$$a\,(\sin i + \sin \delta) = n\lambda,$$

where n is positive for the diffracted rays on the same side of the reflected ray as the normal, and negative for those on the other side, δ_n is the deviation of the line reckoned from the normal to the grating, and positive when situated on the same side of the normal as the incident ray; i is the inclination of the normal of the grating to the incident ray. This is, of course, at once determined from the position of the grating, when its normal is parallel to the incident ray. This position was found in the following way. A piece of plate-glass was placed in the small mahogany box at the end of the collimator tube (for a description of which see the paper already referred to in the Scientific Proceedings of the Royal Dublin Society, p. 10) in such a way as to allow the light from the slit to pass through it to the grating, and to reflect upwards the image of the slit reflected back from the grating. The grating was moved about till the image coincided with the slit. A wide slit was used to ascertain approximately the position of the grating, the slit was then narrowed as far as necessary, and the grating accurately adjusted. After this adjustment neither slit nor grating was moved during the time the whole of the series of photographs were being taken.

The deviation δ_n is not measured directly. One measures $\frac{i-\delta}{2}$ or $\frac{\delta-i}{2}$ according as the spectrum observed is to the right or to the left of the regularly reflected image of the slit. The following calculation of the wave-lengths corresponding to the fiducial line I. for the spectra to the left will serve as an example.

Position of grating when the normal was parallel to the incident ray	51° 3′ 18″
Position of grating when the spectrum was being photographed.	42° 22′ 25″
Point on the arc corresponding to fiducial line I. . .	32° 43′ 41″

$$51^\circ\ 3'\ 18''-42^\circ\ 22'\ 25''=8^\circ\ 40'\ 53''$$
$$=i$$
$$42^\circ\ 22'\ 25''-32^\circ\ 43'\ 41''=9^\circ\ 38'\ 44''$$
$$=\frac{\delta-i}{2}$$
$$\text{whence } \delta=27^\circ\ 58'\ 21''$$

δ is negative and n is negative, $a=\cdot00146859$

therefore $\lambda=4671\cdot7$

The constant of the grating was determined by means of the sodium lines D_1 and D_2 of the first and second order of spectra on both sides of the regularly reflected rays, with the following result:—

	D_1.	D_2.
First order to the right	0·00146831	0·00146855
Second order.	0·00146858	0·00146861
First order to the left.	0·00146853	0·00146900
Second order.	0·00146861	0·00146855

Mean=0·00146859

The following table gives the wave-lengths corresponding to all the fiducial lines, together with their angular measurements.

SPECTRA to the left.

Fiducial lines.	Angular measurements.	Corresponding wave-lengths.
	° ′ ″	
I.	32 43 41	4671·7
II.	33 4 21	4515·27
III.	33 25 1	4357·85
IV.	33 45 41	4199·49
V.	34 6 21	4040·2
VI.	34 27 1	3880·0
VII.	34 47 41	3718·93
VIII.	35 8 21	3557·00
IX.	35 29 1	3394·22
X.	35 49 41	3230·63
XI.	36 10 21	3066·27
XII.	36 31 1	2901·14
XIII.	36 51 41	2735·26
XIV.	37 12 21	2568·68
XV.	37 33 1	2401·41
XVI.	37 53 41	2233·46
XVII.	38 14 21	2064·87
XVIII.	*	

SPECTRA to the right.

Fiducial lines.	Angular measurements.	Corresponding wave-lengths.
	° ′ ″	
IX′.	37 33 1	3426·02
X′.	37 12 21	3250·71
XI′.	36 51 41	3075·71
XII′.	36 31 1	2901·00
XIII′.	36 10 21	2726·61
XIV′.	35 49 41	2552·57

The fiducial lines I. to X. inclusive, photographed on the spectra to the left of the regularly reflected image of the slit, were contained on Plates 6–12, IX. to XV. on Plates 11–18, and XII. to XVII. on Plates 17–26.

Photographs of the portion 11–18 only were taken of the spectrum to the right.

It will be seen from the table that the differences between consecutive readings to the left are the same as those to the right for the same part of the spectrum.

The reading on the goniometer when the spectra to the right were being photographed was 30° 39′ 38″.

Determinations of the wave-lengths of the spectral lines.—The linear positions, or as these will be hereafter termed the scale numbers of the fiducial lines, and spectral lines of each plate were carefully measured off by means of the dividing engine in hundredths of an inch and fractions thereof. Interpolation curves were constructed from the wave-lengths and the scale numbers of the fiducial lines. The wave-lengths of the metallic lines were determined from these curves.

The scale numbers of the fiducial lines of the various plates are given in the following tables.

TABLES giving in linear measurements the positions of the fiducial lines on the several spectra photographed.

Spectra to the left.

Portions 6–12. Measurements in hundredths of an inch.

(1)

Fiducial lines.	Indium.	Aluminium.	Thallium.	Copper.	Carbon.	Magnesium.	Arsenic.	Mean.
I.	0·000	0·060	..	..	..	..	..	0·030
II.	37·250	37·250	37·250	37·250	..	37·250	37·250	37·250
III.	74·305	74·185	74·365	74·260	74·300	74·375	74·380	74·300
IV.	111·585	111·505	111·575	111·575	111·595	..	111·54	111·575
V.	148·825	148·840	148·795	148·745	148·835	148·845	148·780	148·815
VI.	186·225	186·225	186·190	186·380*	186·165	186·186	186·250	186·210
VII.	223·695	223·725	223·620	223·660	223·675	..	223·740	223·685
VIII.	261·400	261·365	261·295	261·455	261·335	261·240	261·450	261·350
IX.	299·155	299·175	299·09	299·135	299·110	299·110	..	299·145
X.	337·260	337·200	337·175	337·350	337·280	..	..	337·255

(2)

Fiducial lines.	Lead.	Tellurium.	Tin.	Mean.
I.				
II.	37·250	37·250	..	37·250
III.	74·555	74·440	74·345	74·500
IV.	111·700	111·750	111·660	111·700
V.	149·070	149·025	149·030	149·050
VI.	186·430	186·465	186·370	186·450
VII.	223·980	223·985	224·020	223·985
VIII.	261·700	261·660	261·695	261·680
IX.	299·570	299·585	299·540	299·575
X.	337·655	337·530	337·600	337·590

(3)

Fiducial lines.	Iron.	Nickel.	Mean.
I.			
II.	37·250	37·250	37·250
III.	74·350	74·240	74·295
IV.	111·485	111·440	111·465
V.	148·665	148·620	148·645
VI.	185·965	186·050	186·010
VII.	223·490	223·510	223·500
VIII.	261·110	261·265	261·190
IX.	298·885	299·100	298·995
X.	336·880	337·050	336·965

TABLES giving in linear measurements the positions of the fiducial lines on the several spectra photographed (continued).

Spectra to the left.

Portions 11–18. Measurements in hundredths of an inch.

(4)

Fiducial lines.*	Magnesium.	Silver.	Tin.	Iron.	Carbon.	Bismuth.	Mean.
XV.	0·000	0·000	0·000	0·000	0·000	0·000	0·000
XIV.	48·625	48·640	48·760	48·650	48·585	48·600	48·650
XIII.	98·345	98·035	98·260	98·305	98·180	98·350	98·275
XII.	148·925	148·965	149·040	148·965	148·895	149·150	148·965
XI.	200·750	200·590	200·795	200·665	200·860	200·710	200·730
X.	253·650	253·620	253·605	253·715	253·500	253·710	253·620
IX.	308·045	308·085	308·090	308·250	308·110	308·150	308·115

(5) (6)

Fiducial lines.*	Tellurium.	Arsenic.	Zinc.	Mean.	Lead.	Antimony.	Mean.
XV.	0·000	0·000	0·000	0·000	0·000	0·000	0·000
XIV.	48·815	48·705	48·750	48·757	48·685	48·805	48·745
XIII.	98·730	98·610	98·515	98·670	98·560	98·650	98·605
XII.	149·520	149·500	149·350	149·457	149·320	149·560	149·440
XI.	201·395	201·405	201·250	201·350	201·050	201·380	201·215
X.	254·515	254·365	254·450	254·443	254·360	254·250	254·305
IX.	308·975	308·880	308·865	308·907	308·815	308·715	308·765

(7) (8) (9)

Fiducial lines.*	Indium.	Thallium.	Mean.	Aluminium.	Copper.
XV.	0·000	0·000	0·000	0·000	0·000
XIV.	49·170	49·010	49·090	48·705	48·640
XIII.	99·055	99·160	99·108	98·310	98·180
XII.	150·205	150·230	150·218	149·065	148·780
XI.	202·320	202·500	202·410	200·970	200·640
X.	255·645	255·740	255·693	254·010	253·530
IX.	310·305	..	310·305	308·340	308·040

* These lines were measured in the reverse to the usual order.

TABLES giving in linear measurements the positions of the fiducial lines on the several spectra photographed (continued).

Spectra to the left.

Portions 17–26. Measurements in hundredths of an inch.

(10)

Fiducial lines.	Iron.	Antimony.	Mean.
XII.	0·000	0·000	0·000
XIII.	85·805	85·790	85·800
XIV.	168·005	168·020	168·010
XV.	246·635	246·700	246·700
XVI.	322·030	321·960	322·000
XVII.	394·570	394·150	394·000

	(11)	(12)	(13)
Fiducial lines.	Tin.	Thallium.	Indium.
XII.	0·000	0·000	0·000
XIII.	85·725	85·685	85·625
XIV.	168·080	167·810	167·715
XV.	246·435	246·400	246·055
XVI.	321·800	321·615	321·310
XVII.	393·910	393·880	393·140

Spectra to the right.

Portions 11–18. Measurements in hundredths of an inch.

	(1)	(2)	(3)
Fiducial lines.	Magnesium.	Thallium.	Zinc.
I.	0·000	0·000	0·000
II.	101·530	101·660	102·140
III.	198·000	198·630	198·965
IV.	289·540	290·175	290·220
V.	376·500	377·270	377·430
VI.	458·905	459·760	459·950

TABLE of means and numbers adopted as correct measurements of the fiducial lines on each plate.

Spectra to the left.

Portions 6–12. Measurements in hundredths of an inch.

(1)

Fiducial lines.	Mean measurements.	Adopted numbers.	Intervals.
I.	0·03	0·15	
II.	37·25	37·20	37·05
III.	74·30	74·30	37·10
IV.	111·575	111·50	37·20
V.	148·815	148·80	37·30
VI.	186·21	186·20	37·40
VII.	223·685	223·70	37·50
VIII.	261·35	261·35	37·65
IX.	299·145	299·15	37·80
X.	337·255	337·15	38·00

(2)

Fiducial lines.	Mean measurements.	Adopted numbers.	Intervals.
I.	..	..	
II.	37·25	37·25	..
III.	74·50	74·40	37·15
IV.	111·70	111·65	37·25
V.	149·05	149·00	37·35
VI.	186·45	186·45	37·45
VII.	223·985	224·00	37·55
VIII.	261·68	261·70	37·70
IX.	299·575	299·55	37·85
X.	337·59	337·60	38·05

TABLE of means and numbers adopted as correct measurements of the fiducial lines on each plate (continued).

Spectra to the left.

Portions 6–12 (continued). Measurements in hundredths of an inch.

(3)

Fiducial lines.	Mean measurements.	Adopted numbers.	Intervals.
I.	..	..	
II.	37·25	37·20	..
III.	74·295	74·275	37·075
IV.	111·465	111·45	37·175
V.	148·645	148·725	37·275
VI.	186·01	186·10	37·375
VII.	223·50	223·575	37·475
VIII.	261·19	261·20	37·625
IX.	298·995	298·975	37·775
X.	336·965	336·95	37·975

Portions 11–18. Measurements in hundredths of an inch.

(4)

Fiducial lines.	Mean measurements.	Adopted numbers.	Intervals.
XV.	0·000	0·000	
XIV.	48·65	48·65	48·65
XIII.	98·275	98·30	49·65
XII.	148·965	148·95	50·65
XI.	200·73	200·70	51·75
X.	253·62	253·70	53·00
IX.	308·115	308·05	54·35

Table of means and numbers adopted as correct measurements of the fiducial lines on each plate (continued).

Spectra to the left.

Portions 11–18 (continued). Measurements in hundredths of an inch.

(5)

Fiducial lines.	Mean measurements.	Adopted numbers.	Intervals.
XV.	0·000	0·000	
XIV.	48·757	48·80	48·80
XIII.	98·67	98·60	49·80
XII.	149·457	149·40	50·80
XI.	201·35	201·30	51·90
X.	254·443	254·45	53·15
IX.	308·907	308·95	54·50

(6)

Fiducial lines.	Mean measurements.	Adopted numbers.	Intervals.
XV.	0·000	0·000	
XIV.	48·745	48·775	48·775
XIII.	98·605	98·55	49·775
XII.	149·44	149·325	50·775
XI.	201·215	201·20	51·825
X.	254·305	254·325	53·125
IX.	308·765	308·80	54·475

TABLE of means and numbers adopted as correct measurements of the fiducial lines on

Spectra to the left.

Portions 11–18 (continued). Measurements in hundredths of an inch.

(7)

Fiducial lines.	Mean measurements.	Adopted numbers.	Intervals.
XV.	0·000	0·000	
			49·05
XIV.	49·09	49·05	
			50·05
XIII.	99·108	99·10	
			51·05
XII.	150·218	150·15	
			52·15
XI.	202·410	202·30	
			53·40
X.	255·695	255·70	
			54·75
IX.	310·305	310·40	

(8) (9)

Fiducial lines.	Aluminium readings.	Adopted numbers.	Intervals.	Copper readings.	Adopted numbers.	Intervals.
XV.	0·000	0·000		0·000	0·000	
			48·70			48·625
XIV.	48·705	48·70		48·64	48·625	
			49·70			49·625
XIII.	98·31	98·40		98·18	98·25	
			50·70			50·625
XII.	149·065	149·01		148·78	148·875	
			51·80			51·725
XI.	200·97	200·90		200·64	200·60	
			53·05			52·975
X.	254·01	253·95		253·53	253·575	
			54·40			54·325
IX.	308·34	308·35		308·04	307·90	

TABLE of means and numbers adopted as correct measurements of the fiducial lines on each plate (continued).

Spectra to the left.

Portions 17–26. Measurements in hundredths of an inch.

	(10)			(11)		
Fiducial lines.	Mean measurements.	Adopted numbers.	Intervals.	Tin measurements.	Adopted numbers.	Intervals.
XII.	0·000	0·000		0·000	0·000	
XIII.	85·80	85·80	85·80	85·725	85·75	85·75
XIV.	168·01	168·00	82·20	168·08	167·90	82·15
XV.	246·67	246·70	78·70	246·435	246·55	78·65
XVI.	322·00	322·00	75·30	321·80	321·80	75·25
XVII.	394·36	394·00	72·00	393·91	393·75	71·95

	(12)			(13)		
Fiducial lines.	Thallium measurements.	Adopted numbers.	Intervals.	Indium measurements.	Adopted numbers.	Intervals.
XII.	0·000	0·000		0·000	0·000	
XIII.	85·685	85·70	85·70	85·625	85·625	85·625
XIV.	167·81	167·80	82·10	167·715	167·65	82·025
XV.	246·40	246·40	78·60	246·055	246·175	78·525
XVI.	321·615	321·60	75·20	321·31	321·30	75·125
XVII.	393·88	393·50	71·90	393·14	393·125	71·825

TABLE of means and numbers adopted as correct measurements of the fiducial lines on each plate (continued).

Spectra to the right.

Portions 11–18. Measurements in hundredths of an inch.

(1)

Fiducial lines.	Magnesium measurements.	Adopted numbers.	Intervals.
I.	0·000	0·000	
			101·475
II.	101·53	101·475	
			96·475
III.	198·00	197·95	
			91·625
IV.	289·54	289·575	
			86·925
V.	376·50	376·50	
			82·375
VI.	458·905	458·875	

(2) (3)

Fiducial lines.	Thallium measurements.	Adopted numbers.	Intervals.	Zinc measurements.	Adopted numbers.	Intervals.
I.	0·000	0·000		0·000	0·000	
			101·65			101·70
II.	101·66	101·65		102·14	101·70	
			96·65			96·70
III.	198·63	198·30		198·965	198·40	
			91·80			91·85
IV.	290·175	290·10		290·22	290·25	
			87·10			87·15
V.	377·27	377·20		377·43	377·40	
			82·55			82·60
VI.	459·76	459·75		459·95	460·00	

It will be observed that there are differences in the measurements of the fiducial lines which cannot be accounted for by errors of observation. These differences may be due to two causes: first, the difficulty of placing every photographic plate in exactly the same position in the camera; second, to the alteration in length of the screw of the dividing engine by change of temperature when the different plates were measured. That the temperature has some effect upon these measurements appears probable from the fact that the first dozen or so of photographs which were measured on consecutive days give those readings which agree most closely. The dividing engine could not be kept always in the same room, and on its removal it was probably subjected to small

changes of temperature such as would very nearly account for the discrepancy in the readings. Thus, taking the linear coefficient of expansion of wrought iron to be 0·0000122 for 1° C., the length of screw to be 20 inches, and the change of temperature 5°, the change in entire length would amount to 0·001 inch, or in 5 inches, which is about the measured length of each photograph, to 0·000305. From this must be deducted the coefficient of expansion of white glass, which amounts to 0·000215 inch, which reduces the error to 0·00019. The error is generally an increasing one as the distance from zero increases.

We have taken advantage of our numerous measurements to reduce the error to a minimum: first, by taking the mean of several readings; secondly, by observing the intervals occurring between the lines and adopting numbers which, while they accord with the progressive increase in these intervals, closely approximate the numbers obtained by measurement. On comparison of the two sets of figures, it will be found that the mean of all readings and the mean of all adopted numbers agree. The greatest difference between any two series of mean adopted numbers and mean readings amounts to 0·0005 inch, the average error being 0·00023 inch. The value of these fractions in wave-length for any portion of the spectrum is not greater than 0·2 tenth-metre for the less refrangible, and probably as little as 0·05 for the more refrangible rays.

A difference in the linear measurements of the fiducial lines could occur by an unequal contraction of the gelatine film on drying. This was never the case with our plates for the reason that the central portion of each film only was used, the films were dry when the photographs were taken, and would presumably remain in the same position on the glass after developing unless some artificial mode of desiccation were employed, such as the elimination of water by steeping in alcohol. This latter method was never resorted to.

In the case of the photographs of the spectra to the right of the reflected image of the slit, one series of numbers, those of zinc, do not satisfactorily show the same relation between their intervals as can be traced between those of the other two plates, thallium and magnesium. The images on these photographs are not so sharp, and present consequently a greater difficulty in measurement than those on the spectra to the left. That the numbers adopted for the zinc plate are fairly correct is evident from the values determined for the cadmium lines.

The apparently greater dispersion of the portion 17–26 than of the portion 11–18, and of the latter than of 6–12, is due to the varying inclination of the plates to the axis of the camera lens. The plates, when in focus for the portion 6–12, were but slightly if at all inclined, while the plates including 11–18 and 17–26 were taken at a very considerable inclination; the inclination for the latter being the greater. A screen of cardboard was placed in front of the upper half of the slit when photographing the fiducial lines, in order to prevent them obscuring the spectral lines below to which they referred. When the reflections of the slit were too diffuse it was found advan-

tageous to photograph four or five spectra on one plate, the photographs of the slit falling on the uppermost spectrum only, any error arising from adjustment of the plates for the reception of the different spectra being corrected by means of the cadmium lines.

The fiducial lines of the portions 6–12 were sufficiently close together for distances between any two of them to be taken as proportional to wave-lengths, with a maximum error of about 0·1 *tenth-metre.* Owing, however, to the inclination of the plates for the other two portions of the spectrum, the error in taking the distances as proportional to wave-lengths was too large; three points were therefore interpolated between each pair of fiducial lines. This was easily accomplished, as the intervals between the reflections were simply related to one another. The maximum error of the curves constructed with the fiducial lines and the interpolated points was about 0·1 to 0·25 *tenth-metre* for the portions 11–18 and 17–26 respectively. Corrections were made for the latter error with the plates 17–26.

The curves were constructed on millimetre paper, 4 millims. were allowed for each difference of a *tenth-metre* of wave-length and 1 millim. for each two-thousandth of an inch of the scale numbers.

The wave-lengths of the cadmium lines.

As stated in the introduction to this paper, on taking photographs from metallic electrodes an electrode of cadmium was always used to furnish a spectrum in reference to which all other spectra could be measured. In the adjoining table will be found the values we have adopted for the wave-lengths of the cadmium lines, together with the numbers afforded by each plate employed in their determination. The latter numbers, it will be seen, do not give the true wave-lengths, but require a small correction, which will be found in the table. The correction for the numbers calculated for the lines 12, 17, 18, on the portion 11–18, from photographs of spectra on either side of the reflections of the slit, are shown by the table to be as nearly as possible 2·9 and 3·3 *tenth-metres,* of negative sign for the spectra to the right and positive sign for those to the left. Guided by these corrections, and owing to the fact that the lines 12α, β, γ occur also on the photographs of the portion 6–12, the lines 17 and 18 on the photographs of the portion 17–26, the corrections for the remaining lines have been deduced. Thus, for the portion 17–26 the corrections for the lines 17, 18 were found to be 3·6 and 3·8 respectively, and a correction of 4·0 was consequently made to remaining lines of this portion of the spectrum. In a like manner the corrections for the lines of the portion 6–12 were found. The explanation of the necessity of these corrections will be given later on, when considering the errors introduced by uncorrected lenses.

Spectra to the left of the regularly reflected image of the slit. | *Spectra to the right of the regularly reflected image of the slit.*

Portion of the spectrum 6–12.

No. of line of cadmium spectrum (MASCART's notation).	Plates. Indium.	Thallium.	Aluminium.	Lead.	Mean.	No. of line of cadmium spectrum (MASCART's notation).	Corrections.	Wave-lengths adopted.
7	4411·4	4411·2	4411·4	4411·6	4411·5		+3·0	4414·5
8 (air)	3991·4	3990·99	3991·6	3991·4	3991·5		+3·0	3994·5
9	3608·7	3608·4	3609·0	3608·4	3608·6		+3·2	3611·8
	3606·4	3606·2	3606·6	3606·2	3606·4		+3·2	3609·6
10	3463·6	3463·5	3463·7	3463·4	3463·6		+3·2	3466·8
	3462·3	3461·6	3462·2	3461·9	3462·2		+3·2	3465·4
11	3399·9	3399·6	3399·8	3399·6	3399·7		+3·2	3402·9
12 α	3256·8	3256·8	3257·3	3257·2	3257·0		+3·2	3260·2
„ β	3248·6	3248·4	3248·8	3248·5	3248·6		+3·2	3251·8
„ γ	3246·4	3246·2	3246·6	3246·2	3246·4		+3·2	3249·6

Portion of the spectrum 11–18.

No. of line	Iron.	Thallium.	Magnesium.	Carbon.	Mean.	No. of line	Magnesium.	Thallium.	Zinc.	Mean.	Corrections.	Wave-lengths adopted.
12 α	3257·23	3257·67	3257·03	3257·3	3257·31	12 α	3263·23	3262·7	3262·7	3262·88	±2·9	3260·2
„ γ	3246·63	3246·95	3246·4	3246·6	3246·65	„ γ	3252·59	3252·0	3252·03	3252·21	±2·9	3249·5
17	2744·4	2745·0	2744·0	2744·15	2744·39	17	2751·4	2751·0	2751·07	2751·16	±3·3	2747·7
18	2568·8	2569·4	2568·5	2569·0	2568·92	18	2575·7	2575·2	2575·38	2575·43	±3·3	2572·2

Portion of the spectrum 17–26.

No. of line	Iron.	Thallium.	Tellurium.	Indium.	Mean.	Corrections.	Wave-lengths adopted.
17	2744·1	2744·1	2744·1	2744·1	2744·1	+3·6	2747·7
18	2568·2	2568·5	2568·2	2568·8	2568·4	+3·8	2572·2
22	2317·5	2318·0	2317·6	2317·7	2317·6	+4·0	2321·6
23	2309·6	2309·8	2309·6	2309·6	2309·6	+4·0	2313·6
Isolated ray (CORNU).	2284·8	2285·2	2284·8	2285·0	2284·9	+4·0	2288·9
24	2261·9	..	2261·9	2262·1	2261·9	+4·0	2265·9
25	2192·3	2192·5	2192·3	2193·0	2192·4	+4·0	2196·4

When the corrections in the preceding tables are seen to vary slightly it is owing to small errors which are introduced by the re-adjustment of the grating. The corrections applicable to each spectrum are found by converting the values obtained for the cadmium lines given by the fixed electrode into the wave-lengths given above.

In the following tables the wave-lengths of a number of lines are compared with measurements of previous observers. That these numbers agree in a very satisfactory manner gives us some confidence in our method of working and in the probable accuracy of the wave-lengths we have adopted for the cadmium lines, as derived from our several measurements. With the exception of the lines 3402·9, 2196·4, and the triple group 3260·2, 3251·8, 3249·5, our numbers agree well with those of M. CORNU, but at these points the difference is considerable. We have carefully measured these lines directly and we are inclined to consider our numbers correct. M. CORNU did not directly measure these lines, but derived the numbers he has given from a comparison of his photographs with his map of the solar spectrum. But the reason why the wave-lengths we have given are probably the more correct lies in the fact that about this region there are many iron lines, and our measurements of iron agree with those of M. CORNU, while, if our cadmium lines departed from accuracy as far as they differ from those of M. CORNU, this difference would occur in the iron lines.

COMPARISON of the wave-lengths of lines from the spectrum of cadmium.

	MASCART.	CORNU.	HARTLEY and ADENEY.	LECOCQ.	THALÉN.
7	4414·5	..	4414·6	4414	
8	3985·6	..	3994·5	..	3995
9	3607·5	3609	3609·6		
10	3464·5	3465·5	3465·4		
11	3403·0	3401·5	3402·9		
12α	..	..	3260·2		
„ β	..	..	3251·8		
„ γ	3287·5	3247·0	3249·5		
17	2763·4	2747·7	2747·7		
18	2574·2	2572·3	2572·2		
23	2318·2	2313·5	2313·6		
24	2265·6	2265·5	2265·9		
25	2217·6	2194·5	2196·4		

No. 8 is an air line.

Nos. 9 and 10 we discovered to be doublets, but the numbers here given are those assigned to the more refrangible lines of each.

No. 12 is a triple group; this fact was recognised by M. CORNU. The wave-lengths of two iron lines close to this group are the following:—

CORNU.	HARTLEY and ADENEY.
3246·2	3246·3
3242·7	3243 0

LINES of iron compared with those in the solar spectrum measured by MM. THALÉN and CORNU.

Hartley and Adeney.	Thalén.	Cornu.
4403·7	4404	
4382·6	4383	
4325·0		
4307·1	4307	
4298·3		
4293·3	4294	
4281·7		
4271·0	4271	
4259·9		
4249·8	4250	
4201·4	4201	
4198·4	4198	
4143·0	4143	
4071·5	4071	4071·2
4063·0	4063	4062·9
4045·4	4045	4045·0
3968·7		
3933·1	..	3932·9
3929·7	..	3929·7
3927·6	..	3927·2
3902·6	..	3902·0
3899·3		
3895·1	..	3894·8
3888·1	..	3888·0
3885·7	..	3885·1
3878·1	..	{ 3877·6 3877·3
3872·2	..	{ 3871·4 3871·2
3865·2	..	{ 3864·8 3865·2 3865·5
3859·6		
3856·1	..	3855·8
3849·1	..	3849·8
3840·3	..	{ 3840·0 3840·5
3834·0	..	3833·6
3827·4	..	3827·7
3825·5	..	3825·2
3824·0	..	3824·1
3820·3	..	3819·8 L
3815·8	..	3815·3
3812·6	..	3812·7
3808·9		
3804·4	..	3805·0
3798·4	..	3798·7
3794·6	..	3795·0
3767·0	..	3766·8
3765·3	..	3765·0
3763·3	..	3763·3
3757·9	..	3757·8
3749·4	..	3749·4
3745·4	..	3745·4

Hartley and Adeney.	Cornu.
3742·7	3743·0
3736·9	3736·5
3734·7	3734·4
3719·7	3719·8
3709·0	3709·0
3705·5	3705·6
3687·3	3687·2
3679·5	3680·3
3676·5	
3647·6	3647·0
3631·0	3630·9
3618·6	3617·8
3594·9	
{ 3586·3 3584·8	3586·2 3584·9 }
3581·1	3581·5 N
3569·6	
3565·0	3564·2
3558·1	3558·2
3554·2	3554·0
3540·9	3541·4
3525·9	3525·8
3520·7	3520·7
3513·3	3513·8
3443·3	3443·7 3443·1 }
3440·2	3440·0 O
3436·9	
3389·5	
3370·2	
3366·1	
3364·9	
3358·7	3359·2 P
3353·3	
3305·4	3304·8
3291·5	3290·8
3288·8	3289·3
3285·4	3284·8 Q
3279·9	
3276·2	
3258·2	
3246·3	3246·2
3243·0	3242·7
3227·0	3226·6
3221·5	3221·0
3212·7	3212·2
3209·5	
3204·6	3204·3
3199·9	3199·8
3195·7	3196·3
3192·7	
3186·2	
3182·3	
3179·1	3179·8 R

LINES of iron compared with those in the solar spectrum measured by MM. THALÉN and CORNU (continued).

HARTLEY and ADENEY.	CORNU.	HARTLEY and ADENEY.	CORNU.
3176·8		3066·6	
3153·6		3046·9	3046·5
3143·9	3144·1 3144·4	3036·4	3036·1
3134·6		3024·8	{3025·2 3024·8}
3116·1		3020·1	3019·9 T
3113·4		3002·1	{3002·0 3002·3}
3104·8		2993·6	2994·3
3099·5	3099·8 S_2	2964·0	
3096·7		2948·4	2947·8
3082·8		2946·9	
3070·3			

VARIOUS metallic lines measured by other observers compared with the new measurements.

	HARTLEY and ADENEY.	THALÉN.	CORNU.	LIVEING and DEWAR.	LECOCQ DE BOISBAUDRAN.	ÅNGSTRÖM and THALÉN.
Magnesium lines	4480	4481				
	3837·9	..	3837·7			
	3832·1	..	3831·6			
	3829·2	..	3829·0			
	*{ 3336·2	..	3334·0			
	3331·8	..	3330·0			
	3329·1 }	..	3327·0			
	{ 3096·1	..	3095·7			
	3091·9	..	3092·0			
	3090·0 }	..	3090·1			
	2935·7	..	2934·9			
	2928·0	..	2926·7			
	2913·7					
	2851·3	..	2850·3†			
	2848·0					
	2846·0					
	{ 2801·6	..	2801·3			
	2796·9	..	2797·1			
	2794·1	..	2794·5			
	2789·6 }	..	2789·9			
	{ 2781·8	..	..	2782·2		
	2780·2					
	2778·7	..	..	2779·5		
	2776·9					
	2775·5 }	..	..	2776·9		

* Our numbers here differ by about 2 *tenth-metres*, but an iron line in this part of the spectrum has given the following wave-lengths, 3358·7 (H. and A.) and 3359·2 (CORNU).

† Messrs. LIVEING and DEWAR give 2852·0 for this line. ("On the Disappearance of some Spectral Lines," Proc. Roy. Soc., vol. xxxiii., p. 429.)

VARIOUS metallic lines measured by other observers compared with the new measurements (continued).

	HARTLEY and ADENEY.	THALÉN.	CORNU.	LIVEING and DEWAR.	LECOQ DE BOISBAUDRAN.	ÅNGSTRÖM and THALÉN.
Aluminium lines	4477·2	..	..	..	4478	
	3961·0	..	3960·7	..	3962	
	3943·4	..	3943·3	..	3943	
	3092·2	..	3091·6			
	3081·5	..	3081·7			
Indium lines . .	4510·2	4509	..	..	4511	
	4101·3	4101	..	..	4101	
Lead lines . . .	4386·4	4386	..	..	4386	
	4245·2	4246	..	..	4245	
	4061·5	4062				
	4057·5	4058	..	..	4056	
Bismuth lines .	4301·4	4302	..	..	4303	
	4259·1	4259	..	..	4259	
	4121·2	4119	..	..	4118	
Carbon lines . .	4266·3	..	..	..	..	4266
	3919·5	..	..	3919·3		
	3875·7	..	..	3876·5		
	2993·3	..	..	2995·0		
	2967·5	..	..	2968·0		
	2836·8	..	..	2837·2		
	2836·0	..	..	2836·3		
	2746·6	..	..	2746·5		
	2640·0	..	..	2640·7		
	2511·6	..	..	2511·9		
	2508·7	..	..	2509·0		
	2478·3	..	..	2478·3		
	2297·7	..	..	2296·5		

Errors.

There are four sources of error to which the measurements from our photographs are open. The total error is easily eliminated, but it is difficult to give precise values to the individual errors. They are :—

First—The lens of the collimator was uncorrected. It was focussed for white light, and the adjustment was unaltered during the photographing of the spectra. On this account the rays striking the grating would not be parallel to one another, but be slightly convergent. Further, the rays corresponding to the different fiducial lines would be, after reflection, diverging from points not in the axis of rotation of the grating. To this source of error is mainly due the correction we have to make in our measurements.

Second.—M. CORNU found during his researches on the "Ultra-Violet Solar Spectrum" (Annales Scientifiques de l'École Normals, iii., 1874, second series, p. 430), that in order to make his numbers obtained by direct vision and by photography

comparable one with the other, he had to make a correction of −1·0 *tenth-metre* to those obtained by the latter means. An explanation of this will be found in M. CORNU's paper.

Third.—The error arising from the adjustment of the grating to its position for photographing the spectrum. This does not exceed 0·5 *tenth-metre*, and, it should be noted, appears in all the measurements of the plate. This may be eliminated by taking the mean of the measurements of the cadmium lines.

Fourth.—The error in the measurements arising from change of temperature affecting the dividing engine. This is within $\pm$ 0·2 for well defined lines, and it is always corrected by the cadmium lines. The errors of observation in the measuring of lines are also, for well-defined lines, at most $\pm$ 0·2.

The first and second errors are of opposite sign for the spectra, to the right and left of the regular reflection of the slit, and are therefore easily eliminated by taking the mean of the spectra on the two sides.

The wave-lengths given in the accompanying tables are all comparable with one another. If an error occurs in any of the values for the cadmium lines, it will be common to all the lines in that part of the spectrum in which the particular line occurs, and will be easily eliminated. If the wave-lengths assigned to the cadmium lines are correct, there are only two errors to which the wave-lengths in the tables that have been calculated from grating spectra are liable. These are that incurred in the measurement of the lines by the microscope and dividing engine, and that due to the interpolation curve. The errors of measurement mentioned above are, for well-defined lines, in no case greater than $\frac{1}{2000}$th of an inch; this, in terms of wave-length, equals 0·2, 0·17, and 0·12 *tenth-metres* for the portions 6–12, 11–18, and 17–26, respectively. The error of the interpolation curve is not more, we believe, than 0·1 *tenth-metre.* The maximum for well-defined lines probably does not exceed $\pm$ 0·3 *tenth-metre.* In the case of faint lines, the general error is larger, but it seldom rises to more than 0·5. Thus in the accompanying table of air lines, the wave-lengths of which were all determined directly by measuring diffraction spectra, two sets of numbers taken from different photographs are given, and the numbers for the following six, which are all very feeble and diffuse lines, and therefore difficult to measure, are the only ones differing by more than the general error :—

Aluminium plate. Portion 6–12.	Copper plate. Portion 6–12.
4402·0	4403·1
4215·9	4217·1
4025·9	4024·7
3851·0	3850·0
3842·2	3841·2
3325·3	3324·1

The subjoined lists give the numbers of some of the lines of tin, lead, indium, and copper that occur between the cadmium lines 11 and 12, and near the cadmium line 18. Each of these lines occurs therefore on two different plates, and has been twice measured.

Some Lines of Tin, Lead, Indium, and Copper, of which duplicate measurements have been made.

Copper.				Tin.			
Plate 6-12.	Plate 11-18.		Plate 17-26.	Plate 6-12.	Plate 11-18.		Plate 17-26.
3306·8	3307	2544·5	2544·6	3329·9	3329·6	2705·6	2705·9
3289·9	3289·9	2528·8	2528·9	3282·9	3282·9	2664·6	2665·2
3282·2	3282·1	2526·1	2526·3	3261·7	3261·5	2660·2	2660·6
3273·3	3273·2	2522·6	2522·7	3174·3	3174·1	2657·8	2658·1
3246·8	3246·9	2506·3	2506·2			2645·2	2645·7
		2491·1	2491·7			2643·0	2643·3
		2489·3	2489·0			2631·4	2631·5
		2485·5	2485·7			2617·9	2617·8
		2481·8	2481·9			2593·4	2593·9
		2477·9	2478·5			2570·6	2570·5
		2468·1	2468·7			2545·5	2545·7
						2488·1	2488·8
						2482·8	2482·9

Lead.				Indium.			
Plate 6-12.	Plate 11-18.		Plate 17-26.	Plate 6-12.	Plate 11-18.		Plate 17-26.
3277·9	3277·5	2716	2716·7	3236·5	3236·0	2713	2712·9
3242·4	3242·4	2697·8	2697·7	3246·5	3245·7	2709·1	2709·5
		2662·4	2662·6	3255·3	3255·8	2706·0	2707·0
		2650·0	2650·0	3257·7	3257·9	2630·6	2631·7
		2627·1	2627·8	3273·8	3274·0	2600·3	2600·3
		2613·2	2613·5			2559·6	2559·4
		2576·3	2576·5			2553·9	2554·3
		2566·8	2567·6			2527·2	2527·0
		2561·3	2561·8				
		2475·5	2476·0				

There is no distinction made in these tables between strong and weak or sharp and diffuse lines, and the numbers therefore represent the degree of accuracy common to our measurements of lines of different characters.

Determination of wave-lengths from prismatic spectra.—Owing to the large portion of the spectrum being focussed on one plate, and the very fine definition of the lines, it was advantageous to take measurements from grating spectra, and determine the wave-lengths by interpolation between the fiducial lines. Faint lines, and some of the weak ones that were indistinguishable under the microscope, were marked by a fine needle-point, and a drawing of the line and point, as seen under a hand lens, was

taken to serve as a guide when measuring with the microscope. In this way, very accurate measurements were obtained, even of lines that could not possibly have been measured otherwise. Nearly all lines given in the accompanying tables, up to and including a wave-length of 2265, have been measured from grating spectra. Between λ 2265 and λ 2145·7, the very well marked lines only were measured from the grating spectra. The numbers not obtained from grating spectra for the other lines given in the tables were determined by means of an interpolation curve, constructed from measurements taken from prism spectra.

The curve was laid down on millimetre paper, four millimetres being allowed for each difference of a *tenth-metre* in wave-length, and one millimetre for each $\frac{1}{1000}$th of an inch of difference in the scale numbers. The wave-lengths were taken as normals, and the scale numbers as abscissæ; the aggregate length of the curve from $\lambda = 4800$ to λ 2020 was about nine metres. The spectra were taken from one quartz prism of 60°, composed of two halves each of 30°, one of right-handed and the other of left-handed rotation. The prism was fixed for the minimum angle of deviation of the cadmium line 2747·7, and the photographs were similar to those published in the Journal of the Chemical Society (Transactions, vol. xii., p. 85, W. N. HARTLEY).

In the tables, besides the wave-lengths, the scale numbers from the prismatic spectra are in every case given. These numbers are expressed in hundredths of an inch and fractions thereof. The numbers for the various metals are strictly comparable with each other, since the measurements from each spectrum have been reduced to a standard spectrum of an alloy of tin and cadmium. This was accomplished in the following way. The spectrum of each metal was photographed with that of the tin-cadmium alloy. The same electrode of the alloy was employed for all the spectra, and was not moved during the time the whole series was being photographed. An alloy of tin-cadmium was used because it gives a large number of well-defined lines, equally distributed. Notwithstanding the care taken, and that twelve spectra were photographed on the same plate, in only four spectra are the tin and cadmium lines coincident in position. The mean of the readings for these four spectra was taken for the standard spectrum, and all others were reduced to it by finding the corrections for the tin and cadmium lines, and interpolating corrections for lines between them.

For the construction of the curve, 180 lines from the different spectra were employed. The whole of these lines were cut by the curve. A few lines were left a little to the one side or the other, but these are not included in the above number. The lines employed for the portion of the spectrum beyond the cadmium line 2146·8 were those of zinc, M. CORNU's numbers for their wave-lengths being made use of. This portion of the curve was made as continuous as possible with the other. Not all the points were cut by it, and to this is owing the slight difference between some of the numbers in our table of the zinc lines and those of M. CORNU.

The curve is a very regular one, and might be drawn from a very few accurate points.

The value of the error of $\frac{1}{2000}$th of an inch in measurement for different parts of the curve for prism spectra, in terms of wave-length, is given in the following table:—

Portion of curve between the wave-lengths given.	Value of error in tenth-metres of $\frac{1}{2000}$th of an inch.
4780 to 4440	1·1 to 1·0
4440 to 3990	1·0 to 0·7
3990 to 3600	0·7 to 0·5
3600 to 3200	0·5 to 0·3
3200 to 2800	0·3 to 0·25
2800 to 2400	0·25 to 0·16
2400 to 2020	0·16 to 0·08

Well defined lines can with certainty be measured to $\frac{1}{2000}$th inch; for weak and indistinct lines the error of measurement amounts to $\frac{1}{1000}$th, but even with this error most satisfactory determinations can be obtained from the curve.

Great care is necessary in measuring these prismatic spectra. It is difficult to accurately adjust the cross hairs to the lines, owing to the latter being somewhat, though very slightly, curved; consequently the readings may not be quite true with reference to the tin and cadmium lines, but either a little too small or a little too large.

The danger of this error is reduced to a minimum if the photographs are taken with the electrodes rather close together, or where only strong lines are to be observed by making the spark cross the slit.

In the list of lines given in the tables for each metal, the greatest care has been taken to eliminate those due to foreign metals. This has been done as completely as possible by taking a large number of photographs of the spectra of several elements for the purpose of comparison. In cases of doubt, spectra of very strong solutions of the purest salts were taken. It has been found that lines due to impurities generally present a decidedly different character to any of the lines due to the metal under examination. This will be referred to again further on. The wave-length of every line identified with a given metal has been determined either from the grating or prismatic spectrum. In the present state of our knowledge this has been thought important. The short lines of cadmium and zinc, for instance, have not been measured by previous observers; they are, however, a very characteristic feature in each spectrum.

Certain weak lines do not appear in the diffraction spectra, but are plainly visible in those obtained by means of a prism. Such lines have been measured with the interpolation curve, and are distinguished in the following tables by their wave-lengths being printed in italic figures.

It was originally intended that these spectra should be drawn on the same scale as CORNU'S map of the solar spectrum; this would have necessitated scales and drawings eight feet in length for each of the sixteen spectra. The work was actually commenced, but the mapping of every line proved too laborious; accordingly enlarged photographs of the prism spectra, about thirty-six inches in length, have been utilized by writing the wave-length over each line. These photographs are intended to serve for particular reference. In addition, each line has been carefully described, and its position on the photographed spectra has been very carefully determined and recorded in the scale numbers. For the purposes of chemical analysis, small maps and actual photographs, showing the characters of the lines, are of most value, and accordingly the principal lines have been drawn on the scale of wave-lengths on sheets of a size convenient for reference and comparison with a series of prism photographs ten inches in length. The scale numbers refer to spectra of about one-half these dimensions. Should it be found necessary at any time to rectify any of the wave-lengths given in this paper, this may be easily accomplished by the use of an interpolation curve, derived from the scale numbers and true wave-lengths.

In all cases where the wave-lengths on the maps differ from the numbers in the tables the latter must be considered as the more correct, the drawings being on too small a scale to admit of great accuracy, and moreover some of the numbers were slightly altered after the maps had been drawn.

DESCRIPTIONS OF SPECTRA AND TABLES OF WAVE-LENGTHS.

Full particulars concerning the method of producing the prism spectra, together with an account of the electrodes employed, have been already published in the Scientific Transactions of the Royal Dublin Society, and the characters of the various lines observed is there defined.

A peculiar feature of certain lines in the spectra of cadmium and indium has been observed, we believe, for the first time by us. The lines are continuous lines, but they do not extend from the point of one electrode to the other, but occupy only an intermediate position, commencing and terminating at some distance from the metallic points. A similar character is observable in certain air lines, when strong metallic lines occur in close proximity on either side. In both air lines and metallic lines the central portions become stronger, and the ends fade away as the temperature is increased. Lines which show this in a marked degree are those of indium with wave-lengths 2429·0, 2389·8, 2332·2, and that of cadmium with wave-length 2544·5. In a less remarkable manner the following lines represent this character in cadmium: 3080·2, 2868·0, 2832·3, 2774·5, 2763·1, 2658·5, 2635·3; in indium, 2956·1, 2709·3, 2602·5, and 2520·9. Air lines altered by the proximity of metallic lines are the following: 3408·0, 3329·3, 3007·0, and 2733·2.

The Spectrum of Air.

Scale-numbers.	Description of lines.	Electrodes employed.			Remarks.
		Copper.	Aluminium.	Mean.	
		Wave-lengths.	Wave-lengths.	Wave-lengths.	
7·97	Faint	4674·2	..	4674·2	The air lines being difficult to measure, we have taken the mean wave-lengths derived from two or three different photographs.
8·60	Faint	4660·2	..	4660·2	
9·08	Weak	4647·2	..	4647·2	
9·48	Weak	4641·2	..	4641·2*	*THALÉN gives this line as double.
10·00	Strong	4629·0	4628·7	4628·9	
10·40	Weak	4619·9	..	4619·9	
10·74	Weak	4612·3	..	4612·3	
11·10	Weak	4605·6	..	4605·6	
11·40	Weak	4600·1	..	4600·1	
11·85	Weak	4595·0	..	4595·0	
11·89	Weak	4589·3	..	4589·3	
13·80	Faint nebulous band	4553·2	..	4553·2	
	Faint	4543·4	..	4543·4	
14·97	Weak band	4530·1	..	4530·1	
15·25	Faint, nebulous	4523·0	..	4523·0	
15·7	Weak, fine	4513·7	..	4513·7	
15·9	Weak	4506·6	..	4506·6	
16·96	Weak, fine	4476·6	..	4476·6	
17·48	Weak band	4466·1	..	4466·1	
18·23	Weak, fine	4458·7	..	4458·7	
19·25	Strong	4446·3	4445·8	4446·0	
20·03	Weak nebulous band	4433·0	4432·2	4432·6	
20·30	Weak, nebulous	4426·3	4425·5	4425·9	
20·89	Weak	..	..	4415·5	From the platinum plate.
21·01	Weak	..	..	4413·6	
21·80	Faint	4403·11	4402·0	4402·6	
21·93	Weak	4395·0	4394·8	4394·9	
22·5	Very faint	..	4386·3	4386·3	
22·83	Faint	4378·1	4377·9	4378·0	
23·61	Weak	4365·9	4365·7	4365·8	
24·16	Very faint, nebulous	..	4356·4	4356·4	
24·53	Weak, fine	4350·7	4350·2	4350·5	
24·67	Strong	4348·4	4348·0	4348·2	
24·83	Weak, fine	4343·8	4344·0	4343·9	
25·32	Faint	4336·0	4335·8	4335·9	
25·63	Faint	4330·9	4330·6	4330·8	
25·78	Faint	4327·2	4326·6	4326·9	
25·88	Faint	4324·6	4324·6	4324·6	
26·31	Weak	4319·0	4318·5	4318·7	
26·44	Weak	4316·3	4316·1	4316·2	
27·15	Very faint, nebulous	4306·1	4306·9	4306·5	
	Very faint	..	4302·0	4302·0	
27·16	Faint, nebulous	4290·3	4289·7	4290·0	
28·93	Faint, nebulous	..	4275·3	4275·3	
	Very faint, sharp	..	4274·3	4274·3	
29·49	Very faint	4265·8	4264·9	4265·4	
30·21	Faint	4253·4	4253·3	4253·4	
30·95	Strong, nebulous	4240·2	4241·0	4240·6	
31·21	Strong, nebulous	4236·7	4236·0	4236·4	
31·68	Fairly strong, nebulous	4229·1	4228·7	4228·9	
31·98	Faint, nebulous	4222·3	4222·8	4222·6	
32·38	Faint	4217·1	4215·9	4216·5	

THE Spectrum of Air (continued).

Scale-numbers.	Description of lines.	Electrodes employed.			Remarks.
		Copper.	Aluminium.	Mean.	
		Wave-lengths.	Wave-lengths.	Wave-lengths.	
33·16	Faint, nebulous	4206·4	4206·2	4206·3	
33·63	Faint, nebulous	4197·5	4198·3	4197·9	
34·16	Weak, sharp	4189·3	4189·4	4189·3	
34·43	Weak, sharp	4185·2	4185·0	4185·1	
35·09	Weak, broad, nebulous	4176·4	4177·3	4176·8	
35·41	Weak, nebulous	4169·0	4169·5	4169·2	
35·97	Very faint, nebulous.	..	4157·9	4157·9	
36·47	Weak, fine	4152·8	4152·6	4152·7	
37·03	Fairly strong, fine	4145·4	4145·3	4145·4	
37·29	Fairly strong, fine	4132·8	4132·7	4132·8	
38·50	Faint, fine	4123·6	4123·7	4123·7	
38·73	Fairly strong, fine	4118·8	4119·1	4119·0	
39·23	Faint, fine	4110·5	4111·2	4110·9	
39·68	Fairly strong, fine	4103·6	4104·3	4104·3	
39·77	Fairly strong, fine		4102·6	4102·6	
40·25	Strong, fine.	4096·2	4096·7	4096·5	
40·36	Very faint, fine	..	4092·6	4092·6	
40·97	Faint, fine	4084·4	4085·3	4084·8	
41·68	Strong, fine.	4074·9	4075·3	4075·1	
41·98	Strong, fine.	4071·3	4071·5	4071·4	
42·13	Strong, fine.	4069·1	4069·2	4069·2	
42·58	Very faint, fine	..	4063·5	4063·5	
42·94	Very faint, fine	..	4057·2	4057·2	
44·09	Strong, broad, nebulous	4041·8	4041·5	4041·7	
44·55	Weak, nebulous	4034·3	4034·5	4034·4	
45·18	Faint, nebulous	4024·8	4025·9	4025·3	
47·33	Very strong, sharp	3994·6	3994·5	3994·5	
47·83	Very faint	..	3988·5	3988·5	
48·23	Faint, sharp, fine.	3982·9	3983·1	3983·0	
48·97	Strong, fine	3972·6	3972·4	3972·5	
49·33	Faint, fine	..	3967·3	3967·3	
50·30	Strong, fine	3954·8	3954·9	3954·8	
51·03	Faint, nebulous	3944·1	3945·0	3944·5	
51·43	Weak, nebulous	3939·0	3939·5	3939·2	
51·88	Very faint	3932·4	3933·5	3932·9	
52·13	Very faint	..	3929·0	3929·0	
53·03	Strong, fine.	3918·5	3918·4	3918·5	
53·57	Weak, fine	3911·8	3911·6	3911·7	
54·95	Very faint	..	3892·4	3892·4	
55·85	Weak, fine	3881·8	3882·0	3881·9	
57·27	Faint, fine	3864·2	3863·4	3863·8	
57·86	Weak, nebulous	3856·4	3856·1	3856·2	
58·43	Faint, fine	3850·0	3851·0	3850·0	
58·99	Faint, nebulous	3841·2	3842·2	3841·7	
59·20	Weak, nebulous	3839·5	3839·1	3839·3	
59·86	Weak, fine	3831·0	3831·1	3831·0	
60·46	Very faint	..	..	3824·0	An approximation.
61·97	Faint, fine	3803·7	3804·4	3804·0	
63·07	Faint, fine	3791·6	..	3791·6	
63·97	Faint, fine	3781·8	3782·4	3782·1	
64·92	Faint, fine	3771·6	3771·3	3771·5	
65·95	Faint, fine	3759·4	3759·4	3759·4	

The Spectrum of Air (continued).

Scale-numbers.	Description of lines.	Electrodes employed. Copper.	Aluminium.	Mean.	Remarks.
		Wave-lengths.	Wave-lengths.	Wave-lengths.	
66·35	Faint, fine	3753·7	3753·7	3753·7	
66·85	Strong, fine	3749·0	3748·9	3749·0	
67·57	Very faint, fine	3739·3	3740·1	3739·7	
68·78	Strong, fine	3726·6	3726·6	3726·6	
70·01	Fairly strong, fine . . .	3712·2	..	3712·2	
70·89	Very faint	..	..	3702·0	An approximation.
71·89	Faint, fine, sharp. . . .	..	3639·0	3639·0	
79·21	Faint, fine	..	3613·6	3613·6	
79·63	Weak, fine	..	3610·0	3610·0	
81·08	Weak, fine	3595·0	3594·9	3595·0	
81·62	Weak, fine	3589·6	3589·7	3589·6	
82·29	Weak, fine	3583·1	3584·3	3583·7	
82·90	Weak, fine	3575·9	3576·4	3576·2	
84·38	Weak, nebulous	3560·4	3560·9	3560·6	
85·10	Very faint	..	3550·3	3550·3	
85·93	Weak, nebulous	3545·6	3544·8	3545·2	
89·21	Very faint, fine	..	..	*3514·1*	
90·76	Very faint, fine	..	..	*3499·7*	
91·61	Weak, fine	..	..	3490·7	
93·05	Faint, fine	3478·2	3478·1	3478·1	
93·83	Weak, fine	..	3471·2	3471·2	
95·51	Very faint, fine	..	...	*3456·2*	
96·40	Very faint, fine	..	..	*3448·0*	
97·63	Strong, fine	3436·8	3437·1	3436·9	This line is much altered in the palladium spectrum by two strong neighbouring lines. It appears strong in the centre, thinning away towards each end.
100·96	Fairly strong, fine . . .	3407·8	3408·2	3408·0	
103·08	Fairly strong, fine . . .	3389·7	3390·0	3389·9	
104·51	Weak, fine	3376·6	3377·2	3376·9	
105·00	Faint, fine, sharp. . . .	..	3373·6	3373·6	
105·28	Faint, fine, sharp. . . .	3370·2	3370·4	3370·3	
105·65	Fairly strong, fine, sharp .	3366·1	3366·4	3366·7	
105·77	Fairly strong, fine, sharp .			3365·7	
107·28	Fairly strong, fine . . .	3353·7	..	3353·7	
108·63	Very faint, nebulous. . .	..	..	*3342·7*	
109·88	Strong, fine	3331·8	3331·2	3331·5	In the antimony spectrum only the central portion of this double line is visible, the two ends gradually thinning away. A metallic line fails near it.
110·23	Strong, fine	3329·1	3329·4	3329·3	
110·79	Faint, fine	3324·1	3325·3	3324·7	
111·49	Weak, fine	3320·1	3319·9	3320·0	
112·97	Very faint, fine	..	3313·3	3313·3	
113·07	Very faint, fine	..	3307·1	3307·1	
113·75	Very faint, fine	..	3301·1	3301·1	
115·13	Faint, nebulous, broad . .	..	3289·9	3289·9	Possibly a triplet.
117·26	Faint, nebulous	..	3274·6	3274·2	
118·33	Weak, fine	..	3265·2	3265·2	
119·05	Weak, fine	..	3259·9	3259·9	
124·57	Very faint	..	3219·7	3219·7	This and the following lines are from the indium and thallium plates, 11–18.
132·97	Very faint, fine	3157·4	..	3157·4	
135·61	Fairly strong, fine . . .	3139·0	3139·6	3139·3	
136·51	Fairly strong, fine . . .	3134·2	..	3134·2	
138·36	Very faint	3122·4	..	3122·4	
148·14	Very faint, fine	..	3058·4	3058·4	
150·00	Faint, fine	..	3046·3	3046·3	
150·73	Very faint, fine	..	3042·5	3042·5	

THE Spectrum of Air (continued).

Scale-numbers.	Description of lines.	Electrodes employed.			Remarks.
		Copper.	Aluminium.	Mean.	
		Wave-lengths.	Wave-lengths.	Wave-lengths.	
152·01	Faint, fine	..	3034·9	3034·9	
153·77	Weak, fine	..	3024·1	3024·1	
154·86	Faint, fine	..	3016·1	3016·1	
156·27	Strong, fine	3006·7	3007·4	3007·0	In the antimony and arsenic spectra a metallic line falls near this, and its character is altered. In other spectra it generally appears stronger, and in cadmium it is strong throughout its length, but in the case of the above spectra it is strongest in the centre.
160·45	Weak, fine	2983·0	2982·6	2982·8	
164·59	Faint, fine	2959·7	2959·3	2959·5	
178·35	Faint, fine, sharp. . . .	2884·6	..	2884·6	
178·87	Faint, fine, sharp. . . .	2880·4	..	2880·4	
189·975	Weak, nebulous	2822·8	2823·3	2823·1	
194·935	Faint, fine	2799·0	2800·0	2799·5	
205·615	Very faint, fine	2748·8	..	2748·8	This and following numbers from the indium, 17–26 plate.
208·955	Faint, fine	..	..	2733·2*	
213·875	Faint, broad, nebulous . .	..	..	2710·1	
241·375	Very faint	..	..	*2598·4*	*The line 2733·2 is strongest in the centre, thinning away at each end.
243·105	Faint, fine	..	..	*2591·8*	
246·175	Faint, fine	..	..	*2580·0*	
261·85	Weak, nebulous, broad . .	..	..	2522·1	
274·78	Weak, fine, sharp . . .	..	..	2478·1	
279·42	Very faint, nebulous. . .	..	..	2463·0	
282·29	Faint, fine	..	..	2453·8	
284·80	Fairly strong, fine . . .	..	..	2445·2	
288·56	Fairly, strong, fine . . .	..	..	2433·6	
291·58	Weak, nebulous	..	..	2423·8	
293·37	Faint, fine	..	..	2418·6	
294·15	Very faint, fine	..	..	*2416·2*	
295·60	Very faint, fine	..	..	*2411·7*	
296·95	Very faint, fine	..	..	*2407·7*	
299·975	Very faint, fine	..	..	*2398·3*	
302·555	Very faint, fine	..	..	*2390·7*	
322·86	Very faint, broad, nebulous	..	..	2332·2	
328·21	Fairly strong, broad . .	..	..	2318·1	
333·25	Very faint, fine, sharp . .	..	..	*2314·4*	
334·23	Faint, fine, sharp . . .	*2301·3*	*2301·8*	*2301·8*	From the zinc and bismuth prism spectra.
335·62	Faint, fine, sharp . . .	*2297·6*	*2298·0*	*2298·0*	
337·14	Faint, fine, sharp . . .	*2294·1*	*2294·2*	*2294·2*	
338·35	Very faint, fine, sharp . .	..	..	*2291·0*	
339·00	Very faint, fine, sharp . .	..	..	*2289·3*	
354·51	Very faint, nebulous . .	..	..	*2250·2*	
381·19	Very faint, nebulous . .	..	..	*2186·0*	An approximation.

The Spectrum of Magnesium.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
17·46	Very strong, discontinuous, broad, with nimbus	4480·1	All the strongest lines of magnesium are much extended and surrounded by a very strong nimbus. A nebulous air-line occurs at $\lambda=3856{\cdot}1$.
54·72	Weak, short	*3896·0*	
55·03	Weak, short	*3892·0*	
57·90	Weak, short	*3855·5*	
58·58	Weak, short	*3849·5*	
59·30	Strong, continuous, extended	3837·9	A very strong nimbus is seen around these lines.
59·83	Strong, continuous, extended	3832·1	
60·07	Strong, continuous, extended	3829·2	
65·45	Faint, short, broad, nebulous	*3765·3*	
109·52	Weak, fine, continuous, extended, faint in centre	3336·3	
110·04	Weak, fine, continuous	3331·8	
110·36	Weak, fine, continuous	3329·1	
135·67	Faint, short	3139·3	
136·33	Faint, short	3134·3	
140·55	Weak, short, broad, nebulous	*3107·0*	
142·30	Strong, continuous, fine, extended	3096·2	A large nimbus is seen here.
142·85	Fairly strong, continuous, fine, extended	3091·9	
143·18	Fairly strong, continuous, fine, weak in centre	3089·9	
145·85	Weak, short, fine	3071·6	
150·09	Faint, short, fine	*3046·0*	
167·74	Faint, very fine, discontinuous	*2941·6*	
168·70	Very strong, continuous, broad, with nimbus extended	2935·8	
170·18	Very strong, continuous, broad, with nimbus extended	2928·1	
172·58	Strong, fine, short	2913·8	
178·25	Faint, short, broad, nebulous	*2884·3*	
184·63	Very strong, continuous, broad, with nimbus, extended	2851·2	
185·23	Very faint, fine, continuous	*2847·9*	
185·61	Very faint, fine, continuous	*2845·9*	
191·72	Faint, discontinuous, nebulous	*2815·3*	
192·80	Faint, discontinuous, nebulous	*2810·0*	
194·55	Very strong, continuous, fine, extended	2801·6	The first and third lines of this quadruple group are more persistent than the second and fourth, so that the former lines are visible in solutions too dilute to show the latter. The nimbus to this group of lines is remarkably strong.
195·39	Very strong, continuous, fine, extended	2796·9	
195·95	Very strong, continuous, fine, extended	2794·1	
196·92	Very strong, continuous, fine, extended	2789·6	
198·64	*Fairly strong*, continuous, fine	2781·8	A nimbus surrounds the extremities of these lines.
198·96	*Fairly strong*, continuous, fine, weak in centre	2780·2	
199·30	*Strong*, continuous, fine	2778·7	
199·61	*Fairly strong*, continuous, fine, weak in centre	2776·9	
199·97	*Fairly strong*, continuous, fine	2775·5	
208·48	Faint, fine, discontinuous	*2736·0*	
208·80	Faint, fine, discontinuous	*2734·3*	
226·42	Short, weak, broad, nebulous	*2658·4*	

The Spectrum of Zinc.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
5·72	Faint, short, fine	*4725·0*	The lines of zinc are much extended.
7·72	Faint, short, fine	*4680·0*	
61·35	Very faint, short	*3813·5*	
61·53	Very faint, short	*3811·5*	
66·12	Faint, very short	*3757·5*	
69·34	Weak, very short	*3720·5*	
69·98	Weak, very short	*3713·5*	
70·79	Weak, very short	*3704·5*	
71·75	Weak, very short	*3694·0*	
72·75	Weak, very short	*3683·0*	
74·13	Weak, very short	*3668·0*	
76·01	Faint, very short	*3645·4*	
77·49	Weak, very short	*3632·2*	
78·25	Weak, very short	*3623·4*	
82·57	Faint, very short	*3578·2*	
84·35	Faint, very short	*3560·8*	
86·84	Very faint and short	*3536·8*	
87·56	Faint, very short	*3529·8*	
89·73	Very faint and short	*3509·2*	
91·61	Faint, very short	*3491·8*	
108·49	VERY STRONG, CONTINUOUS, broad, with nimbus, extended	3344·4	
113·75	VERY STRONG, CONTINUOUS, broad, with nimbus, extended	3301·7	
116·30	STRONG, CONTINUOUS, with nimbus, extended	3281·7	
119·53	Faint, very short	*3255·8*	
121·77	Faint, very short	*3238·7*	
122·30	Faint, very short	*3234·6*	
145·69	STRONG, CONTINUOUS, fine, sharp, extended	3075·6	
146·22	STRONG, DISCONTINUOUS, fine, extended	3071·7	
152·03	STRONG, DISCONTINUOUS, slightly extended	3035·4	
153·89	Faint, short	3024·1	Coincident with an air line.
154·85	Weak, discontinuous, fine, sharp	3017·5	
158·21	Faint, very short	2996·7	
164·86	Faint, very short	2959·5	Coincident with an air line.
177·86	Faint, very short	*2886·4*	
183·60	Faint, very short	*2856·3*	
194·98	STRONG, CONTINUOUS, fine, with a nimbus	2800·1	
198·54	Very faint, short	*2782·5*	
199·30	Faint, short	*2778·4*	
201·23	STRONG, CONTINUOUS, with a nimbus	2770·2	
204·37	*Fairly strong*, discontinuous	2754·5	
212·17	Faint, short	*2719·7*	
213·97	Faint, long	2711·5	
220·45	Faint, long	2683·8	
226·64	Faint, short	2657·0	
238·96	Weak, discontinuous, fine	2607·6	
242·97	Very faint and short	*2592·3*	
243·75	Very faint and short	*2589·3*	
244·84	Very faint and short	*2585·1*	
245·81	Weak, discontinuous, fine	*2581·4*	
247·58	Weak, short	*2574·8*	
249·01	Weak, short	*2569·4*	
252·31	VERY STRONG, CONTINUOUS, broad, with a nimbus much extended	2557·3	

THE Spectrum of Zinc (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
258·42	Faint, very short	*2535·0*	These lines appear as dots on the photographs.
259·17	Faint, very short	*2532·3*	
260·85	STRONG, very short	*2526·3*	
262·26	STRONG, very short	*2521·3*	
264·12	STRONG, very short	*2514·7*	
265·85	STRONG, very short	*2508·7*	
267·95	Very strong, broad, sharp, continuous, with a nimbus, extended	2501·5	
269·23	Very faint, short	*2497·0*	The lines more refrangible than this, described as very short, appear as dots on the photographs.
269·34	Very faint, short	*2496·5*	
271·10	STRONG, short, nebulous	*2490·4*	
272·44	STRONG, very short	*2485·9*	
272·70	Weak, very short	*2485·0*	
273·10	Faint, very short	*2483·7*	
274·43	Faint, very short	*2479·2*	
276·56	Weak, very short	*2472·2*	
277·70	Faint, very short	*2468·3*	
278·38	Weak, very short	*2465·9*	
279·40	Faint, very short	*2462·8*	
279·88	Weak, very short	*2461·3*	
280·37	Faint, very short	*2459·8*	
283·44	Weak, very short	*2450·0*	
286·02	Weak, very short	*2441·6*	
287·29	Weak, very short	*2437·7*	
288·51	Faint, very short	*2433·9*	
290·73	STRONG, very short	*2427·0*	
291·91	Weak, very short	*2423·3*	
292·75	Faint, very short	*2420·7*	
293·35	STRONG, very short	*2418·8*	
296·69	Weak, very short	*2408·4*	
297·71	Weak, very short	*2405·3*	
298·79	Very faint and short	*2401·9*	
299·81	Very faint and short	*2398·7*	
300·60	Very faint and short	*2396·4*	
301·68	Very faint and short	*2393·3*	
302·76	Very faint and fine, rather long	*2390·1*	
304·79	Very faint and short	*2384·2*	
305·30	Very faint, fine, discontinuous	*2382·8*	
309·10	Very faint, short	*2371·7*	
310·43	Very faint, short	*2367·8*	
317·10	Weak, very short	*2348·7*	
317·79	Very faint, fine, discontinuous	*2346·7*	
324·04	Very faint and short	*2329·3*	
329·30	Weak, very short	*2315·0*	
331·58	Weak, very short	*2308·8*	
347·76	Faint, very short, fine, sharp	*2267·0*	
352·62	Faint, very short, fine, sharp	*2255·0*	
402·91	Weak, continuous, nebulous	*2138·5*	
419·05	Faint, discontinuous, fine	*2104·2*	CORNU measured this quadruple group as a double line, λ of the more refrangible line =2098·8.
420·04	Faint, discontinuous, fine	*2102·0*	
421·51	Faint, continuous, nebulous	*2099·0*	
422·95	Very faint, discontinuous	*2095·9*	
428·02	Faint, short, nebulous	*2085·4*	CORNU did not measure these lines.
431·87	Faint, short, fine	*2077·6*	
436·50	Very faint, short	*2068·4*	
439·37	Faint, discontinuous, nebulous	*2062·8*	
440·35	Faint, continuous, nebulous	*2060·8*	
460·30	Faint, continuous, nebulous	*2024·2*	

THE Spectrum of Cadmium.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
2·45	Faint, fine, discontinuous	*4799·0*	The strongest lines of cadmium are remarkable for being much extended, more so than those of zinc or even magnesium. The nimbus surrounding the electrodes is larger than that of zinc, but much less than that of the magnesium lines.
7·85	Weak, fine, discontinuous	4676·7	
20·93	Weak, fine, continuous, extended	4414·5	
32·53	Faint, very short	*4215·8*	
36·15	Faint, very short	*4158·0*	
37·29	Faint, very short	*4141·0*	
38·19	Faint, very short	*4127·4*	
39·01	Faint, very short	*4115·2*	
47·93	Faint, very short	*3987·6*	
48·75	Weak, very short	*3976·3*	
49·89	Weak, very short	*3974·5*	
51·39	Weak, very short	*3940·0*	
58·29	Faint, very short	*3851·0*	
61·64	Faint, very short	*3810·0*	
(72·59)	(Faint, fine)	(3682·6)	Line of doubtful origin. Like an impurity, as it proceeds from one electrode only.
79·37	STRONG, CONTINUOUS, greatly extended, with a nimbus	3611·8	A pair appearing like a single line.
79·68	VERY STRONG, continuous, much extended, with a nimbus	3609·6	
87·06	Weak, fine, discontinuous	3535·0	
90·88	Weak, short, nebulous	3498·2	
94·30	STRONG, continuous, much extended, with a nimbus	3466·8	A pair appearing like a single line.
94·50	VERY STRONG, continuous, much extended, with a nimbus	3465·4	
101·45	VERY STRONG, continuous, extended	3402·9	
103·56	Weak, very short	*3384·7*	These lines appear somewhat nebulous, being very short and crowded together.
115·76	Faint, very short	*3285·3*	
116·06	Weak, very short	*3282·9*	
116·87	Weak, very short	*3276·4*	
118·45	Weak, very short	*3264·1*	
119·00	STRONG, continuous, extended, fine	3260·2	
120·04	Weak, fine, continuous	3251·8	
120·42	STRONG, FINE, continuous, extended, weak in centre	3249·5	
122·21	Faint, very short	*3233·6*	
123·91	Faint, very short	*3222·6*	
124·30	Faint, very short	*3219·9*	
124·77	Weak, very short	3216·0	
125·39	Faint, very short	*3211·8*	
125·79	Weak, very short	3209·0	
126·91	Faint, very short	*3200·6*	
127·49	Weak, very short	*3196·8*	
127·78	Weak, very short	*3194·9*	
129·21	STRONG, very short	*3185·1*	
129·62	Faint, very short	*3181·5*	
130·13	Faint, very short	*3177·9*	
130·40	Faint, very short	*3176·1*	
130·87	STRONG, very short	*3172·9*	
132·60	STRONG, very short	*3161·0*	
133·30	Weak, very short	*3156·6*	
133·78	Faint, very short	*3152·7*	
136·80	Weak, continuous, fine	3132·5	

The Spectrum of Cadmium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
137·34	STRONG, very short	3129·4	
138·05	Weak, very short	3123·6	
138·47	Weak, very short	3120·9	
138·92	Fairly strong, very short	3117·8	
139·80	Weak, very short	*3112·0*	
142·38	STRONG, very short	3095·0	
143·05	Faint, very short	*3090·5*	
143·48	Faint, very short	*3087·7*	
144·10	STRONG, very short	3084·3	
144·60	Weak, continuous, fine	3080·2	This line appears strongest in the centre.
145·24	Weak, very short	*3076·7*	
145·78	Faint, very short	*3073·2*	
146·61	Fairly strong, very short	*3067·8*	
147·21	STRONG, very short	*3064·0*	
148·13	Fairly strong, very short	3058·4	Coincident with an air line.
149·10	Fairly strong, very short	3052·3	
149·75	Fairly strong, very short	3048·2	
151·88	Faint, very short	*3034·9*	
153·68	Faint, very short	*3023·8*	
154·93	Weak, very short	*3016·1*	
155·48	Faint, very short	*3013·8*	
157·16	Faint, very short	*3002·5*	
158·47	Weak, very short	*2994·8*	
159·93	Weak, very short	*2986·1*	
161·05	STRONG, CONTINUOUS, extended	2979·9	
162·70	Weak, very short	*2970·2*	
163·95	Faint, very short	2964·5	
165·96	Faint, very short	2951·4	
166·75	*Fairly strong*, short	2947·1	
173·50	Weak, very short	*2909·9*	
179·07	STRONG, CONTINUOUS, extended	2880·1	
181·37	Weak, continuous, fine	2868·0	In some photographs this line appears strongest in its central portion, more especially when the spark is strong and the electrodes near together. Though continuous it does not extend from pole to pole.
187·65	*Fairly strong*, continuous	2836·1	
188·22	Faint, short	*2833·0*	
188·30	Faint, fine, continuous	*2832·3*	Strongest in its central portion.
193·37	Faint, very short	*2807·3*	
194·01	Weak, very short	*2804·0*	
199·10	Faint, very short	*2779·8*	
200·18	Faint, continuous, fine	2774·5	Strongest in its central portion.
202·04	Weak, very short	2766·5	
202·58	Weak, fine, continuous	2763·1	Strongest in its central portion.
205·87	VERY STRONG, CONTINUOUS, broad, with a nimbus, and greatly extended	2747·7	
210·72	Weak, very short	2726·9	
215·34	Faint, very short, fine	2706·0	
222·05	Weak, continuous, fine	2677·2	
226·49	Very faint and short	*2658·5*	Strongest in its central portion.
228·67	Very faint and short	*2649·4*	
229·62	Very faint and short	*2645·4*	
231·05	Very faint and short	*2639·5*	
231·12	Weak, continuous, fine	2639·7	

THE Spectrum of Cadmium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
232·09	Very faint and short	*2635·3*	Strongest in its central portion.
232·75	Very faint and short	*2632·7*	
232·84	Very faint, continuous, fine	*2632·3*	
233·45	Very faint and short	*2630·2*	
233·61	Very faint, continuous, fine	*2629·1*	
234·69	Very faint and short	*2624·8*	
236·35	Weak, very short	2618·0	
237·39	Faint, fine, continuous, sharp	*2614·0*	
238·13	Very faint and short	*2611·0*	
240·79	Very faint and short	*2600·8*	
241·30	Very faint and short	*2598·8*	
242·19	Very faint and short	*2595·3*	
243·08	Very faint and short	*2592·0*	
244·16	Very faint and short	*2587·8*	
244·86	Very faint and short	*2585·0*	
248·24	VERY STRONG, CONTINUOUS, broad, with a nimbus and greatly extended	2572·2	
250·70	Very faint and short	*2563·2*	
252·30	Very faint and short	*2557·4*	
252·96	Very faint and short	*2555·0*	
253·72	Weak, very short	2551·6	
255·08	Very faint and short	*2547·2*	
255·73	Faint, fine, continuous	2544·5	The central portion of the line only visible.
258·50	Weak, short	2499·6	
272·02	Weak, short	2488·2	
277·50	*Fairly strong*, fine, discontinuous	2469·3	
293·32	Weak, short, fine	2418·5	
307·11	Faint, very short, fine	*2377·3*	
307·75	Faint, very short, fine	*2376·6*	
323·80	STRONG, CONTINUOUS, weak in centre	2329·5	
326·80	VERY STRONG, CONTINUOUS, with a nimbus, weak in centre	2321·6	
329·85	VERY STRONG, CONTINUOUS, broad, with a strong nimbus, extended	2313·6	
332·22	STRONG, CONTINUOUS, fine, weak in centre	2307·0	
339·25	VERY STRONG, CONTINUOUS, broad	2288·9	
347·20	Weak, discontinuous	2268·6	
348·15	VERY STRONG, continuous, broad, with a nimbus and slightly extended	2265·9	
354·60	Weak, short	2249·2	
358·35	*Fairly strong*, discontinuous, fine	2241·4	
364·73	Weak, short	2227·0	
373·55	Weak, continuous	2206·2	
377·48	STRONG, BROAD, continuous, nebulous, weak in centre	2196·4	
400·20	STRONG, BROAD, continuous, nebulous	2146·8	
415·60	Faint, very short, nebulous	*2111·5*	

Five lines of zinc appear in the photographs, which proceed from one electrode only, their wave-lengths are 3344·7, 3302·1, 3282·0, 2557·3, 2501·5.

The Spectrum of Aluminium.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
15·5	Strong, short	4511·0	A comparison of the map with the photograph of this spectrum will show that most of the short lines photographed are due to iron or other impurities.
15·85	Strong, short	4518·3	
17·66	Strong, very short	4477·2	
19·42	Strong, very short	4445·2	
49·85	Very strong, continuous, sharp, extended	3960·9	
51·16	Very strong, continuous, sharp, extended	3943·4	
70·02	Strong, short	3713·4	
71·03	*Fairly strong*, short	3701·5	
79·17	A triplet, very strong	3612·4	
80·50	The lines are short and extended, the	3601·1	
82·07	most refrangible being the strongest and most extended	3584·4	
142·86	Very strong, continuous, sharp, extended	3091·9	
144·5	Very strong, continuous, sharp, extended	3081·2	
147·10	*Fairly strong*, short, fine	*3065·0*	
147·40	*Fairly strong*, short, fine	*3062·8*	
148·12	*Fairly strong*, short, fine	*3058·5*	
148·50	Strong, short, fine, sharp, slightly extended	3056·4	
148·90	*Fairly strong*, short, fine	*3053·6*	
149·60	*Fairly strong*, short, fine	*3049·1*	
179·00	Strong, fine, discontinuous, slightly extended	2879·9	
191·76	Very strong, discontinuous, broad, sharp, much extended	2815·3	
226·30	*Fairly strong*, discontinuous, fine	2659·3	
228·26	*Fairly strong*, discontinuous, fine	2651·2	
233·25	Very strong, short, broad, with a nimbus, extended	2630·6	
247·75	Strong, discontinuous, extended.	2574·1	
249·66	Strong, discontinuous, extended.	2566·9	
308·55	Strong, discontinuous, nebulous.	*2373·3*	
309·00	Strong, short, nebulous	*2372·0*	
309·60	Weak, short, fine	*2370·2*	
309·94	Weak, short, fine	*2367·2*	
310·62	Strong, discontinuous, nebulous.	*2364·5*	

This spectrum was from carbon electrodes, kept moistened with a strong solution of pure aluminium chloride.

The photographs show faintly at one pole the two lines of copper, wave-lengths 3273·2 and 3246·9. A number of iron lines also appear; their wave-lengths have not been determined.

THE Spectrum of Indium.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
7·60	STRONG, short, fine	4681·5	
8·83	STRONG, short, fine	4655·2	
9·58	STRONG, short, fine	4637·0	
15·88	VERY STRONG, CONTINUOUS, fine, extended	4510·2	
30·10	STRONG, and very short	4253·1	
39·91	VERY STRONG, similar to line 4510·2	4101·3	
41·84	VERY STRONG, very short	4071·6	
42·53	Weak, very short	*4063·5*	
44·61	VERY STRONG, very short	4032·7	
45·20	*Fairly strong*, very short	*4025·6*	
58·10	VERY STRONG, short	3852·8	
	Fairly strong, short	3840·5	
59·53	VERY STRONG, short	3834·7	
62·92	Faint, short, nebulous	*3794·8*	
106·56	Faint, short	*3359·5*	
119·31	VERY STRONG, CONTINUOUS, fine, sharp, extended	3257·8	
119·68	STRONGER, CONTINUOUS, broader, more extended, with a nimbus	3255·5	
120·74	Weak, continuous, fine	3246·1	
122·15	Weak, short, fine, sharp	3236·2	
129·02	Weak, discontinuous, fine	3186·2	
130·72	Faint, very fine in centre, thins away at each end	3174·1	Appears to be a tin line.
132·81	Weak, short nebulous	3159·7	
134·26	Weak, short nebulous	3148·6	
149·97	Weak, discontinuous, fine	3047·0	Appears to be a tin line.
151·35	VERY STRONG, CONTINUOUS, broad, extended	3038·7	
156·38	VERY STRONG, short, broad, with a nimbus	3008·0	
160·70	VERY STRONG, short, broad, with a nimbus	2982·3	
165·115	Faint, continuous, very fine	2956·1	The central portion of this line only is visible.
168·00	VERY STRONG, CONTINUOUS, broad, extended, fine in the centre	2940·8	
169·56	STRONG, CONTINUOUS, fine, extended	2932·3	
177·34	VERY STRONG, discontinuous, sharp, extended	2889·8	
183·50	Faint, continuous, very fine	2857·1	
187·00	Very faint and fine, continuous	2839·2	
187·70	Weak, fine, continuous	2836·0	
188·38	Very faint and fine, continuous	2832·1	Between these two lines two or three broad nebulous dots occur. They are too faint to measure.
204·94	*Fairly strong*, continuous, fine	2752·8	
205·34	Weak, short, nebulous	2750·7	
208·01	Faint, short, fine	*2738·1*	
210·56	Faint, short, broad, nebulous	*2727·0*	
213·70	Weak, continuous, fine	2712·9	
214·56	STRONG, sharp, continuous, extended	2709·3	In some photographs this line is strongest in its central portion.
215·32	Very faint and fine, continuous	2706·4	
233·27	Weak, short, nebulous	2631·2	
238·21	Very faint and fine, continuous	*2610·8*	
240·74	Weak, fine, continuous	2602·5	Strongest in its central portion.
241·34	Weak, fine, short	2600·2	
243·32	Weak, fine, short	2591·0	
244·42	Weak, fine, short	2586·6	
250·32	Weak, discontinuous, fine	2564·7	
251·76	STRONG, continuous, fine, extended	2559·5	
253·30	Weak, short, fine	2554·1	

The Spectrum of Indium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
255·78	Weak, short	2545·8	
260·74	Strong, very short	2527·1	
262·45	Weak, fine, continuous	2520·9	Strongest in the central portion.
270·93	Faint, short.	2492·7	
272·65	Faint, short, very fine	2485·5	
272·90	Faint, short, very fine	2485·1	
274·67	A very faint nebulous dot	2478·3	
277·34	*Fairly strong*, fine, discontinuous	2470·2	
278·00	Weak, very fine, sharp, only visible in central portion	2468·4	
279·68	Faint, very short dot	2462·5	
280·30	*Fairly strong*, continuous	2460·8	
280·42	Faint, very short dot	2460·3	Appears to be nearly coincident with previous line.
284·25	Faint, very short dot	*2447·4*	
285·40	Faint, very short dot	*2443·7*	
288·63	Weak, very short dot	2433·6	
289·43	Weak, very short dot	*2431·0*	
289·89	Very faint and fine, continuous	2429·0	The central portion only of this line is visible.
290·18	Weak, discontinuous, fine	2428·6	
291·99	Weak, fine, sharp, discontinuous	2423·2	
292·11	Weak, fine, sharp, discontinuous	2422·8	
294·16	Weak, very short.	*2416·3*	
298·25	Weak, very short.	*2403·5*	
300·21	Weak, very short.	*2397·6*	
302·86	Faint, very fine, continuous	*2389·8*	The central portion only of this line is visible.
303·50	Faint, short	*2388·0*	
304·54	Weak, very short	*2385·9*	
305·87	Weak, very short	*2381·0*	
309·44	Weak, short, very fine, one half stronger than the other	*2370·7*	
314·24	Faint, very short	*2357·0*	
314·65	Very faint and fine, continuous	*2355·8*	
314·75	Faint, very short	*2355·4*	
315·30	Faint, very short	*2353·8*	
316·12	Strong, discontinuous	2351·3	
322·96	Faint, very fine, continuous	*2332·2*	The central portion only of this line is visible.
332·20	Very strong, sharp, continuous, extended	2306·9	
338·96	Faint, short	*2289·3*	
339·65	Faint, short	*2287·8*	
(343·44)	Weak, fine, sharp, half line	(*2278·0*)	This is probably an impurity.
348·82	Weak, short, fine, sharp	*2264·4*	
349·06	Weak, very short, nebulous	*2263·8*	
354·93	Weak, short, fine, sharp	*2249·2*	
356·74	Weak, short, fine, sharp	*2245·7*	
(361·82)	Faint, short, fine, sharp, half line	*2232·2*	These are probably due to impurities.
(362·32)	Faint, short, fine, sharp, half line	*2231·0*	
(366·80)	Weak, short, fine, sharp, one half stronger than the other	*2220·2*	These are probably due to an impurity.
(370·04)	Weak, short, fine, sharp, one half stronger than the other	*2212·4*	
372·92	Faint, discontinuous, fine	*2205·5*	
374·44	Faint, short, nebulous	*2202·0*	
377·82	Weak, fine, short	*2194·0*	
378·94	Weak, fine, short	*2191·2*	
383·42	Weak, short, fine, sharp	*2181·0*	
394·81	Faint, very short, nebulous	*2155·8*	
403·20	Faint, short, fine	*2137·8*	
431·61	Faint, discontinuous, broad, nebulous	*2078·1*	

The Spectrum of Thallium.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
5·09	Faint, short	*4740·0*	Very great extension is characteristic of some of the strongest lines in this spectrum.
29·18	Weak, very short	4270·5	
36·43	Weak, very short	*4152·7*	Coincident with an air line.
39·36	STRONG, very short	4109·4	
42·92	Weak, discontinuous, fine, sharp	4057·2	Coincident with an air line.
46·38	Weak, very short	*4009·2*	
51·90	STRONG, very short	3932·7	
63·37	Weak, very short	*3790·0*	
64·55	VERY STRONG, CONTINUOUS, sharp, very much extended	3775·6	
72·80	Weak, very short	*3682·2*	
73·50	Weak, very short	*3674·6*	
74·95	Faint, very short	*3658·9*	
75·50	Faint, very short	*3652·9*	
87·75	VERY STRONG, FINE, CONTINUOUS, extended	3528·8	
88·70	VERY STRONG, broad, continuous, sharp, very much extended, with a nimbus on more refrangible side	3518·6	
89·44	Weak, very short	3512·7	
89·90	Weak, very short	*3507·8*	
95·55	STRONG, short	3455·8	
104·03	STRONG, short, fine	3381·3	
105·44	Very faint and short	*3369·1*	
108·06	Weak, very short	*3347·4*	
113·95	Faint, very short	*3299·6*	
114·72	Faint, very short	*3293·6*	
115·34	Very faint and short	*3288·6*	
117·45	Faint, very short	*3271·6*	
120·74	Faint, fine, sharp, discontinuous	*3246·6*	
123·22	STRONG, CONTINUOUS, fine, sharp, extended	3229·0	
125·06	Weak, short	*3214·2*	
127·59	Faint, short	*3195·6*	
128·87	Faint, short, broad, nebulous	*3186·6*	
132·43	STRONG, short	3162·6	
134·68	Faint, short	*3146·7*	
138·72	Faint, short	*3119·4*	
139·90	Faint, short	*3111·4*	
140·73	Faint, short	*3105·7*	
143·02	VERY STRONG, CONTINUOUS, sharp, weak in the centre	3091·0	
171·50	STRONG, CONTINUOUS, fine, sharp	2920·8	
172·21	VERY STRONG, CONTINUOUS, broad, extended	2917·7	
176·49	Very faint, fine, continuous	*2893·9*	
185·06	Faint, short, nebulous	*2848·6*	
187·43	Faint, short	*2836·7*	
189·73	Weak, continuous, sharp, fine	2825·4	
192·32	Faint, short, nebulous	*2812·5*	
201·87	VERY STRONG, broad, with faint nimbus, much extended	2767·1	
214·63	Faint, continuous, fine	*2709·4*	
214·89	STRONG, CONTINUOUS, fine	2708·6	
217·72	Faint, short	2700·1	
224·075	Faint, short	2669·1	

THE Spectrum of Thallium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
224·98	Weak, continuous, fine, sharp . . .	2665·0	
237·82	Weak, continuous, nebulous	2608·7	
246·30	STRONG, CONTINUOUS, fine, sharp . . .	2579·7	
253·80	Faint, long, fine	2551·6	
259·86	STRONG, long, with faint nimbus . . .	2530·0	
274·93	Weak, fine, continuous	2477·7	
277·68	*Fairly strong*, short	2468·9	
282·85	STRONG, short	2451·9	
301·30	*Fairly strong*, short	2394·8	
306·18	STRONG, CONTINUOUS, with faint nimbus	2380·0	
311·27	*Fairly strong*, short	2364·8	
319·32	Faint, short, nebulous	2343·1	
328·61	Weak, discontinuous, fine, sharp . .	*2257·0*	
335·27	STRONG, LONG, fine, with faint nimbus .	2299·3	
351·77	Faint, short	*2257·0*	
357·10	Faint, short	*2243·7*	
359·05	Faint, continuous	*2239·0*	
368·36	Faint, short	*2217·0*	
371·08	Faint, short	*2210·0*	Coincident with a tin
373·80	Weak, short	*2208·5*	
402·75	Faint, short	*2189·0*	

THE Spectrum of Copper.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
	Weak, short	4274·2	No scale number was recorded for this line.
80·63	Weak, short	*3598·9*	
80·85	Weak, short	*3596·6*	
88·21	Faint, short	*3523·6*	
89·51	Faint, short	*3510·4*	
92·55	Faint, short	*3483·2*	
93·02	Faint, short	*3478·8*	
93·81	Faint, short	*3471·6*	
95·56	Faint, short	*3455·8*	
96·17	Faint, short	*3450·1*	
(104·01)	Very faint, short, half line	(*3381·0*)	Probably a silver line with wave-length 3382·1.
113·10	*Fairly strong*, short	3306·8	
115·10	*Fairly strong*, short	3289·9	
116·10	Weak, about one half scarcely visible .	3282·1	
(116·41)	Faint, one half only visible	(3280·1)	Probably the silver line with wave-length 3280·1.
117·25	VERY STRONG, SHARP, continuous, much extended	3273·2	
118·38	Weak, short	3265·2	Coincident with an air line.
119·05	Faint, short	3260·2	Coincident with an air line.
120·705	A LITTLE STRONGER, more extended than line 3273·2; in other respects similar	3246·9	
(121·10)	Faint, short, half line	(*3243·9*)	Probably an impurity.
(122·35)	Faint, short, half line	(*3233·4*)	Probably an impurity.

The Spectrum of Copper (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
135·63	Faint, short	3139·7	There are five faint half lines slightly less refrangible than this, too faint to be measured.
136·40	Faint, short	3134·2	
138·08	Weak, short	*3123·7*	Approximations.
139·28	Faint, short	*3115·7*	
140·48	Weak, short	*3107·4*	
141·92	Faint, short	*3097·8*	
151·35	Faint, short	3035·6	
153·90	Faint, short	3023·4	
164·53	Weak, discontinuous	2959·6	
178·55	Faint, short	2882·4	
179·56	Weak, short	2877·4	
187·55	Weak, short	2836·5	
190·13	Weak, short	2823·2	
201·36	Strong, short, sharp	2769·1	
201·81	Weak, short	2766·2	
206·35	Weak, short	2745·9	
211·80	A triplet of short fine lines, the least refrangible weak, the other two strong	2721·2	
212·55		2718·4	
213·71		2713·1	
216·10	Strong, short, fine line	2702·7	
216·58	Strong, short, fine line	2700·5	
219·37	Strong, short, fine line	2688·8	
224·70	Weak, short	2666·0	
230·31	Very faint and short	2643·5	
236·45	Weak, fine, discontinuous, fine . . .	2617·8	
238·87	Faint, short, fine	2608·9	
241·10	Strong, short, fine line	2599·7	
241·58	Strong, short, fine line	2598·3	
243·66	Weak, short, fine	2590·1	
248·00	Very faint, short, nebulous	*2573·0*	There are two or three very faint lines similar in character to this line, less refrangible, too faint to be measured.
248·28	Very faint, short, nebulous	*2572·0*	
248·61	Faint, short	2570·9	
250·11	Faint, short, nebulous	*2565·3*	
253·29	Very faint, short, nebulous	*2553·7*	
253·70	Faint, short, nebulous	*2552·2*	
255·94	Very strong, discontinuous	2544·6	
257·65	Faint, very short, nebulous	2538·2	Approximations.
258·53	Faint, very short, nebulous	2533·9	
259·25	Faint, very short, nebulous	2531·4	
260·25	A pair of strong short lines, the more refrangible line slightly weaker	2528·8	
261·00		2526·2	
261·95	Very faint and short, nebulous . . .	2522·7	Approximations.
262·21	Very faint and short, nebulous . . .	2522·1	
263·22	Very faint and short, nebulous . . .	2518·3	
263·79	Very faint and short, nebulous . . .	2517·5	
264·79	Very faint and short, nebulous . . .	2513·2	
265·33	Very faint and short, nebulous . . .	2512·2	Approximations.
266·20	Weak, very short	2508·7	
266·77	Strong, short, sharp	2506·2	
269·29	Very faint and short, fine	2497·4	
269·71	Very faint and short, fine	2495·9	
270·91	Weak, very fine, sharp, long line . .	2491·4	
271·65	Strong, sharp, short, fine line . . .	2489·1	
272·72	Short, strong line	2485·6	
273·74	Weak, very fine, and short	2481·8	

THE Spectrum of Copper (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
274·89	Faint, very fine and short	2478·2	
275·87	Very faint, short	2475·1	
276·45	*Fairly strong* short line	2473·2	
277·88	Weak short line	2468·4	
278·57	Very short, nebulous	*2465·2*	
279·81	Very short, nebulous	*2461·5*	
280·85	Very short, nebulous	*2458·2*	
282·63	Very short, nebulous	*2452·5*	
284·45	Very short, nebulous	*2446·7*	
285·31	Weak, very short	2444·1	An approximation.
286·15	Weak, very fine, discontinuous	2441·6	
286·63	Very faint and short	*2439·9*	
287·93	Very faint and short	*2435·7*	
289·63	Very faint and short	*2430·3*	
290·35	Very faint and short	*2428·2*	
291·51	Weak, short, fine	2425·1	
292·33	Very faint, short	*2422·0*	
295·51	Weak, very short, fine	2412·2	
297·68	Weak, very short, fine	2404·8	
298·31	STRONG, SHORT, fine line	2403·3	
299·40	STRONG, SHORT, fine line	2400·1	
301·90	Very faint, fine	2393·0	
302·21	Very faint and fine, very short	2392·2	
304·39	Very faint, short, fine	2385·2	
307·35	Weak, short, sharp, fine	2376·7	
309·17	Faint, fine, short	2371·6	
309·57	VERY STRONG, LONG, broad	2370·1	
310·13	Faint, fine, short line	2368·7	
314·07	Fairly strong, short	2357·2	
314·59	Faint, short	2355·0	
316·83	Faint, fine, short	2348·8	
317·79	Faint, fine, short	2346·2	
321·25	Weak, short	*2336·6*	
333·46	Very faint, short, and very fine	*2303·8*	
334·7	Very faint, short	*2300·5*	
335·88	Very faint, short	*2297·5*	
336·80	STRONG, SHARP, FINE, discontinuous, extended	2295·0	
336·98	Weak, sharp, fine, discontinuous, less extended	2294·6	
338·02	Weak, short, fine	2291·4	
339·66	Weak, short, fine	2286·7	
342·87	Faint, short, fine	*2279·6*	
343·67	STRONG, FINE, discontinuous, extended, somewhat nebulous on more refrangible side	2277·0	
347·92	Faint, short line	2265·8	
348·55	Weak, fine, short, nebulous	2263·9	
348·78	Weak, fine, short, nebulous	2263·2	
352·05	Faint, short	2257·7	
354·41	Faint, short	2250·0	
355·27	A PAIR of double lines, each consisting of a rather long, VERY STRONG, sharp, fine, and extended line, and a weak, discontinuous, nebulous line	2248·2	
355·50		2247·7	
357·10		2244·0	
357·32		2243·5	

The Spectrum of Copper (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
361·52	A group of six fine sharp lines, *the third and fourth, fairly strong,* rather long and extended, are the strongest and longest of the group. A faint nimbus about the central portion of the group	*2233·0*	
361·82		*2232·2*	
362·23		*2231·2*	
362·76		*2230·0*	
363·12		*2229·1*	
363·52		*2228·1*	
364·00	Very faint, short, and very fine line .	*2227·0*	
364·37	Very faint, short, and very fine line .	*2226·0*	
367·20	STRONG, FINE, sharp, discontinuous . .	*2219·3*	
367·50	Weak, nebulous, discontinuous . . .	*2218·5*	
368·34	Weak, short, nebulous	*2216·5*	
368·62	Faint, short, fine	*2215·8*	
369·34	Faint, short, fine	*2214·1*	
370·47	STRONG, FINE, sharp, discontinuous . .	*2211·3*	
370·74	Weak, nebulous, discontinuous . . .	*2210·8*	
374·56	Faint, very short	*2208·8*	
375·06	Weak, short, fine	*2200·3*	
375·28	Very faint, short, nebulous	*2199·8*	
376·68	Weak, short	*2196·5*	
378·67	*A pair* of double lines, each consisting of a STRONG, short, fine, sharp, extended, and a weak, short, nebulous line	*2192·0*	
378·96		*2191·2*	
379·61		*2189·6*	
380·12		*2188·5*	
383·37	A very faint, short line	*2181·0*	
384·30	*Rather strong,* short line	*2179·0*	
384·68	Weak, nebulous, short line	*2178·0*	
386·29	Weak, short, fine	*2174·5*	
398·07	Weak, short, fine	*2148·8*	
404·18	Weak, short, fine	*2135·8*	
404·93	Faint, short, nebulous	*2134·2*	
409·08	A pair of double lines, each consisting of a weak, fine, short, and a faint, nebulous, short line	*2124·4*	
409·29		*2124·0*	
410·63		*2122·1*	
410·92		*2121·5*	
413·45	Very faint, short, fine	*2116·0*	
416·07	Very faint, short, fine	*2110·5*	Possibly a double line.
419·61	Very faint, short, fine	*2103·0*	

THE Spectrum of Silver.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
86·39	Faint, short	*3541·3*	
101·42	Faint, short	3404·2	
103·06	Faint, short	3398·7	
103·94	VERY STRONG, sharp, continuous, much extended	3382·3	
107·49	Faint, short, nebulous	3351·8	Coincident with Sn. 3351·8
112·47	Faint, short	*3311·6*	
113·15	Faint, short	*3306·1*	
113·85	Double, two similar lines, faint and short	*3300·6*	
114·05		*3299·0*	
114·89	Faint, short, nebulous	*3292·3*	
115·35	Faint, short, nebulous	*3288·6*	
116·45	VERY STRONG, forming a pair with previous strong line, with which it is similar in character but a little stronger	3280·1	
117·30	Faint, short, nebulous	*3272·8*	
118·47	Faint, short, nebulous	3265·2	
119·03	Faint, short, nebulous	3260·2	
120·05	Faint, short, nebulous	*3251·8*	
121·10	Weak, short	*3243·8*	
122·69	Faint, short, nebulous	*3231·8*	
123·13	Faint, short, nebulous	*3228·6*	
123·95	Faint, rather longer than foregoing, nebulous	*3222·3*	
124·81	Faint, short, nebulous	*3216·0*	
125·89	Faint, short, nebulous	*3208·1*	
127·16	Faint, short, nebulous	*3198·8*	
128·30	Faint, short, nebulous	*3190·6*	
129·30	Faint, short, nebulous	*3183·7*	All the foregoing short lines except the first really form a group of lines remarkably similar in character, and for the most part equi-distant from one another.
129·96	Faint, short, nebulous	*3179·2*	
130·70	Faint, short, nebulous	3174·3	
136·40	Very faint, short	*3134·9*	
137·25	Very faint, short	*3129·2*	
168·50	*A triplet* of short lines, the least refrangible being weak, the other *two fairly strong*	2937·4	
169·30		2933·5	
170·17		2928·2	
171·75	Weak, short	2919·1	
175·025	*A pair* of *fairly strong* short lines	2901·6	
176·075		2895·6	
180·44	*Fairly strong*, short	2872·7	
191·82	*Fairly strong*, short	2814·5	There are 7 or 8 very faint nebulous dots between these lines too faint to be measured.
195·03	*Fairly strong*, short	2798·9	
201·81	STRONG, SHORT, sharp, fine, much extended	2766·4	
204·20	STRONG, FINE, shorter and less extended	2755·5	
207·06	Faint, very fine, short	2742·9	
211·96	Very faint, short, very fine	*2720·6*	
214·22	STRONG, SHORT, broad, much extended	2711·3	
221·20	*Fairly strong*, short	2680·5	
226·27	STRONG, SHORT, extended, fine, sharp	2659·6	
227·08	Weak, short, fine	2656·2	
234·10	*Pair* of short fine lines	2627·3	
234·77		2625·2	

The Spectrum of Silver (continued).

Scale-numbers.	Description of lines.	* Wave-lengths.	Remarks.
237·57	Pair of weak, fine, short lines . .	2613·7	
239·75		2605·4	
241·43	Very faint, short, fine	*2598·2*	
242·38	Faint, short, fine	*2594·7*	
246·30	STRONG, SHORT, fine	2579·9	
249·98	Faint, short, nebulous	*2565·8*	
250·70	Faint, short, nebulous	*2563·2*	
251·16	Stronger but shorter, fine	*2561·5*	
253·73	Very faint, short, fine	*2552·0*	
258·73	STRONG, SHORT, extended, very fine, sharp	2534·5	
268·81	STRONG, SHORT, extended, sharp . . .	2506·0	
267·50	Weak, short, fine.	2503·6	
272·63	Faint, short, fine	2486·4	
272·87	Weak, short, fine	2485·4	
274·52	*Fairly strong*, short, fine	2479·9	
275·41	*A little stronger*, short fine.	2476·8	
276·41	STRONGER, LONGER, and extended, fine on less, but nebulous on more refrangible side.	2473·3	
277·69	Faint, very fine, sharp	*2469·0*	
279·92	*A pair of fairly* strong, short lines.	2462·2	
280·52		2459·8	
282·60	STRONG, short	2453·0	
284·38	VERY STRONG, discontinuous, extended, sharp	2447·4	
284·79	Weak, fine, short.	*2445·7*	
285·41	Fairly strong, short	2443·9	
287·46	VERY STRONG, continuous, broad, extended, forming a pair with line 2447·4	2437·3	
290·00	VERY STRONG, sharp, extended . . .	2429·8	
290·14	Weak, short, fine.	2428·8	
292·18	Faint, short, very fine	2422·8	
293·08	STRONG, short, fine	2419·9	
294·72	Very faint, short.	*2414·5*	
295·35	VERY STRONG, continuous, extended, broad, nebulous on more refrangible side, sharp on less	2413·3	
295·94	Discontinuous, broad, less strong, and extended, otherwise similar in character	2411·3	
296·41	Very faint, short	*2409·3*	
297·94	Faint, short, fine	2406·4	
298·85	Faint, short, fine	2404·5	
301·10	Faint, short, fine	2395·7	
301·92	Very faint, short, fine	2393·3	
302·74	*Fairly strong*, short, fine	2390·8	
304·07	Faint, short, fine	2386·7	
304·20	Faint, short, fine	2386·2	
305·25	Faint, short, fine	2383·6	
307·94	*Fairly strong*, short, broad, nebulous .	2375·5	
311·05	Weak, short, fine	2365·8	
311·70	*Fairly strong*, fine	2364·3	
312·34	*Fairly strong*, fine	2362·3	
313·47	*Fairly strong*, short, fine, slightly extended	2359·2	

The Spectrum of Silver (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
313·88	STRONG, SHORT, extended, fine	2358·1	
318·93	Very faint, fine, short	*2343·7*	
319·42	Very faint, fine, short	*2342·1*	
320·47	Very faint, fine, short	*2339·2*	
322·81	Very faint and short, fine	*2332·5*	
323·35	VERY STRONG, discontinuous, broad, much extended, sharp on less refrangible edge, and *nebulous on more refrangible side*	2331·7	
325·53 325·73	DOUBLE, consisting of a STRONG broad NEBULOUS, and a VERY STRONG FINE line, *nebulous on more refrangible side*, but discontinuous and much extended	2325·8 2325·3	
326·65	Weak, short, fine	2322·3	
327·37	VERY STRONG, discontinuous, broad, much extended, fine sharp on less, and *nebulous on more refrangible side*	2320·6	
328·05	Faint, short, very fine	2319·5	
328·59	VERY STRONG, discontinuous, much extended, broad, sharp on less, and *nebulous on more refrangible side*	2317·4	
331·27	Weak, fine, short	2310·1	
336·13	Faint, very short, fine	*2296·8*	
340·04	Very faint, short, fine	*2286·7*	
342·55	VERY STRONG, broad, short, sharp on less, and *nebulous on more refrangible side*	2280·7	
343·50	Faint, very short, fine	*2277·8*	
344·40	Faint, very short, fine	*2275·3*	
352·96	Weak, short, very fine	2254·1	
354·95 355·90	PAIR OF STRONG, BROAD, short lines, sharp on less, and *nebulous on more refrangible side.*	2249·9 2247·6	
362·86	*Fairly strong*, broad, short, sharp on less, and *nebulous on more refrangible side*	2230·6	
372·73 374·44	A pair of very faint and short fine lines	*2206·0* *2202·0*	
381·21	Weak, short, fine on less, and *nebulous on more refrangible side*	*2186·0*	
390·33	Faint, short	*2165·8*	
392·33	Very faint, and very short	*2161·3*	
399·65	Weak, short, fine on less, and *nebulous on more refrangible side*	*2145·4*	
412·05	Very faint, short, nebulous	*2119·0*	
415·38	Very faint, short, nebulous	*2112·0*	

The strong lines of silver described as nebulous on the more refrangible side may be double, like those of copper, to which they are perfectly similar in character. There is, however, no appearance of their being double with the dispersion we have employed.

The Spectrum of Carbon.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
29·47	VERY STRONG, short, slightly extended	4266·3	
52·93	STRONG, very short	3919·5	
·55·85	Weak, discontinuous, extended	3881·9	This line is not given by Messrs. LIVEING and DEWAR. Coincident with an air line.
56·32	*Fairly strong*, very short	3875·7	
56·70	Weak, fine, discontinuous	3870·7	These lines are not given by Messrs. LIVEING and DEWAR.
81·57	Weak, short	3589·9	
82·05	Weak, fine, discontinuous	3584·8	
82·28	Weak, fine, discontinuous	3583·3	
131·65	Weak, very short	3167·7	
131·91	Weak, very short	3166·0	
158·70	Weak, very short, broad, nebulous	2993·1	
163·23	Weak, very short	2967·3	
187·43	STRONG, SHORT, extended	2836·7	
187·65	STRONG, SHORT, extended	2835·9	
206·20	*Fairly strong*, very short, *nebulous*	2746·6	
230·34	Weak, very short, nebulous	2640·0	
264·84	STRONG, SHORT, fine, extended	2511·6	
265·88	STRONG, SHORT, fine, extended	2508·7	
274·78	Sharp, fine, barely discontinuous, extended	2478·3	This line is all but continuous. It is the longest line in this spectrum.
335·89	STRONG, short, with a nimbus	2297·7	

This spectrum was taken from a piece of very pure Ceylon graphite, which contains only traces of iron and of magnesium as determined by an analysis of the ash. No iron lines appear in this spectrum, and only four lines of magnesium, namely, those with wave-lengths 2801·6, 2796·9, 2794·1, and 2789·6.* There are certain lines in Messrs. LIVEING and DEWAR'S spectra which are absent from ours, viz.: those with wave-lengths 2733·2, 2541·5, 2528·2, 2523·6, 2518·7, 2515·8, 2514·0, and 2506·6.

* These have since been shown to belong to the spectrum of silicon. See "Line Spectra of Boron and Silicon," Proc. Roy. Soc., vol. xxxv., p. 301; also report presented to the British Association, Chemical News, vol. xlviii., p. 1 (W. N. HARTLEY, Nov. 1, 1883). Of the lines attributed to the arc spectrum of carbon by Messrs. LIVEING and DEWAR that to which they assign a wave-length of 2478·3 is the only line belonging to this element. Their measurement is identical with that which we have obtained from the longest line in the spark.

The Spectrum of Tin.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
12·20	Strong, very short	4584·3	The lines 3351·8 and 3282·9 are short and very broad, with a strong nimbus, so that they have a somewhat nebulous appearance. In a less degree this remark also applies to the lines 2657·9, 2643·2, and 2631·5, and to several other lines in this spectrum described as very short.
15·18	Weak, discontinuous, fine	4524·0	
25·93	Weak, very short	4324·6	
32·50	Weak, very short	4215·3	
42·93	Weak, short, fine	*4057·0*	
49·75	*Fairly strong*, very short	3961·8	
50·88	Weak, discontinuous	*3947·0*	
53·92	Strong, very short	3906·6	
57·62	Strong, very short	3859·0	
62·40	Strong, continuous, extended, fine	3800·3	
63·80	Strong, very short	3783·4	
64·29	Strong, very short	3779·0	
65·55	*Fairly strong*, very short	3763·9	
67·15	Very strong, very short	3745·1	
68·10	Strong, very short	3734·4	
68·40	*Fairly strong*, very short	3727·0	
70·50	Strong, very short	3707·6	
72·36	Weak, very short	3686·7	
74·15	Weak, very short	3667·6	
75·23	Weak, discontinuous, fine	3655·5	
78·19	Faint, very short	3623·9	
78·90	Faint, very short	3616·9	
79·80	Strong, very short	3609·3	
80·60	*Very strong*, very short	3598·3	
83·12	Strong, very short	3574·0	
85·50	Fairly strong, very short	3549·7	
86·57	Faint, very short	*3539·3*	
89·13	Faint, very short	*3514·8*	
92·08	Faint, very short, fine	*3487·3*	
93·83	Weak, very short	3471·1	
100·35	Strong, short	3412·7	
102·88	Faint, very short	3390·4	
107·51	Very strong, short, extended, broad, with a nimbus	3351·8	A very characteristic group. These lines with those at 3174·3, 3033·1, 3007·9, are the principal lines in this spectrum.
110·25	Very strong, continuous, fine	3330·0	
112·21	Weak, short	3314·6	
116·03	Very strong, short, broad, with a nimbus extended	3282·9	
118·83	Very strong, continuous, extended, fine	3261·6	
120·89	Weak, very short	3245·0	
124·14	Faint, very short	3219·6	
124·62	Faint, discontinuous	3218·0	
130·70	Very strong, continuous, extended, fine	3174·3	
135·50	Weak, discontinuous, fine	3140·6	
138·24	Weak, very short	3122·3	
139·50	Faint, very short	3131·0	
142·31	Faint, very short	3095·2	
146·12	Strong, very short	3070·6	
150·06	Weak, discontinuous	3046·5	
152·18	Very strong, continuous, extended, fine	3033·1	
156·29	Very strong, continuous, extended, fine	3007·9	
173·05	Weak, continuous, faint in centre	2911·9	
176·18	Strong, very short	2895·0	
177·80	Strong, very short	2886·9	
179·50	Weak, very short	2877·4	
180·07	Faint, very short	*2874·7*	

The Spectrum of Tin (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
182·47	VERY STRONG, CONTINUOUS, extended, fine	2862·1	
184·99	STRONG, CONTINUOUS, extended, fine . .	2849·3	
185·45	STRONG, very short	2847·6	
187·01	VERY STRONG, CONTINUOUS, extended, sharp	2838·9	
192·30	STRONG, continuous	2812·5	
192·40	Faint, discontinuous, fine	2811·5	
197·61	Faint, discontinuous	2787·3	
198·28	Fairly strong, continuous, weak in centre	2784·0	
199·34	STRONG, CONTINUOUS, extended . . .	2778·8	
202·23	Faint, long, continuous	*2765·0*	
204·52	Faint, short	*2754·0*	
205·01	Faint, short	*2751·8*	
205·70	Faint, short	2749·0	
206·24	Faint, short	2746·0	
207·94	Faint, short	*2738·4*	
209·17	Faint, long	*2733·0*	
215·35	VERY STRONG, CONTINUOUS, extended, sharp	2705·8	
224·95	STRONG, very short	2664·9	
225·98	STRONG, CONTINUOUS, fine	2660·2	
226·56	VERY STRONG, short, nebulous . . .	2657·9	
229·67	STRONG, very short	2645·4	
230·23	VERY STRONG, short, nebulous . . .	2643·2	
233·17	VERY STRONG, short, nebulous . . .	2631·5	
236·40	STRONG, very short	2617·9	
237·47	Faint, very short	2613·8	
238·18	Faint, short	*2611·0*	
239·38	Faint, short	*2606·3*	
241·31	Faint, short	2598·5	
242·65	*Fairly strong, continuous*	2593·6	
243·10	*Fairly strong*, short	2591·7	
248·70	STRONG, CONTINUOUS, extended . . .	2570·5	
250·70	Faint, very short, nebulous	*2563·2*	
252·20	Faint, continuous, fine	*2557·7*	
255·45	STRONG, CONTINUOUS, extended . . .	2545·6	
259·59	Faint, discontinuous, fine	*2530·8*	
261·57	Weak, discontinuous, fine	2523·4	
264·35	Faint, very short	*2514·0*	
266·61	Faint, very short	*2506·0*	
268·55	Faint, very short	*2499·3*	
269·80	STRONG, CONTINUOUS, extended . . .	2495·0	This is a characteristic group, which is repeated, but with a lesser intensity, in the three lines immediately following.
271·85	STRONG, short, broad, nebulous . . .	2488·0	
273·40	STRONG, CONTINUOUS, extended . . .	2482·9	
281·83	Weak, discontinuous	2455·5	
283·37	*Fairly strong*, short, *nebulous*	2449·4	
284·82	Weak, continuous	2445·2	
287·55	STRONG, very short	2436·4	
288·65	Faint, short	2433·3	
289·95	VERY STRONG, CONTINUOUS, extended .	2429·3	
292·37	VERY STRONG, CONTINUOUS, extended .	2421·8	
296·80	Weak, discontinuous, fine	2408·0	
300·82	Faint, short	*2395·8*	
301·54	Faint, short	*2393·7*	
305·40	Faint, short	*2382·3*	

The Spectrum of Tin (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
305·84	Weak, discontinuous	*2381·1*	
310·11	Strong, discontinuous	2368·3	
314·85	Very strong, continuous, extended	2355·0	
321·94	Strong, discontinuous	2335·3	
328·34	Strong, continuous, *nebulous*	2317·9	
339·65	*Fairly strong*, discontinuous	2288·1	
346·58	Strong, discontinuous, *nebulous*	2270·0	
347·25	Faint, short	2268·6	
347·63	Weak, short	2267·1	
355·83	Strong, discontinuous	2247·0	
361·43	Faint, discontinuous, fine	*2233·2*	
362·92	Strong, very short	*2229·6*	
366·29	Strong, very short	*2221·5*	
368·88	Weak, very short	*2215·2*	
371·01	*Fairly strong*, discontinuous	*2210·1*	
375·50	Weak, short	*2199·2*	
377·48	Weak, short	2195·0	
396·95	Weak, short	*2151·2*	
411·70	Faint, short	*2119·2*	
414·65	Faint, discontinuous	*2113·6*	
430·84	Faint, short	*2079·3*	
437·64	Faint, short	*2066·1*	

The Spectrum of Lead.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
21·76	Faint, short	4399·4	The very short lines in this spectrum, which are also strong, are surrounded by a nimbus which gives them a somewhat nebulous appearance. There are several nebulous lines, as for instance those with wave-lengths 3591·9, 3555·9, 3278·5, 3016·5, 2949·2, 2650, and a broad nebula extending from 2540 to 2523·4.
22·50	Very strong, short, sharp, extended	4386·4	
29·12	Weak, very short	4271·4	
30·71	Very strong, discontinuous, broad, sharp, extended	4245·3	
34·68	Faint, short	4180·9	
42·63	Weak, discontinuous, fine	4061·5	
42·93	Strong, continuous, extended, sharp	4057·5	
45·58	Faint, discontinuous, very fine	*4020·5*	
49·72	Faint, very short	*3961·5*	
50·47	Weak, very short	3951·7	
51·84	Faint, very short	3934·0	
52·29	Faint, very short	3927·5	
53·60	Weak, short	3910·4	
58·13	Strong, very short	3853·2	
58·97	Strong, very short	3842·9	
59·77	Strong, very short	3832·5	
60·18	Weak, very short	3827·5	
63·61	Strong, short	3785·9	
67·61	Strong, continuous, fine, extended	3738·9	
68·15	Faint, very short	*3734·3*	
69·67	Weak, short, broad, nebulous	*3717·0*	
70·38	Faint, short, nebulous	*3709·0*	

The Spectrum of Lead (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
72·13	Weak, very short	3688·8	
72·69	STRONG, CONTINUOUS, fine, extended	3682·9	
73·75	STRONG, CONTINUOUS, fine, with a nimbus at ends	3671·0	
75·23	STRONG, very short	3656·1	
76·80	STRONG, CONTINUOUS, extended, fine	3639·2	
81·35	*Fairly strong*, very short	3591·9	
81·77	*Fairly strong*, very short	3590·5	
83·31	STRONG, CONTINUOUS, extended, fine	3572·6	
94·11	Weak, very short	3563·9	
94·47	Weak, very short	3562·2	
92·41	Weak, very short	3484·3	
95·66	Weak, short, nebulous	3455·9	
112·74	Faint, very short	3308·9	
114·04	Weak, very short	3296·8	
116·60	*Fairly strong*, very short	3278·5	
116·98	*Fairly strong*, very short	3276·9	
121·38	*Fairly strong*, short	3242·4	
122·22	Weak, continuous, with a nimbus at the ends	3219·9	
130·46	VERY STRONG, short, with a nimbus	3176·0	
136·09	STRONG, short, with a nimbus	3137·3	
143·45	Weak, very short	3088·5	
143·68	Weak, very short	3086·7	
149·20	Weak, very short	3051·1	
150·52	STRONG, short, with a nimbus	3043·3	
152·56	Weak, very short	3030·2	
154·78	Weak, discontinuous, nebulous	3016·5	
161·27	Faint, very short	2978·8	
166·33	Weak, short, nebulous	2949·2	
170·45	STRONG, CONTINUOUS, fine, extended	2872·2	
181·06	Faint, very short, fine	2867·8	
182·25	STRONG, very short	2863·2	
188·37	STRONG, CONTINUOUS, fine, extended	2832·2	
190·30	STRONG, CONTINUOUS, fine, extended	2822·1	
194·67	VERY STRONG, CONTINUOUS, fine, extended, with a nimbus	2801·4	
212·87	Weak, short	2716·3	
217·26	Weak, short	2697·2	
225·41	STRONG, CONTINUOUS, extended, fine	2662·5	
228·47	*Fairly strong*, *nebulous*, short	2650·0	
231·54	Faint, very short	*2637·5*	
233·91	Faint, short, fine	2627·4	
237·48	VERY STRONG, CONTINUOUS, extended, with a nimbus	2613·4	
247·08	STRONG, CONTINUOUS, fine	2576·4	
249·44	Weak, very short	2567·2	
251·01	STRONG, very short, with a nimbus	2561·6	
259·11	Weak, short, broad, and *nebulous band*	2539·9 2523·4	These measurements are taken on each side of the band. This band is a remarkable peculiarity of the lead spectrum.
269·85	Faint, short, nebulous	2496·0	
275·39	STRONG, CONTINUOUS	2475·7	
279·41	Faint, very short, nebulous	*2462·8*	
284·61	Weak, discontinuous	2445·7	
285·29	Weak, continuous, fine, faint in the centre	2443·6	

The Spectrum of Lead (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
288·61	Faint, short	2432·3	
289·89	Faint, short	2427·8	
295·49	Faint, discontinuous, fine	2411·2	
298·73	Weak, discontinuous, fine	2402·1	
301·43	STRONG, CONTINUOUS	2393·7	
302·53	Faint, fine, short	*2390·8*	
303·15	Faint, fine, short	*2389·0*	
322·60	Faint, fine, discontinuous	*2333·3*	
335·82	Faint, short	*2297·7*	
355·32	STRONG, CONTINUOUS	2247·9	
359·40	Faint, continuous	*2238·2*	
373·43	STRONG, CONTINUOUS, somewhat *nebulous*, with a nimbus	2204·3	
388·35	Weak, discontinuous	*2170·0*	

The Spectrum of Tellurium.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
6·48	Faint, short	*4707·5*	There is a nimbus throughout the whole extent of this spectrum where the points of the electrodes have made an impression upon the plate, but it may be remarked that the continuous lines show no distinct nimbus, neither are they, as a rule, nebulous. The large number of short lines in this spectrum is remarkable. But few of the lines are extended, they are those with wave-lengths 3382·4, 3280·0, 3273·4, 2413·3, 2386·3, 2383·8, 2247·3, and 2243·3.
7·12	Faint, short	*4693·0*	
11·31	Weak, short	*4602·0*	
17·10	Weak, short	*4487·0*	
17·51	Weak, short	*4480·0*	
19·88	Weak, short	*4436·0*	
21·76	Weak, short	*4400·0*	
22·94	Weak, fine, short	4378·0	
23·69	Weak, short	4364·5	
24·33	Weak, short	*4353·0*	
25·94	Faint, short	4324·6	
27·32	*Fairly strong*, short	4301·5	
27·78	Faint, short	4292·7	
28·10	Faint, short	4287·3	
28·94	*Fairly strong*, short	4274·4	
29·72	*Fairly strong*, short	4259·8	
32·18	*Fairly strong*, short	4221·1	
34·70	Weak, short	*4180·7*	
35·36	Faint, short	*4170·3*	
38·70	Faint, short	4119·7	
41·83	Weak, short	4072·7	
42·64	*Fairly strong*, short	4061·3	
43·10	Fairly strong, short	4054·2	
43·60	Faint, short	4048·3	
46·60	STRONG, short	4006·0	
48·20	*Fairly strong*, short	3983·8	
49·30	*Fairly strong*, short	3968·6	
50·80	*Fairly strong*, short	*3948·0*	
51·96	Weak, short	*3932·5*	
53·76	Weak, short, nebulous	*3908·7*	

The Spectrum of Tellurium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
59·04	Strong, short	3841·3	
62·04	Faint, short	3803·0	
62·75	Weak, short	3796·9	
63·40	Faint, short	*3789·0*	
64·54	Faint, short	*3776·0*	
64·96	Faint, short, fine	*3771·0*	
65·48	Faint, short, fine	*3765·0*	
65·96	Faint, short, fine	*3759·0*	
66·42	Faint, short, fine	*3754·0*	
68·00	Strong, short	3735·5	
68·89	Strong, short	3726·2	
69·74	Faint, short	*3716·0*	
71·30	Faint, short	*3698·7*	
72·70	Faint, short	*3683·3*	
73·30	Faint, short	*3676·7*	
73·98	Faint, short	*3670·4*	
75·28	Faint, short	*3656·4*	
75·84	*Fairly strong*, short	3649·2	
76·30	*Fairly strong*, short	3644·3	
77·04	Faint, short	*3636·3*	
77·98	Faint, short	3626·7	
78·88	*Fairly strong*, short	3617·0	
79·45	Faint, short	3611·0	
80·37	Faint, fine, short	3601·7	
80·66	Faint, fine, short	3599·6	
81·35	Faint, short	3594·5	
82·19	Faint, short	3589·4	
85·32	Strong, short	3551·6	
86·32	Very faint, short	*3541·8*	
87·22	Very faint, short	*3533·1*	
88·55	Strong, short	3520·3	
89·57	Weak, short	*3510·8*	
91·18	Strong, short	3496·3	
92·48	Weak, short	3483·7	
92·81	Faint, short	*3480·8*	
93·50	Weak, short	*3474·4*	
94·46	Faint, short	3465·5	
95·52	Strong, short	*3456·0*	
96·14	Weak, short	*3450·4*	
97·07	Strong, short	3441·2	
99·26	Faint, short	*3422·2*	
100·04	Faint, short	*3415·3*	
100·95	Strong, short	3407·5	
103·90	Very strong, continuous, extended, sharp	3382·4	The principal lines in this spectrum are 3382·4, 3307·1, 3280·0, 3273·4, and 3246·8; there are also two others, 2386·3 and 2383·8.
104·83	Faint, short	*3374·1*	
106·27	Strong, short	3362·4	
107·50	*Fairly strong*, short	3352·1	
110·35	*Fairly strong*, short	3329·0	
111·09	Faint, short	*3322·7*	
111·95	Faint, short	*3315·8*	
113·05	Strong, continuous	3307·1	
115·16	Weak, continuous	3289·6	
116·43	Very strong, continuous, extended, sharp	3280·0	
117·35	Very strong, continuous, extended, sharp	3273·4	

The Spectrum of Tellurium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
118·09	Weak, short	3267·4	
118·35	Weak, short	3264·6	
119·55	Strong, short	3256·3	
120·10	Faint, short	3250·8	
120·77	Very strong, continuous, extended, sharp	3246·8	
121·33	Faint, fine, short	*3242·1*	
122·36	Faint, short	*3234·2*	
123·02	Weak, short	*3229·4*	
124·03	Faint, short	*3221·8*	
124·61	Faint, short	*3217·6*	
125·19	Faint, short	*3213·3*	
125·63	Weak, short	3210·4	
128·09	Faint, fine, continuous	*3192·2*	
128·69	Faint, short	*3188·1*	
129·33	Weak, short	*3183·7*	
130·75	Weak, fine, continuous	3174·4	
131·50	Faint, short	*3168·5*	
132·95	Weak, short	*3158·4*	
133·60	Faint, short	*3154·1*	
134·81	Faint, short	*3145·7*	
136·90	Weak, short	*3131·7*	
137·89	Weak, short	*3124·7*	
138·70	Faint, nebulous, short	*3119·5*	
140·46	*Fairly strong*, discontinuous	3107·5	
141·80	Faint, fine, short	*3098·7*	
142·28	Faint, short	*3095·5*	
143·43	Faint, short	*3088·0*	
145·85	*Fairly strong*, short	3072·7	
147·50	Weak, short	3063·2	
149·03	Weak, short	*3052·8*	
150·00	Strong, continuous, *nebulous*, weak in the centre	3046·0	
153·86	Weak, short	3022·1	
154·78	Strong, short	3016·6	
155·68	Faint, short	*3012·1*	
156·91	Faint, short	*3004·1*	
158·18	Faint, short	*2996·4*	
159·49	Faint, short	*2988·8*	
161·70	Faint, short	2976·2	
161·78	Faint, short	2975·5	
162·26	Weak, short	2973·1	
163·36	Strong, short	2966·1	
164·40	Weak, fine, continuous	2960·3	
165·14	Faint, short	*2956·3*	
166·14	Faint, short	*2950·6*	
166·46	Faint, short	*2948·8*	
167·10	Faint, short	*2945·3*	
167·82	Strong, short	2940·8	
168·44	Faint, short	*2937·7*	
169·08	Faint, short	2932·5	
170·00	Weak, short	2928·1	
171·00	Faint, short	*2923·4*	
171·75	Weak, short	2918·9	
174·27	Weak, short	2905·9	
174·96	Faint, short	*2901·9*	

THE Spectrum of Tellurium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
176·24	STRONG, discontinuous, *nebulous*	2894·3	
176·44	*Fairly strong*, short, *nebulous*	2893·3	
179·54	Very faint and fine, short	*2877·4*	
180·28	Very faint and fine, short	*2873·6*	
181·25	STRONG, discontinuous, *nebulous*	2867·7	
182·78	*Fairly strong*, fine, discontinuous	2859·9	
183·40	STRONG, discontinuous, *nebulous*	2857·0	
185·62	*Fairly strong*, short, fine	2844·9	
186·72	*Fairly strong*, short, fine	2840·0	
187·42	Very faint and fine, short	*2836·9*	
187·90	Very faint and fine, short	*2834·4*	
190·02	*Fairly strong*, continuous, fine, short	2823·2	
191·74	Very faint and fine, short	*2815·3*	
192·18	Very faint and fine, short	*2813·0*	
194·96	Faint, very fine, short	2799·1	
195·52	Faint, very fine, short	2795·5	
196·40	STRONG, discontinuous, nebulous	2791·9	
201·32	*Fairly strong*, continuous, fine, sharp	2768·6	
201·85	*Fairly strong*, short, fine, sharp	2766·5	
202·00	Faint, very fine, continuous, sharp	2766·0	
204·12	Weak, fine, sharp, short	*2756·0*	
205·08	Very faint, short, nebulous	*2751·5*	
206·45	Faint, short, fine	*2745·0*	
206·90	Faint, short, fine	*2743·0*	
207·70	Faint, short	*2739·5*	
208·06	Faint, short	*2738·0*	
211·36	Weak, short, nebulous	*2723·2*	
211·94	Weak, short, fine	*2720·7*	
212·54	Weak, short, fine	*2718·0*	
213·67	Weak, fine, short	*2713·0*	
214·23	STRONG, short, nebulous	2710·2	
216·10	Weak, fine, short	2702·3	
216·58	Weak, fine, short	2700·3	
217·36	*Fairly strong*, short, nebulous	2696·6	
217·98	*Fairly strong*, short, nebulous	2694·1	
218·83	Weak, fine, short	2690·2	
219·40	Weak, fine, short	2688·2	
220·50	Weak, short, nebulous	2683·2	
221·35	Weak, short, nebulous	2679·8	
222·55	Weak, continuous, fine, sharp	2674·6	
224·68	Faint, short, fine	*2666·0*	
226·13	Weak, short, fine, with a nimbus on the less refrangible side	2659·4	
226·85	Faint, short, nebulous	*2657·1*	
228·75	Weak, short, nebulous, fine	2648·7	
229·15	Weak, short, nebulous, fine	2647·0	
230·37	Very faint, short, nebulous	*2642·3*	
231·67	Weak, very short	*2637·0*	
232·20	*Fairly strong*, short, *nebulous*	2634·7	
233·30	Weak, short, nebulous	2630·5	
233·94	Faint, short, fine	*2627·8*	
234·83	Faint, short, fine	*2624·3*	
235·54	Faint, short, fine	*2621·4*	
236·56	Weak, fine, continuous, sharp	2617·4	
237·46	Faint, fine, short	*2613·7*	
238·07	Faint, fine, short	*2611·3*	

THE Spectrum of Tellurium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
239·86	Weak, short, nebulous	2604·4	
241·13	Weak, short, fine	2599·4	
241·48	Weak, short, fine	2598·1	
242·56	Very faint, short	*2594·0*	
243·56	Weak, short, nebulous	2590·1	
244·88	Weak, short, nebulous	*2585·0*	
246·16	Weak, short, nebulous	*2580·1*	
246·69	Weak, short, nebulous	*2578·0*	
247·58	Faint, fine, short	*2574·8*	
248·24	Faint, short, nebulous	2572·4	
249·44	Very faint, short, nebulous	*2567·8*	
250·44	Very faint, short, nebulous	*2564·1*	
251·93	Very faint, short, nebulous	*2558·7*	
254·40	Weak, short, nebulous	*2549·7*	
255·96	*Fairly strong*, fine short	2543·7	
257·90	Weak, short, broad, nebulous	2536·8	
258·71	Weak, fine, sharp, short	2533·8	
260·00	STRONG, CONTINUOUS, fine, sharp	2529·4	
260·28	Weak, short, fine, nebulous	*2528·3*	
261·03	Weak, short, fine, nebulous	*2525·6*	
266·84	*Fairly strong*, short, fine, sharp	2505·2	
267·56	Very faint and short, fine, sharp	*2502·7*	
268·74	*Fairly strong*, short, nebulous	2498·6	
270·83	Weak, continuous, fine	2491·3	The next line, which is nebulous, appears to overlap this, which is a fine line.
270·90	Weak, short nebulous	2490·8	
271·60	Weak, short, fine, sharp	*2488·7*	
272·60	Weak, short, nebulous	*2485·3*	
273·92	Faint, short, fine	*2480·9*	
274·30	Faint, short, fine, nebulous	*2479·6*	
275·16	Faint, short, nebulous	*2476·7*	
276·26	*Fairly strong*, short, sharp	2473·2	
277·53	Weak, short, nebulous	*2469·0*	
279·69	Faint, short, fine, nebulous	*2462·0*	
280·25	Faint, short, fine, nebulous	*2460·2*	
282·43	Weak, short, nebulous	*2452·8*	
284·15	*Fairly strong*, discontinuous, sharp	2447·8	
285·21	Faint, short, nebulous	*2444·3*	
286·05	Faint, fine, continuous, sharp	*2441·7*	
287·30	STRONG, CONTINUOUS, slightly extended with a *faint nimbus*	2438·0	
289·10	Weak, fine, slightly continuous, nebulous	*2432·0*	
289·86	Weak, short, nebulous	*2429·7*	
290·33	Weak, fine, continuous, sharp	*2428·2*	
290·80	Weak, short, nebulous	*2426·7*	
291·36	Faint, short, nebulous	*2425·0*	
292·88	Weak, short, nebulous	*2420·3*	
293·44	Faint, short, nebulous	*2418·5*	
295·08	STRONG, CONTINUOUS, slightly extended	2413·3	
295·66	*Fairly strong*, slightly continuous and extended	2411·4	
298·20	*Fairly strong*, broad, nebulous, short	*2403·7*	
299·36	*Fairly strong*, fine, short	*2400·0*	
301·83	Faint, short, fine, nebulous	*2392·8*	
302·58	Faint, short, fine, nebulous	*2390·7*	

THE Spectrum of Tellurium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
304·10	VERY STRONG, CONTINUOUS, extended, with a nimbus	2386·3	
304·92	VERY STRONG, CONTINUOUS, extended, with a nimbus	2383·8	
307·26	Weak, short, nebulous	*2377·0*	
307·84	Weak, short, nebulous	*2375·3*	
309·54	STRONG, SLIGHTLY continuous and extended, sharp	2370·3	
311·50	Faint, short, fine, nebulous	*2364·7*	
312·18	Faint, short, fine, nebulous	*2362·8*	
313·22	Faint, fine, short, nebulous	*2359·8*	
313·68	*Fairly strong*, fine, short	2358·6	
314·16	Faint, fine, short, nebulous	*2357·0*	
316·08	Weak, a broad nebulous dot	*2351·7*	
318·64	Faint, a rather broad nebulous dot . .	*2344·3*	
320·06	Faint, short, nebulous	*2340·3*	
321·32	Weak, short, nebulous	*2336·8*	
323·10	STRONG, short, slightly extended, sharp	2332·0	
325·50	STRONG, SHORT, slightly extended, sharp	2325·5	
327·17	STRONG, SHORT, slightly extended, sharp	2321·0	
328·32	STRONG, SHORT, slightly extended, sharp	2317·8	
331·08	Weak, short, nebulous	*2310·1*	
333·52	Faint, short, nebulous	*2303·7*	
334·48	Faint, short, nebulous	*2301·1*	
335·89	Very faint, short, nebulous	*2297·5*	
336·84	*Fairly strong*, continuous, nebulous .	*2295·0*	
338·01	Very faint, short, nebulous	*2291·8*	
339·24	Faint, a broad nebulous dot	2288·6	
342·34	*Fairly strong*, short, nebulous. . . .	*2280·6*	
343·74	*Fairly strong*, short, nebulous. . . .	*2277·2*	
340·40	*Fairly strong*, a rather broad nebulous dot	*2285·7*	
348·09	*Fairly strong*, continuous, nebulous .	2266·2	
348·90	Faint, short, nebulous	*2264·2*	
350·50	*Fairly strong*, continuous, nebulous .	*2260·4*	
351·93	*Fairly strong*, continuous, nebulous .	*2256·6*	
354·60	*Fairly strong*, short, fine, nebulous . .	*2250·0*	
355·18 355·36	*A pair of fairly strong, very fine, continuous lines, with a fairly broad nimbus on the more refrangible portion of the pair.*	2248·0 2247·3	
357·18	*Fairly strong, rather long, nebulous on more and sharp on less refrangible side.*	2248·3	
358·34	Faint, very short, broad, nebulous . .	*2240·7*	
362·20 362·21 363·20	*A triplet of weak slightly continuous, fine, nebulous lines with a nimbus, making them appear one broad nebulous line*	*2231·3* *2230·3* *2229·0*	
364·08	Weak, short, nebulous	*2226·8*	
365·58	Faint, very short, broad nebulous . .	*2223·2*	
367·42	*Fairly strong*, continuous, nebulous on more refrangible side	*2219·3*	
368·58	Weak, continuous, nebulous	*2216·0*	
370·53	*Fairly strong*, discontinuous, nebulous	*2211·2*	
371·22	*Fairly strong*, continuous, nebulous .	*2209·5*	

THE Spectrum of Tellurium (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
374·08	Very faint, very short, nebulous . .	*2202·8*	
375·16	Weak, continuous, nebulous	*2200·1*	
376·67	Very faint, short, nebulous	*2196·5*	
378·53	*Fairly strong, continuous*, broad, nebulous .	*2192·2*	
379·60	*Fairly strong*, discontinuous, broad, nebulous	*2189·7*	
380·82	Weak, broad, very short, nebulous . .	*2186·9*	
382·95	Faint, short, nebulous	*2182·0*	
384·18	*Fairly strong, continuous*, broad, nebulous	*2179·2*	
385·90	Faint, very short, nebulous	*2175·3*	
389·60	Faint, very short, nebulous	*2167·2*	
390·31	Faint, very short, nebulous	*2165·7*	
393·05	Faint, short, nebulous	*2159·7*	
397·64	Weak, short, nebulous	*2149·7*	
398·54	Weak, continuous, nebulous	*2147·8*	
399·07	Weak, short, nebulous	*2146·7*	
400·83	Weak, continuous, broad, nebulous .	*2142·7*	
403·81	Weak, short, nebulous	*2136·5*	
404·56	Weak, short, nebulous	*2135·0*	
409·06	Weak, short, nebulous	*2125·5*	
410·46	Weak, short, nebulous	*2122·5*	
412·04	Very faint, very short, nebulous . .	*2119·0*	
413·34	Very faint, very short, nebulous . .	*2116·3*	
414·75	Weak, short, nebulous	*2113·3*	
416·02	Weak, short, nebulous	*2110·5*	
417·42	Faint, very short, nebulous	*2108·4*	
419·34	Faint, very short, nebulous	*2103·6*	
420·91	Very faint, very short, broad, nebulous .	*2100·2*	
431·43	Weak, short, broad, nebulous	*2078·5*	
445·45	Weak, short, broad, nebulous	*2050·8*	
451·60	Weak, short, broad, nebulous	*2039·2*	
455·40	Weak, short, broad, nebulous	*2032·7*	

A certain number of lines of copper are coincident with those in tellurium. They are given in the table of coincidences, two of these lines are strong both in copper and tellurium.

The Spectrum of Arsenic.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
13·90	Weak, very short, fine	*4550·0*	There are several lines in this spectrum which are continuous and nebulous. Most of the strong lines have a nimbus.
14·48	Weak, very short, fine	*4538·4*	
16·73	STRONG, discontinuous	4494·3	
	Strong, short	*4474·0*	
	Strong, discontinuous	*4466·3*	
	Strong, discontinuous	*4458·7*	
20·14	STRONG, discontinuous	*4431·0*	
20·95	Weak, discontinuous	*4415·0*	
23·45	Weak, discontinuous	*4368·7*	
24·54	Weak, discontinuous	*4349·0*	
25·34	Weak, discontinuous	*4335·2*	
26·52	Weak, discontinuous	*4315·2*	
	Weak, discontinuous	*4301·0*	An approximation.
30·79	Weak, discontinuous	*4244·0*	
31·66	Weak, discontinuous	*4229·3*	
33·02	Weak, discontinuous	4207·3	
33·68	*Fairly strong*, discontinuous	4197·7	
34·19	Weak, discontinuous	*4188·9*	
38·70	Weak, short	*4120·0*	
41·15	*Fairly strong*, discontinuous	4081·8	
42·49	Weak, short	4064·3	
44·50	STRONG, short	4036·0	
46·54	Weak, short	*4007·0*	
48·13	Faint, short	*3985·0*	
50·80	*Fairly strong*, discontinuous	3948·5	
52·07	*Fairly strong*, discontinuous	3930·7	
52·88	STRONG, discontinuous	3921·6	
58·99	STRONG, discontinuous	3842·5	
62·50	Faint, discontinuous	3800·7	
63·62	*Fairly strong*, discontinuous	3784·4	
64·86	Faint, discontinuous	*3772·0*	
73·82	Weak, discontinuous	3671·2	
78·34	Faint, very short	*3622·4*	
81·32	Weak, very short	*3591·9*	
85·38	Weak, discontinuous	3551·6	
85·88	Weak, discontinuous	3545·8	
89·54	Faint, short	*3510·8*	
93·78	Weak, discontinuous	3471·1	
118·98	Faint, very short	*3260·1*	
119·46	Weak, short	3256·2	
128·70	Faint, short	*3187·7*	
129·60	Faint, short.	*3181·7*	
137·66	Faint, continuous, fine, very faint in centre	3125·4	
138·86	STRONG, fine, continuous, with a nimbus	3119·2	
139·34	STRONG, fine, continuous, with a nimbus	3116·1	
140·42	Faint, short, nebulous	*3107·7*	
145·56	Fairly strong, continuous, fine . . .	3075·0	
148·40	STRONG, CONTINUOUS, fine, weak in centre, with a nimbus	3057·3	
149·10	STRONG, CONTINUOUS, fine, weak in centre, with a nimbus	3052·6	
152·38	STRONG, CONTINUOUS, sharp	3032·2	
157·12	Weak, continuous, faint in centre . .	3008·2	
159·22	*Fairly strong*, fine, continuous . . .	2990·2	

THE Spectrum of Arsenic (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
160·80	Weak, continuous, nebulous, weak in centre	2981·1	The strong lines here form a characteristic group.
164·82	VERY STRONG, with a nimbus, discontinuous	2958·7	
170·67	Weak, discontinuous	2925·6	
175·75	STRONG, CONTINUOUS, extended	2898·2	
177·22	Weak, continuous, faint in centre	2889·1	
178·40	Weak, continuous	2884·2	
183·04	VERY STRONG, EXTENDED, CONTINUOUS	2859·7	
185·37	Faint, discontinuous	2843·6	
187·01	Faint, discontinuous	2836·9	
188·81	STRONG, CONTINUOUS, NEBULOUS	2829·8	
196·93	Faint, discontinuous	2788·5	
199·22	VERY STRONG, CONTINUOUS, extended, with a nimbus	2779·5	
201·07	Faint, discontinuous	2770·4	
206·77	VERY STRONG, CONTINUOUS, extended	2744·1	
218·85	Faint, discontinuous	*2690·5*	
222·01	Faint, discontinuous	*2677·0*	
222·80	Faint, discontinuous	*2673·8*	
223·79	Faint, discontinuous	*2669·5*	
225·10	Faint, discontinuous	*2663·5*	
228·14	Faint, fine, discontinuous	*2651·5*	
233·33	Faint, fine, discontinuous	*2630·2*	
238·14	Faint, fine, discontinuous	*2611·2*	
240·68	STRONG, CONTINUOUS, *nebulous*	2600·8	
241·67	Weak, fine, continuous	2597·1	
241·96	Faint, continuous	2593·9	
247·90	Faint, discontinuous	2576·0	
248·38	Faint, discontinuous	2571·6	
250·98	Weak, discontinuous	2559·5	
260·37	STRONG, CONTINUOUS	2527·9	
265·68	STRONG, CONTINUOUS	2526·0	
269·02	Weak, continuous	2496·9	
270·65	STRONG, CONTINUOUS, extended	2491·9	
271·28	Weak, discontinuous	2489·1	
278·92	Weak, continuous, nebulous, faint in centre	2464·1	
279·94	Weak, continuous, nebulous, faint in centre	2461·0	
281·58	STRONG, CONTINUOUS, extended, sharp	2456·2	
287·58	STRONG, CONTINUOUS, extended, sharp	2436·9	
287·93	Weak, nebulous, discontinuous	2435·0	
288·65	Weak, continuous, fine	2432·5	
293·89	Weak, very short, nebulous	2415·8	
297·52	Weak, short, nebulous	2403·4	
297·82	Weak, short, nebulous	2402·6	
305·77	STRONG, CONTINUOUS	2381·0	These strong lines form a characteristic group.
309·32	STRONG, CONTINUOUS, with a nimbus	2370·8	
309·74	STRONG, CONTINUOUS, with a nimbus	2369·7	
311·99	Weak, continuous, fine	2362·8	
316·60	VERY STRONG, CONTINUOUS, extended, with a nimbus	2350·1	
318·66	STRONG, CONTINUOUS, extended, sharp	2344·3	
330·40	Weak, continuous, fine	*2320·7*	
339·14	VERY STRONG, NEBULOUS, with a nimbus, and slightly *nebulous*, extended	2288·9	

The Spectrum of Arsenic (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
343·00	Faint, short, nebulous, broad. . .	*2279·0*	
345·66	*Fairly strong*, continuous	*2272·3*	
347·54	Weak, discontinuous	*2267·5*	
362·80	*Fairly strong*, continuous, nebulous	*2230·0*	
372·30	Weak, discontinuous	*2207·0*	
382·74	Weak, discontinuous	*2182·5*	
385·24	Weak, discontinuous	*2176·8*	
390·40	STRONG, CONTINUOUS, broad, nebulous	*2165·4*	
394·44	STRONG, SHORT	*2156·7*	
397·06	STRONG, SHORT	*2151 0*	
398·64	STRONG, SHORT	*2147·8*	
400·13	STRONG, CONTINUOUS, broad, nebulous	*2144·5*	
404·75	STRONG, discontinuous, broad, nebulous	*2135·2*	
415·22	STRONG, continuous, broad, nebulous, weak in the centre	*2112·2*	

The Spectrum of Antimony.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
6·23	Weak, discontinuous . .	*4714·0*	There are very many strong and nebulous lines in this spectrum. Two of the longest and strongest lines which are not nebulous are those with wave-lengths 2597·2 and 2527·6.
7·15	Weak, short	*4692·5*	
11·46	Weak, fine, discontinuous .	*4599·0*	
11·90	Weak, short	*4590·0*	
16·09	Weak, fine, discontinuous .	*4506·5*	
18·75	Weak, short	*4457·0*	
20·33	Faint, short	*4427·5*	
23·10	Weak, short	*4375·0*	
24·55	STRONG, short	4351·5	
26·62	Weak, short	4316·1	
29·57	STRONG, short	4264·4	
32·37	Weak, short	4218·5	
33·86	Weak, short	4194·5	
35·38	Faint, short	*4170·0*	
37·33	Faint, short	4140·2	
37·82	Weak, short	4132·8	
45·17	Faint, short	*4026·0*	
46·05	Weak, short	3984·9	
49·31	Faint, short	*3968·4*	
49·53	Faint, short	3964·1	
49·87	Faint, short	3960·3	
51·86	Weak, short	3933·2	
53·85	Faint, very short	3907·5	
58·38	*Fairly strong*, short . .	3849·7	
59·07	*Fairly strong*, short . .	3840·2	
60·37	Faint, short	*3825·0*	
64·78	Faint, short	*3771·0*	
67·63	VERY STRONG, short . . .	3739·0	
69·10	Weak, fine, continuous .	3722·4	

The Spectrum of Antimony (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
69·35	Faint, short, fine	3720·5	
72·39	Strong, very short	3686·0	
75·57	Strong, very short	3651·6	
76·92	Strong, continuous, fine	3637·5	
77·65	Strong, short, fine	3629·4	
80·74	Very strong, *nebulous*, discontinuous .	3597·8	
81·81	Strong, nebulous, short	3566·0	
84·64	Strong, nebulous, short	3559·1	
87·13	*Fairly strong*, very short	3533·7	These form a remarkable group of lines.
88·65	*Fairly strong*, very short	3520·3	
90·21	Strong, *nebulous*, short	3504·6	
90·92	Strong, *nebulous*, discontinuous . . .	3498·3	
93·55	Strong, *nebulous*, discontinuous . . .	3473·9	
95·19	Faint, short	*3459·0*	
96·05	Faint, short	*3451·1*	
98·97	Strong, very short	3425·9	
100·13	Faint, short	*3414·7*	
101·45	Weak, very short	3403·0	
102·03	Weak, short	*3397·9*	
103·87	Weak, continuous, fine	(3382·0)	Apparently a tellurium line.
109·36	Very strong, short	3336·4	
113·47	Strong, short	3303·2	
116·47	Weak, continuous, fine	3279·7	Apparently tellurium lines in antimony.
117·38	Strong, continuous, fine	3273·0	
118·21	Strong, continuous, fine	3266·6	
120·80	Strong, continuous, fine	(3246·6)	Apparently a tellurium line.
121·65	Very strong, nebulous, discontinuous.	3240·5	
122·87	Strong, continuous, fine	3231·6	
127·48	Faint, short	*3195·6*	
128·86	Faint, short	*3186·1*	
131·65	Faint, short	*3166·7*	
143·75	Faint, short	*3085·2*	
151·03	Strong, nebulous, discontinuous. . .	3039·8	
152·91	Strong, fine, continuous	3029·0	
153·57	Weak, short	3023·7	
153·99	*Fairly strong*, short	3021·1	
155·83	*Fairly strong*, short	3010·4	
161·03	Strong, discontinuous, nebulous. . .	2979·8	
163·56	Strong, discontinuous, nebulous. . .	2965·2	
171·15	Weak, discontinuous	2921·6	
173·00	Strong, short	2912·6	
177·07	Strong, discontinuous, nebulous. . .	2890·3	
179·62	Week, short	2878·3	
179·29	Strong, continuous, extended, fine . .	2877·1	
182·43	Weak, short	2861·9	
183·63	*Fairly strong*, very short	2855·3	
184·82	Weak, continuous, fine	2849·9	
187·50	Weak, short	2836·0	
189·55	Weak, continuous	2824·7	
195·43	Weak, short	2796·9	
197·05	Very strong, short, *nebulous*	2789·6	
198·03	*Fairly strong*, short	2788·5	
200·07	*Fairly strong*, short	2785·3	
200·1	Weak, short	2775·7	
201·29	Strong, continuous, extended, fine . .	2768·9	
202·37	Weak, short	2763·2	

THE Spectrum of Antimony (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
203·11	Weak, very short	2760·8	
204·45	Weak, very short	2754·9	
207·64	STRONG, very short	2740·1	
210·62	*Fairly strong*, continuous, fine	2726·1	
212·57	STRONG, CONTINUOUS, fine	2717·9	
213·39	Faint, short	2714·0	
216·09	Faint, short	*2702·6*	
216·65	Faint, short	*2700·2*	
218·67	*Fairly strong*, fine, continuous	2691·3	
219·91	Weak, short	2685·5	
220·83	STRONG, CONTINUOUS, fine	2681·7	
222·77	Faint, discontinuous	2674·0	
223·70	STRONG, CONTINUOUS, fine	2668·9	
223·97	Strong, short, nebulous	2668·3	
226·87	*Fairly strong*, nebulous, discontinuous	2656·3	
228·07	STRONG, CONTINUOUS, fine	2651·7	
233·07	STRONG, very short	2631·2	
236·73	STRONG, short	2616·3	
237·40	Faint, fine, continuous	2613·7	
238·05	STRONG, CONTINUOUS, fine	2611·3	
241·65	VERY STRONG, CONTINUOUS, extended, sharp	2597·2	
243·63	VERY STRONG, short	2589·4	
248·01	*Fairly strong*, continuous, fine	2572·7	
248·65	*Fairly strong*, discontinuous	2570·1	
249·64	*Fairly strong*, fine, discontinuous	2566·7	
250·18	STRONG, CONTINUOUS, nebulous, weak in centre	2564·6	
251·97	Faint, nebulous, short	2557·4	
252·32	Faint, short	2556·6	
253·13	Weak, fine, continuous	2553·3	
253·93	Faint, short	2549·8	
256·05	STRONG, discontinuous	2542·9	
260·33	VERY STRONG, CONTINUOUS, extended, sharp	2527·6	
262·75	Very faint, fine, continuous	2519·5	
263·02	Weak, short, nebulous	2518·8	
264·28	Weak, short, nebulous	2514·5	
265·45	Weak, fine, continuous	2509·5	
266·28	STRONG, short	2506·5	
268·20	Faint, short, nebulous	*2500·2*	
270·75	Faint, short, fine	*2490·7*	
271·50	Faint, short, fine	*2489·2*	
272·53	Faint, short	*2485·7*	
273·87	Faint, fine, continuous	2480·4	
274·31	Weak, fine, continuous	2479·4	
274·90	STRONG, CONTINUOUS, fine	2477·3	
275·21	Faint, short, nebulous	2476·7	
276·015	Faint, fine, continuous	2473·4	
277·14	Faint, short, nebulous	*2470·2*	
278·94	Faint, short, nebulous	*2464·4*	
279·66	Faint, short, nebulous	*2462·0*	
280·68	Faint, short, nebulous	*2458·8*	
282·09	Faint, short, nebulous	2454·5	
284·20	Faint, short, nebulous	2445·7	
284·90	STRONG, CONTINUOUS, fine	2444·8	

The Spectrum of Antimony (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
287·35	Fine, discontinuous	2438·0	
290·92	Weak, continuous, fine	2425·7	
291·68	Faint, discontinuous, nebulous	2423·0	
292·40	Weak, continuous, fine	2421·5	
295·71	Faint, short	2410·3	
	Faint, short	2408·3	
297·71	Weak, short, nebulous	2405·3	
298·17	Faint, short	2403·8	
299·44	Faint, short	2399·9	
301·00	Weak, discontinuous	2395·3	
304·90	Strong, continuous	2383·2	
308·28	Strong, continuous	2374·3	
309·61	Strong, short	2370·0	
312·72	Strong, short, nebulous	2361·3	Appears single, but is seen to be
312·93	Weak, fine, continuous	2360·7	double when very highly magnified.
315·60	Faint, discontinuous	2353·0	
316·46	Faint, discontinuous	2350·6	
322·41	Weak, very short, nebulous	2334·2	
323·14	Faint, short, nebulous	2331·8	
323·90	Faint, short, nebulous	2329·7	
325·46	Faint, short, nebulous	2325·3	
327·00	Faint, short, nebulous	2322·1	
328·75	Weak, discontinuous	2316·4	
330·37	Strong, continuous, short	2311·8	
332·25	*Fairly strong*, continuous, sharp	3306·8	
336·05	*Fairly strong*, short	2297·0	
337·17	*Fairly strong*, continuous	2294·0	
339·18	*Fairly strong*, continuous	2288·8	
342·28	Faint, very short	2280·8	
343·25	Faint, very short	2278·3	
343·77	Faint, short	2277·1	
346·11	Faint, short	2271·1	
349·15	Strong, continuous	2263·5	
355·38	Strong, continuous, weak in the centre	2248·0	
357·25	Strong, continuous, weak in the centre	2243·5	
360·93	Faint, short	2234·5	
362·22	Faint, short	2231·3	
362·64	Faint, short	2230·3	
363·15	Faint, short	2229·0	
364·53	Weak, nebulous, continuous, weak in the centre	2226·3	
365·45	Faint, discontinuous	2223·5	
366·29	Weak, nebulous, continuous	2221·5	
367·46	Weak, nebulous, discontinuous	2218·7	
368·39	Weak, nebulous, short	2216·3	
370·48	Faint, short	2211·3	
371·50	Weak, nebulous, continuous	2209·0	
373·61	Faint, short	2203·8	
374·31	Weak, continuous	2202·2	
375·05	Faint, short	2200·3	
378·35	Weak, discontinuous	2192·6	
378·76	Weak, short	2191·6	
379·70	Faint, short	2189·3	
384·24	Strong, continuous, broad	2179·0	
385·71	Strong, continuous, broad	2175·8	
388·35	Strong, short	2170·1	

The Spectrum of Antimony (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
393·19	Faint, continuous	*2159·4*	
394·79	Faint, short	*2156·0*	
398·07	Faint, short	*2148·8*	
400·12	Weak, continuous, nebulous	*2144·4*	
401·31	Faint, continuous, very faint in the centre	*2142·0*	
402·53	Weak, continuous, very weak in the centre	*2139·3*	
404·23	Weak, discontinuous	*2135·7*	
408·73	Faint, discontinuous	*2126·1*	
410·43	Faint, discontinuous	*2122·5*	
412·60	Faint, discontinuous	*2118·0*	
416·10	Faint, discontinuous	*2110·4*	
419·01	Faint, short	*2104·2*	
422·70	Faint, continuous	*2096·4*	
427·61	Very faint, very short	*2086·3*	
433·02	Very faint, very short	*2075.3*	
438·30	Weak, continuous, broad	*2064·8*	
445·62	Faint, discontinuous	*2050·5*	
448·25	Faint, discontinuous	*2045·3*	

The Spectrum of Bismuth.

Scale-numbers.	Descripton of lines.	Wave-lengths.	Remarks.
5·74	Weak, fine, continuous	*4724·5*	The very short, strong, and nebulous rays which abound in the less refrangible region of this spectrum resemble those in the spectrum of antimony.
6·49	Weak, short	*4707·0*	
13·41	STRONG, short, extended	*4560·0*	
17·64	Weak, short	*4477·0*	
22·25	Weak, short	*4391·0*	
25·10	*Fairly strong*, short	4339·4	
25·72	*Fairly strong*, short	4328·7	
27·31	VERY STRONG, discontinuous, broad	4301·5	
29·03	STRONG, short	4271·3	
29·85	VERY STRONG, discontinuous	4259·2	
38·63	STRONG, CONTINUOUS, fine	4121·2	
41·43	STRONG, discontinuous	4079·0	
57·25	STRONG, discontinuous	3863·7	
58·51	*Fairly strong*, short	3848·5	
58·72	Weak, short, fine	3845·4	
61·10	STRONG, discontinuous	3815·9	
61·58	*Fairly strong*, fine, discontinuous	3810·5	
63·10	VERY STRONG, *nebulous, continuous*	3792·7	
64·15	*Fairly strong*, discontinuous	3780·6	
66·16	STRONG, discontinuous	3757·0	
68·26	Weak, short	*3732·7*	
69·38	Weak, fine, short	3711·0	
70·30	*Fairly strong*, very short	3704·0	
71·63	VERY STRONG, short, nebulous	3695·3	
72·91	Very weak, short	3684·5	
75·36	STRONG, fine, discontinuous	3653·9	
76·41	Faint, very short	3647·4	

The Spectrum of Bismuth (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
77·29	Weak, very short	3631·9	
79·21	Very strong, nebulous, short	3613·8	
80·99	STRONG, FINE, CONTINUOUS	3595·7	
86·40	STRONG, short	3541·5	
87·91	*Fairly strong*, very short	3527·9	
88·97	Faint, short, nebulous	3517·9	
89·69	STRONG, FINE, CONTINUOUS	3510·5	
92·37	STRONG, short, nebulous	3485·0	
93·65	STRONG, short, nebulous	3473·5	
95·59	Weak, discontinuous, fine	3454·8	
96·12	STRONG, short	3450·7	
98·40	STRONG, discontinuous	3430·9	
102·25	STRONG, CONTINUOUS, SHARP	3396·7	
102·81	Weak, short, nebulous	3393·2	
103·91	Weak, fine, continuous	(*3381·9*)	A tellurium line probably.
112·03	Faint, very short	*3315·3*	
114·03	Weak, short	3297·9	
115·29	Weak, discontinuous, nebulous	3287·4	
116·45	Weak, fine, continuous	(3279·9)	A tellurium line probably.
119·58	Faint, nebulous, short	*3255·4*	
122·03	Weak, short	*3236·8*	
128·75	Faint, nebulous, short	*3187·7*	
131·28	Faint, short	*3170·0*	
132·73	Faint, short	*3160·0*	
137·01	Faint, short	*3130·8*	
139·42	STRONG, nebulous, short	3114·8	
140·11	Weak, discontinuous, fine	3110·4	
145·42	*Fairly strong*, fine, continuous	3075·7	
146·85	VERY STRONG, BROAD, CONTINUOUS, extended, sharp	3067·1	
150·75	Weak, short	3041·3	
151·91	STRONG, short	3038·0	
152·49	STRONG, FINE CONTINUOUS, SHARP	3034·5	
153·75	VERY STRONG, EXTENDED, CONTINUOUS, SHARP	3023·8	
156·02	Weak, short, nebulous	3009·0	
157·27	Faint, short	3001·2	
158·98	STRONG, CONTINUOUS, sharp	2992·2	
159·67	STRONG, CONTINUOUS, slightly extended, sharp	2988·1	
160·51	Weak, continuous, sharp	*2982·9*	The strong lines in this portion of the spectrum form a characteristic group.
162·68	Short, faint	2973·4	
163·27	Short, faint	2968·9	
166·08	Weak, discontinuous, nebulous	2951·0	
167·29	Faint, very short	2942·4	
168·52	VERY STRONG, CONTINUOUS, extended, sharp	2942·4 2937·5	
169·46	Weak, very short	2931·4	
171·03	Faint, very short	*2923·2*	
172·10	Weak, very short	2917·5	
175·85	VERY STRONG, CONTINUOUS, extended, sharp	2897·2	
182·45	*Long, fine*	2862·5	
183·91	VERY STRONG, short, nebulous	2854·8	
185·49	*Fairly strong*, short	2846·1	
186·61	Weak, very short	2840·1	
187·90	Faint, very short	2832·8	

THE Spectrum of Bismuth (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
190·33	*Fairly strong*, very short	2822·2	
191·47	*Fairly strong*, discontinuous	2816·3	
193·03	STRONG, fine, continuous	2808·4	
193·61	Faint, discontinuous	2805·4	
194·29	STRONG, CONTINUOUS	2802·6	
195·29	Weak, fine, continuous	*2798·0*	
198·15	STRONG, very short	2784·0	
199·08	STRONG, CONTINUOUS, extended, sharp	2779·3	
200·15	Weak, very short	2773·5	
200·55	Weak, very short	2772·5	
202 00	STRONG, very short	2766·3	
203·85	Faint, very short	*2757·3*	
206·24	Weak, discontinuous, fine	2746·0	
209·07	Faint, very short	2733·2	
209·89	STRONG, FINE, CONTINUOUS	2729·3	
210·45	Faint, very short	2727·1	
213·53	Weak, nebulous, discontinuous	2713·1	
217·57	STRONG, FINE, CONTINUOUS	2695·6	
218·09	Faint, nebulous, short	2693·2	
221·31	Faint, nebulous, very short	2679·5	
222·15	Faint, nebulous, very short	*2676·6*	
225·24	Very faint, very short, nebulous	*2663·6*	
227·91	STRONG, very short	2651·8	
230·08	Faint, very short, nebulous	2641·4	
233·75	Weak, short	2628·3	
234·05	STRONG, CONTINUOUS, sharp	2627·0	A prominent line.
245·29	Very faint, short	2583·5	
245·64	Weak, fine, continuous	2581·5	
247·40	Very weak, short, nebulous	2575·5	
255·84	Weak, nebulous, discontinuous	2543·3	
259·20	Weak, nebulous, discontinuous	2531·9	
259·87	Weak, discontinuous	2529·7	
261·47	STRONG, FINE, CONTINUOUS	2523·5	
264·01	Weak, fine, continuous	2514·3	
267·38	Weak, short, nebulous	2503·9	
268·22	SHORT, faint	2500·6	
268·67	Faint, fine, continuous	2499·1	
271·65	Weak, nebulous, continuous	2489·1	
274·30	Weak, short, fine	2479·1	
284·08	Weak, continuous, fine	2447·2	
287·37	Weak, short	2437·5	
289·95	Faint, continuous, fine	2429·3	
294·66	VERY STRONG, short, with a large nimbus	2414·8	A prominent line.
295·11	Faint, short, fine	2412·7	
299·21	STRONG, CONTINUOUS, sharp	2400·7	A prominent line.
306·87	Faint, very short, nebulous, broad	*2378·0*	
310·36	STRONG, CONTINUOUS, sharp	2368·0	
317·73	Faint, short, broad, nebulous	*2347·0*	
323·11	Faint, short, fine	*2331·8*	
324·79	Faint, short, broad	*2327·0*	
325·43	Faint, short, fine	*2325·4*	
326·78	Weak, short, fine	2321·7	
328·30	Faint, short, fine	2317·4	
329·83	Faint, short, broad, nebulous	*2313·7*	
330·91	Faint, long, broad, nebulous	*2310·5*	

THE Spectrum of Bismuth (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
334·40	Weak, continuous, fine	*2301·3*	
335·83	Weak, continuous, fine	*2297·6*	
337·13	Weak, continuous, fine	*2294·1*	
338·09	Very faint, short, nebulous	*2291·6*	
342·20	Short, faint	*2281·0*	
343·73	STRONG, CONTINUOUS, sharp	2276·9	
353·59	Faint, short, nebulous	*2252·5*	
354·41	Weak, short	*2250·5*	
355·78	Weak, short	*2247·0*	
362·20	VERY STRONG, CONTINUOUS, somewhat nebulous	2231·4	Prominent lines.
363·10	VERY STRONG, CONTINUOUS, somewhat nebulous	2229·1	
369·05	STRONG, CONTINUOUS	*2214·8*	
373·83	STRONG, CONTINUOUS, nebulous	*2203·3*	
379·29	Faint, nebulous, continuous	*2190·4*	
380·78	STRONG, CONTINUOUS, nebulous	*2187·0*	
385·35	Very faint, nebulous, short	*2176·6*	
395·00	Very weak, continuous, broad, nebulous	*2168·5*	
400·16	Weak, discontinuous, nebulous	*2144·3*	
405·17	Weak, continuous, broad, nebulous	*2133·8*	
416·40	Faint, continuous, nebulous	*2109·8*	
435·60	Faint, short	*2070·2*	
441·69	Faint, continuous, nebulous	*2058·2*	

Coincidences of lines real or apparent.

Those lines the wave-lengths of which are approximately the same have been tabulated, and a close examination of photographs taken from electrodes of such metals as appear to have coincident lines has been made. The instances where lines appear to coincide are extremely rare. One particular case may be referred to, it is that of the two lines 2307 cadmium, and 2306·9 indium. The latter line is the stronger, which points to the occurrence of indium in cadmium, assuming the difference in the numbers to be accidental. An electrode of the one metal was opposed to the other, and a spectrum photographed with the diffraction spectroscope shows that the indium line is distinctly more refrangible than that of cadmium. Many air lines are coincident with metallic lines : this arises not only from their great numbers, but from their breadth and band-like character. In many parts of the spectrum some of our photographs show a continuous background to the metallic lines, which is formed of what to all appearance are finely and very closely ruled lines, these belong to the spectrum of air.

TABLE of approximate and identical wave-lengths of lines belonging to the spectra of the following elements, viz.:—Copper, Silver, Thallium, Indium, Tin, Lead, Tellurium, Aluminium, Cadmium, Arsenic, Antimony, and Bismuth; together with observations on some apparent coincidences.

Cu.	Ag.	Tl.	In.	Sn.	Pb.	Te.	Al.	Cd.	Observations.
3306·8	..	..	..	..	..	3307·1	..	..	A series of coincidences occurs here. The two strongest and longest lines of copper are apparently identical with two of the strongest and longest lines of tellurium. The lines are of the same character in both spectra. The two weak lines of copper coincide with two weak lines of tellurium, and in length, strength, and other features they are precisely the same in both spectra.
3289·9	..	..	..	..	..	3289·6	..	..	
3280·1	..	..	..	..	..	3280·0	..	..	
3273·2	..	..	..	..	..	3273·4	..	..	
3246·9	..	..	..	..	..	3246·8	..	..	
..	..	..	3174·1	3174·3	..	..	..	..	The two least refrangible of these lines appear to be due to tin. The other lines in the spectrum of indium, approximately coincident with those of tin, are of a totally different character.
..	..	..	3047·0	3046·5	..	..	..	..	
..	..	..	3008·0	3007·9	..	..	..	..	
..	..	..	2956·1	..	..	2956·3	..	..	Lines with a different origin and different wave-lengths.
..	..	..	2940·7	..	..	2940·8	..	..	Possibly coincident lines.
..	..	..	2932·2	..	..	2932·5	..	..	Exceedingly faint in tellurium. Very doubtful.
..	..	..	..	..	..	..	2879·9	2880·1	Both fairly strong lines of a similar character. Possibly coincident.
..	..	..	2836·0	..	..	..	..	2836·1	Lines of different characters. The indium very sharp, the cadmium nebulous. Different lines.
..	..	..	2832·3	..	2832·2	..	..	..	The indium line very weak and nebulous, the lead very sharp and strong. Different lines.
2766·2	2766·4	..	..	..	..	..	..	2766·5	Very doubtful. Line strong and long in silver, very faint and long in copper, exceedingly short and rather strong in cadmium.
..	..	..	2631·7	2631·5	..	..	..	..	Lines with different wave-lengths.
..	2579·9	2579·7	..	..	..	..	..	..	Lines with different wave-lengths.
..	2561·5	..	..	..	2561·6	..	..	..	Very doubtful. Probably lines with different wave-lengths.
..	..	2551·6	..	..	..	..	..	2551·6	Both very faint lines. Doubtful.
..	..	..	2545·8	2545·6	..	..	..	..	Very doubtful. The indium line feeble.
2506·2	2506·0	..	..	..	..	..	..	..	Lines with different wave-lengths.
2485·6	2485·4	..	..	..	..	..	..	..	Faint and very doubtful.
..	..	2477·7	2478·1	..	..	..	..	..	Faint and very doubtful.
2473·2	2473·3	..	..	..	..	..	..	..	Doubtful. Probably a very close approximation.
..	2469·0	2468·9	2468·4	..	..	..	..	..	The silver line very faint. The thallium and indium have different wave-lengths.
..	2445·7	..	..	..	2445·7	..	..	..	Very doubtful. The silver line is very weak and short.
..	2411·3	..	..	..	2411·2	..	..	..	The silver line strong, the lead weak and of a totally different character. These lines are not coincident.
..	..	..	2429·1	2429·3	..	..	..	..	The indium line exceedingly faint. Very doubtful.
..	2393·3	..	..	..	2393·7	..	..	..	The silver line exceedingly faint, the lead very strong. Lines of a totally different character.
..	2364·3	2364·3	..	..	..	..	..	..	Lines not coincident. Very faint in the thallium.
..	..	..	2306·9	..	..	..	..	2307·0	These lines are not coincident, the indium is the more refrangible.
2265·8	..	..	..	..	..	..	..	2265·3	Extremely feeble in copper, but very strong in cadmium. Doubtful.
..	2247·6	..	..	..	2247·9	..	..	..	Lines of similar character. Doubtful.

TABLE of approximate and identical wave-lengths of lines belonging to the spectra of the following elements, viz.:—Copper, Silver, Thallium, Indium, Tin, Lead, Tellurium, Aluminium, Cadmium, Arsenic, Antimony, and Bismuth; together with observations on some apparent coincidences (continued).

Sn.	Pb.	Te.	As.	Sb.	Bi.	Observations.
4324·6	..	4324·6	..	..	..	Very faint lines. Doubtful.
3961·8	3961·5	..	..	..	..	Both very short and weak lines. Doubtful.
..	3842·9	..	3842·5	..	..	Lines totally differing in character. Lines with different wave-lengths.
3800·3	..	..	3800·7	..	..	Lines totally differing in character, that in arsenic probably an air-line.
3471·1	..	..	3471·1	..	..	Lines of a similar character, that of tin the stronger.
..	..	3280·0	..	3279·7	3279·9	A line of tellurium in antimony and bismuth.
..	..	3273·4	..	3273·0	..	A line of tellurium in antimony.
..	..	3256·8	3256·2	..	..	Lines with different characters; strong in tellurium, weak in arsenic.
..	..	3246·8	..	3246·6	..	A line of tellurium in antimony.
3219·6	3219·9	..	..	..	..	Lines with totally different characters, that of lead the stronger. Probably different lines.
3174·3	..	3174·4	..	..	..	Lines of the same character, that of tin much the stronger.
..	3016·5	3016·6	..	..	..	Lines of the same character, that of tellurium much the stronger. Probably different lines.
2877·4	..	..	..	2877·1	..	Different lines with totally different characters, that of antimony being the strongest.
..	..	2859·9	2859·7	..	..	The arsenic line is very strong, the tellurium line very strong. Totally different lines.
..	..	2840·0	..	..	2840·1	Both lines weak and nebulous. The tellurium long, and the bismuth short. Doubtful.
..	2822·1	..	..	..	2822·2	Lines of a totally different character; that of lead strong, bismuth very weak.
..	..	..	2779·5	..	2779·3	Both lines of the same character, that of arsenic the stronger.
..	..	2768·6	..	2768·9	..	A line of tellurium in antimony.
..	..	..	..	2651·3	2651·8	Lines of the same character, that of antimony the stronger.
..	..	..	2597·1	2597·2	..	Similar lines; that of antimony very strong, of arsenic very weak.
..	2576·4	..	2576·0	..	..	Lines with totally different characters, and with different wave-lengths.
..	..	2543·7	..	..	2543·3	Lines of similar character, that of bismuth (the weaker) is nebulous. Probably lines with different wave-lengths, as the numbers indicate.
..	..	2529·4	..	..	2529·7	A line of tellurium in bismuth probably.
2523·4	..	..	..	..	2523·5	Very similar lines, that of bismuth the stronger.
..	..	..	2489·1	..	2489·1	These measurements are not comparable; that for bismuth is the centre of a nebulous band.
..	..	..	..	2479·4	2479·1	Similar lines, that of antimony the less weak.
..	..	2473·2	..	2473·4	..	Tellurium in antimony.
..	..	2438·0	..	2438·0	..	Tellurium in antimony.
..	2411·2	2411·4	..	..	..	Similar lines, that of tellurium the less faint. Doubtful.
..	..	2370·3	..	2370·0	..	Tellurium in antimony.
2368·3	..	..	..	..	2368·0	Lines of the same character, but stronger and longer in bismuth. Different lines.
2317·9	..	2317·8	..	..	2317·4	The lines of tin and tellurium are of similar character, but differing in length and strength, those
2247·0	..	2247·3	..	..	..	of tellurium being the stronger. The bismuth line is of quite a different nature.

THE Spectrum of Mercury.

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
24·05	VERY STRONG, CONTINUOUS, much extended, sharp	*4358·0*	All the lines in this spectrum were measured from a prism photograph, and are therefore printed in Italics.
24·61	Faint, continuous, fine	*4348·0*	Several lines due to tin are visible in the photographs, but they are not recorded here.
25·03	Faint, continuous, fine	*4341·0*	
41·56	Weak, continuous, fine	*4077·5*	
43·76	VERY STRONG, CONTINUOUS, much extended, sharp	*4046·5*	
48·19	VERY STRONG, CONTINUOUS, much extended, sharp	*3984·0*	
57·65	Weak, continuous, nebulous	*3859·0*	
60·85	Weak, continuous, nebulous	*3820·0*	
61·90	Faint, continuous	*3807·0*	
62·45	Faint, continuous	*3800·0*	
63·31	STRONG, continuous	*3790·0*	
65·04	Weak, continuous	*3770·0*	
66·34	Faint, fine, continuous	*3754·7*	
66·65	Strong, continuous	*3751·0*	
72·82	Weak, continuous, nebulous	*3681·9*	
74·60 75·37 77·37	A triplet of fairly strong, continuous, sharp, much extended lines, with a nimbus, the most refrangible line being the strongest and most extended	*3662·9* *3654·4* *3632·9*	
84·47	STRONG, CONTINUOUS, fine	*3560·1*	
86·27	STRONG, CONTINUOUS, fine	*3542·3*	
91·52	Very faint, short	*3492·6*	
93·60	Very faint, short	*3473·4*	
96·04	Very faint, short	*3451·4*	
103·00	STRONG, CONTINUOUS, somewhat nebulous, extended	*3389·5*	
105·84	Faint, discontinuous, fine	*3365·5*	
107·58	Weak, nebulous, continuous	*3351·2*	
108·79	STRONG, CONTINUOUS, extended	*3341·2*	
123·40	Faint, short fine	*3226·4*	
126·02	Weak, continuous, fine, faint in the centre	*3207·1*	
137·08 137·95	A PAIR OF VERY STRONG, continuous, much extended lines, with a nimbus	*3130·4* *3124·5*	
142·50	Faint, fine, continuous	(*3094·0*)	Possibly due to an impurity.
154·12	STRONG, CONTINUOUS, with a nimbus, extended	*3021·0*	
163·37	VERY STRONG, BROAD, with a nimbus, extended	*2966·4*	
166·86	STRONG, CONTINUOUS, fine, extended	*2946·6*	
168·84	Weak, discontinuous, fine	*2935·5*	
170·70	Weak, continuous, fine	*2925·2*	
172·50	Weak, continuous, fine, faint in centre.	*2915·3*	
176·63	STRONG, CONTINUOUS, sharp, extended	*2892·9*	
185·45	VERY STRONG, CONTINUOUS, broad, somewhat nebulous, extended	*2846·8*	
188·34	Faint, continuous, fine	*2832·1*	
191·04	STRONG, CONTINUOUS, nebulous	*2819·7*	
192·80	Faint, discontinuous, nebulous	*2810·0*	
193·90	Fairly strong, continuous, fine, weak in centre	*2804·5*	

The Spectrum of Mercury (continued).

Scale-numbers.	Description of lines.	Wave-lengths.	Remarks.
195·17	Weak, continuous, nebulous, faint in centre	*2798·5*	
196·96	Weak, continuous, fine, faint in centre	*2790·0*	
200·48	Faint, discontinuous, nebulous	*2773·2*	
203·09	Weak, fine, discontinuous	*2760·8*	
205·06	Fairly strong, fine, continuous	*2751·5*	
216·26	Faint, nebulous, discontinuous	*2702·0*	
226·66	Weak, discontinuous, nebulous	*2657·6*	
227·97	Strong, very broad, nebulous	*2652·2*	
229·79	Faint, nebulous, discontinuous	*2644·6*	
230·77	Faint, broad, nebulous, discontinuous	*2640·6*	
240·37	Fairly strong, fine, discontinuous	*2602·3*	
245·21	Faint, discontinuous, fine	*2584·2*	
247·46	Faint, continuous, nebulous	*2575·3*	
258·20	Very strong, sharp, continuous, extended	*2535·8*	
258·75	Strong, nebulous, continuous	*2533·8*	
261·88	Very faint, discontinuous, nebulous	*2522·7*	
264·20	Very faint, discontinuous, nebulous	*2514·3*	
270·83	Strong, fine, continuous, weak in centre	*2491·4*	
273·64	Faint, continuous, broad, nebulous	*2484·2*	
274·85	Very faint, discontinuous, nebulous	*2477·7*	
277·85	Faint, discontinuous, nebulous	*2468·0*	
278·16	Faint, discontinuous, nebulous	*2467·0*	
279·15	Faint, discontinuous, nebulous	*2463·7*	
280·53	Very faint, discontinuous, nebulous	*2459·3*	
294·79	Strong, continuous, fine, weak in centre	*2414·3*	
297·07	Strong, continuous, fine, weak in centre	*2407·3*	
302·78	Very faint, continuous, nebulous	*2390·0*	
314·84	Weak, continuous, nebulous	*2355·2*	
319·38	Very faint, continuous, nebulous	*2342·2*	
320·20	Very faint, discontinuous, nebulous	*2340·0*	
329·21	Very faint, continuous, nebulous	*2315·2*	
336·25	Very faint, continuous, fine	*2296·5*	
337·74	Very faint, continuous, nebulous	*2292·6*	
348·65	Fairly strong, continuous, fine, weak in centre	*2264·2*	
349·27	Fairly strong, continuous, fine, weak in centre	*2263·3*	
350·00	Strong, nebulous, continuous	*2261·4*	
353·00	Strong, fine, continuous	*2254·0*	
362·28	Very faint, fine, continuous	*2231·0*	
364·51	Strong, broad, nebulous, continuous	*2225·7*	
379·09	Very faint, fine, continuous	*2190·9*	
398·45	Very faint, fine, continuous	*2148·0*	

VI. *Experiments upon the Heart of the Dog with reference to the Maximum Volume of Blood sent out by the Left Ventricle in a Single Beat, and the Influence of Variations in Venous Pressure, Arterial Pressure, and Pulse-Rate upon the work done by the Heart.*

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[PLATE 7.]

THE most important factor to be determined before calculating the work done by the heart is the quantity of blood forced from the ventricles at each systole. Most of the efforts to determine this quantity have been based either upon faulty observations upon the dead heart, or upon the uncertain data obtained by estimating the mean velocity of the stream of blood in the aorta. Professor MARTIN accordingly suggested to us that we should attempt to measure it directly on the isolated Dog's heart. The work thus undertaken was carried on during the greater part of the university session, 1881-82, and the results obtained are given in the following pages. The method of isolating the heart was essentially that described in Professor MARTIN'S paper (Phil. Trans., 1883, p. 663).

In the course of this work many unexpected difficulties arose, necessitating changes in the apparatus and the method of operating, and preventing us for a long time from obtaining any successful results. In our experiments it was necessary not only that the heart should live and beat, but that it should be in the best possible physiological condition, and any marked pulmonary œdema made an experiment nearly valueless. This most frequent cause of failure was mainly owing to the fact that, on account of the large quantity of blood required for an experiment, we were obliged to use Calf's blood obtained from the butcher; very often this blood, as Professor MARTIN states in his paper, will bring about œdema of the lungs in a short time; large quantities of exuded serum pour out of the tracheal cannula, the air-passages in the lungs become choked up with liquid, and the circulation from the right to the left side of the heart is greatly impeded. We have succeeded, however, in making a considerable number of experiments in which all the conditions

were favourable, the œdema of the lungs not occurring to any marked extent until after many observations had been made.

In speaking briefly of former attempts to compute the amount of blood thrown out from the left ventricle at each systole, it is hardly necessary to refer to the earlier observations made upon dead hearts, since these are universally allowed to be of but little value. We need only say a few words about the later experiments upon living animals made by VOLKMANN, VIERORDT, and FICK.

The method used by VOLKMANN (1) is too well known to require any extended description. It is based upon the principle that the velocity of the blood stream is inversely as the width of the channel in which it flows. The line of argument used is this—starting from the aorta at its origin, which he calls the first vascular section, we have, first, a division into the innominate and the aorta beyond the innominate; this he calls the second vascular section, and supposes that the velocity of the stream in each of the two divisions of this section is the same, and is as much less than the velocity in the undivided aorta as their united sectional areas are greater than the sectional area of the aorta. The innominate in turn divides into branches making a third vascular section; the velocity in each of these branches is again the same, and as much less than the velocity in the innominate as the sum of their sectional areas is greater than the sectional area of the innominate. Then by determining the velocity of the blood in one of these last divisions by means of his hämodromometer, and measuring the width of the different vascular sections mentioned, the necessary data were obtained for estimating the quantity of blood passing through the sectional area of the ascending limb of the aortic arch in a given time, and therefore, knowing the pulse-rate, the quantity of blood sent out from the ventricle at each systole.

The whole method is evidently subject to many serious errors, the most important of which is that the velocity of the blood stream, as has been shown by DOGIEL, undergoes such great variations that but little positive value can be attached to an experimental determination of it at any one time, or even to the mean deduced from many observations.

Besides this, the method employed to determine the width of the vascular sections is open to objection. The means used for determining this factor were briefly these. After obtaining the velocity in the carotid, the diameter of that vessel was measured; the animal then killed, and the vascular system injected from the abdominal aorta with melted wax, and under such a pressure that the carotid assumed the same diameter as that which it had previously when the velocity in it was measured. After the wax had hardened the diameter of the aorta and of the other vessels was obtained. As VOLKMANN himself says, one is not at all sure in this case that the parts of the vascular system under consideration will be distended in the same proportion to each other that existed during life; he thinks, however, that this will make no difference as long as the carotid possesses the same diameter that it had when the velocity in it was measured, since any over-valuation of the width of

the aorta will be balanced by a proportional under-valuation of the velocity in it, and inversely. While this may be true of the aorta as compared with the carotid alone, it will hardly hold with regard to the intervening vessels, the second branch of the third vascular section for instance, any mistake in the diameter of which will introduce an error into the calculation. VIERORDT's method (2) is based, like that of VOLKMANN, upon the proposition that the mean velocity in the arterial system is inversely as its sectional area. He determined the mean velocity in the carotid by means of his hæmotachometer, obtaining results for the Dog which agree very closely with those obtained by VOLKMANN. Knowing the velocity in the carotid, and accepting the measurements given by KRAUSE for Man of the diameter of the carotid, the sub-clavian, the innominate, and the aorta beyond the point at which the innominate is given off, he deduced the quantity of blood flowing through the sectional area of each of these arterial trunks in a second. The quantity of blood flowing per second through the sectional area of the ascending limb of the aortic arch was taken as equal to the sum of these quantities plus 4 cub. centims. allowed for the coronaries. Knowing the number of systoles occurring in a second, the quantity thrown out at each systole is easily obtained. VIERORDT does not accept the supposition of VOLKMANN that the velocity in each branch of any one vascular section is the same, but thinks it safer to assume that the mean velocity in the aorta beyond the giving off of the innominate is one-fourth greater than that in the innominate. For the ratio of the weight of blood thrown out from the left ventricle at each systole to the body weight, he gets a fraction almost identical with that of VOLKMANN: about $\frac{1}{400}$th.

FICK, in consequence of the unreliable data upon which the results obtained by VOLKMANN and VIERORDT are based, can see in their close agreement only accident. His own method (3) while possessing the advantage, as far as Man is concerned, of being applicable directly to the human subject, rests, however, upon assumptions which cannot be freely allowed. The arm in his method was placed in a sort of plethysmograph, by means of which changes in volume occurring at each systole were registered upon a revolving drum; from this curve of volume he constructed a curve showing the changes in the strength of the stream in the axillary artery as compared with the strength of the stream in the axillary vein, which was taken as a constant. From a comparison of this curve with the curve of changes of velocity in the carotid of the Horse obtained by CHAVEAU, he endeavoured to give absolute values to the ordinates of his curve, and in this way determined the quantity of blood flowing through the sectional area of the axillary artery in a second; to obtain the quantity flowing through the sectional area of the sub-clavian in a second he multiplied by two. Then taking the ratio of the strength of stream in the sub-clavian to that in the aorta as given by VIERORDT, he obtained the quantity of blood flowing through the sectional area of the aorta in a second, from which, knowing the pulse-rate, the quantity of blood thrown out at each systole was deduced. The mean of his two experiments

gave about $\frac{1}{1000}$ as the ratio of the weight of blood thrown out at each systole to the body weight.

Before passing on to a discussion of the results of our work it will be necessary to give a brief description of the manner in which observations were made. The apparatus and the method of operation were the same as those described by Professor MARTIN (Phil. Trans., 1883, p. 663). Our Dogs, however, were always anæsthetized by means of a mixture of chloroform and ether, and great care was taken to introduce into the aorta a cannula with as large a bore as possible.

As quickly as possible after the Dog was in the warm chamber, all the connexions made and the heart going well, observations were begun in order to complete a series before pulmonary œdema commenced to impede the flow of blood from the right to the left side of the heart. The chief point in an observation was to determine the quantity of blood pumped out from the left ventricle in thirty seconds. In order to accomplish this, one of us took charge of the kymograph, upon the roll of paper of which was made to write on the same vertical line the pens of two manometers (one recording mean pressure, the other the pulse rate), and of the chronograph, and a marking pen to indicate the period during which the blood was collected from the outflow tube. This person, when an observation was to be made, after allowing the kymograph to run for a few seconds in order to see that the pens were all writing properly, counted aloud 30 seconds, pushing down the handle of the marking pen at the beginning of that time and holding it in that position until the 30 seconds had been counted. The other person meanwhile collected the blood pumped out from the left ventricle during this time by simply moving the end of the outflow tube S (Plate 7) from the funnel x to a graduated cylinder at the beginning of the 30 seconds, and back again to the funnel at the end. The kymograph was then stopped, the observation numbered upon the roll of the kymograph paper and also in the notebook, and the quantity of blood pumped out, the time of the observation, the venous pressure used during the observation, and the temperature of the blood flowing into the heart as given by the thermometer p, noted down. After the conclusion of an experiment the tracings were carefully examined, and the pulse-rate and the mean arterial pressure during each observation ascertained. After each observation, in place of the blood collected, about an equal amount of warmed blood, a supply of which was kept at hand, was poured back into the receiving flask through the funnel F.

In giving the results of our experiments, it will be convenient to consider them under four different heads, viz.:—

I. The maximum quantity of blood which can be thrown out from the left ventricle at a single systole.

II. The influence of variations of arterial pressure on the work done by the heart.

III. The influence of variations of venous pressure on the work done by the heart.

IV. The influence of variations of pulse-rate on the work done by the heart.

I.

The maximum quantity of blood which can be thrown out from the left ventricle at a single systole.

Our method of working in determining this quantity was as follows:—After the animal had been placed safely in the warm case, and all the connexions had been made, an observation was immediately taken in the way described, at a venous pressure of 10 centims.; the pressure was then raised 5 or 10 centims. and another observation taken, and so on, until a limit was reached, that is, a point beyond which increase of venous pressure did not cause an increased outflow from the aorta. The venous pressure was then brought back to 10 centims., and an observation taken as a control experiment to determine whether or not the condition of the lungs in the meantime had been such as to interfere with the passage of blood to the left auricle.

Below is given a table showing the results of six experiments upon this point; the complete records of these experiments will be found farther on under Section III., with the exception of those of May 9th and May 30th. The control observation in all the experiments, with the exception of the two just named, in which it was not made, showed that the passage through the lungs had not yet become seriously obstructed. This part of the experiment, indeed, was generally completed within a few minutes, and before the lungs had become noticeably œdematous. The temperatures given are those of the blood flowing into the right side of the heart.

The weight of the heart is given together with that of the whole animal, though there does not appear to be any constant relationship between it and the weight of blood pumped out of the ventricle at each systole. In weighing the heart the vessels springing from it were cut off at their origin, superfluous fat removed, and the cavities of the heart cut open and cleaned.

TABLE showing the maximum quantity of blood which can be thrown out from the left ventricle at a single systole.

No. of experiment.	Date. 1882.	Weight of Dog.	Weight of heart.	Temperature of blood flowing into the heart, C.	Heart beats in 30 seconds.	Venous pressure in centims. of blood.	Arterial pressure in millims. of mercury.	Total outflow from the aorta in 30 seconds in cub. centims.	Outflow from the aorta at each systole in cub. centims.
		grms.	grms.	°					
1	April 4	5891	51·5	36	99·5	36·8	109	576	5·79
2	„ 13	8125	92	34·25	86·5	50	135	750	8·67
3	„ „	„	„	35·25	84	58	134	715	8·51
4	„ 18	7725	63	37	103	70	143	850	8·25
5	May 2	8610	76	35·5	89·5	60	138	982	10·97
6	„ 9	5645	73	35	82·5	35	112	490	5·94
7	„ 30	9555	81·5	37	102	59	121	950	9·31

Observations on the preceding table.—In experiment 1, April 4th, it is probable that the limit was not quite reached, as will be seen by referring to the complete record of that experiment (see Section III.); the apparatus at that time being arranged so that 36·8 centims. was the highest venous pressure that could be obtained.

In experiment 4, April 18th, the limit was practically reached at a venous pressure of 60 centims. The right auricle then received all the blood the heart could pump out.

In experiment 7, May 30th, we are not positive that the limit was completely reached, no higher venous pressure than that recorded was tried, on account of the object for which the experiment was performed (see Section IV.). This experiment was introduced into this table, since it had been found in the other experiments that the maximum outflow from the aorta was obtained at or below a venous pressure of 60 centims. Since the Dog in this case was somewhat larger than those usually experimented upon, it is possible that a larger outflow might have been obtained at a higher venous pressure.

The ratio of the weight of blood thrown out at each systole to the weight of the animal is given in the following table. The specific gravity of the defibrinated Calf's blood is taken as 1050; an accurate determination in one case gave 1047.

April 4.	$5{\cdot}79 \times 1{\cdot}05 = 6{\cdot}08 \div 5891 = {\cdot}00103$
„ 13.	$8{\cdot}67 \times 1{\cdot}05 = 9{\cdot}10 \div 8125 = {\cdot}00112$
„ 18.	$8{\cdot}25 \times 1{\cdot}05 = 8{\cdot}66 \div 7725 = {\cdot}00112$
May 2.	$10{\cdot}97 \times 1{\cdot}05 = 11{\cdot}52 \div 8610 = {\cdot}00134$
„ 9.	$5{\cdot}94 \times 1{\cdot}05 = 6{\cdot}24 \div 5645 = {\cdot}00111$
„ 30.	$9{\cdot}31 \times 1{\cdot}05 = 9{\cdot}78 \div 9555 = {\cdot}00102$

The agreement amongst the results is as close as could be expected, when we remember the number of disturbing conditions which may come into play.

One of the most important causes of variation, the value of which we did not fully recognise until our experiments had nearly drawn to a close, is to be found in the pulse-rate; this, as will be shown in Section IV., exercises a marked influence on the amount of blood thrown out at each systole.

Omitting the experiments of April 4th and May 30th, since in both cases the maximum was not quite reached, we find that *the mean ratio of the maximum weight of blood thrown out from the left ventricle at a single systole to the whole body weight is* ·00117 *or* $\frac{1}{855}$, *for a mean pulse-rate of* 180 *per minute.*

Since the slowing influence of the vagi is removed from a heart isolated in this way, the pulse-rate of course is greater than in the normal Dog.

The average pulse-rate in a living Dog may be taken as about 120 per minute. We have as yet only one experiment, that of May 30th (see Section IV.), to show what the maximum outflow at each systole is when the heart is beating at this rate. According to that experiment the ratio of the maximum weight of blood forced out

from the left ventricle at each systole to the weight of the animal, when the heart is beating at 120 per minute, is about ·0014, or $\frac{1}{700}$.

It should be mentioned that care was taken in these experiments that the heart should receive an abundant supply of blood. The possible outflow from the flasks in 30 seconds for each venous pressure used was determined beforehand, and found to exceed by several hundred cubic centimetres the actual quantities pumped out by the heart.

When we come to apply the knowledge obtained in this way from the isolated heart to the heart in the normal animal, we are met with the difficulty of deciding which of the results to take as most nearly representing the actual condition of things in the body. For several reasons we are inclined to believe that the maximum outflow comes closer to the average quantity thrown out during life at each systole than any other we might take; in other words we think it very probable that the left ventricle during life is distended during each diastole to about its maximum capacity. Such a supposition is supported by the work of ROY (4) on the Frog's heart.

ROY found for the Frog that with an intra-ventricular pressure of 15 centims., the capacity of the ventricle had practically reached its limit, and that this is just about the amount of intra-ventricular pressure during life. "The intra-ventricular pressure during diastole, so far as it is governed by the auricles, will vary from 2 to 10 centims., precisely the limit at which the ventricle has its greatest possible distensibility within the limits of elasticity."

Another point which may be brought forward that lends some probability to this view, is found in a consideration of the time required for a complete circulation of the blood to take place. Knowing the proportion of the total weight of blood to the weight of the Dog, and the quantity of blood discharged from the heart at each systole, it is evident that we have a ready means of determining the time necessary for the completion of a circulation. Let us see how the time obtained by supposing that our maximum quantity represents the true capacity of the ventricle during life agrees with the results obtained from Dogs by direct experiment.

Taking a Dog weighing 8000 grms., the total quantity of blood will be equal to $8000 \times {\cdot}076 = 608$ grms. The weight of blood thrown out from the left ventricle at each systole, with a pulse-rate of 120, will be about $8000 \times {\cdot}0014 = 11{\cdot}2$ grms. The number of systoles then that must occur in order for the total weight of blood to make a complete circuit through the left ventricle will be $608 \div 11{\cdot}2 = 54$; and 54 systoles at the given pulse-rate will take 27 seconds.

VIERORDT, in his experiments on Dogs, found that the greatest time required for a salt injected into the jugular vein to be detected in the blood of the femoral vein was 21·76 seconds; his mean result from four experiments was 18·08 seconds.

The number obtained from our calculation is somewhat higher, as in the nature of the case it should be, since VIERORDT's experiments were directed to only one of the many paths open to the blood in normal circulation, some of which will require

a longer time to traverse, as for instance, those in which the portal system is included, some a shorter time.

If we take the ratio of the weight of blood pumped out at each systole to the body weight as given by VOLKMANN, and with which that of VIERORDT is almost identical, viz., ·0025, the time necessary for a complete circulation in the hypothetical case above would be about 15 seconds.

The time obtained by such a calculation represents in reality the average time for all the numerous possible paths. Of the various paths open for the blood to take after leaving the heart, those in the course of which only two capillary regions are interposed will practically take about the same time to traverse, the velocity in the arteries and veins being such that mere distance from the heart will add but little to the time required. For that large quantity of blood which, sent out from the left ventricle, returns to it again only after having passed through three capillary regions, the time required for circulation will naturally be greatly increased. Hence the average time for all the different paths should be somewhat greater than the time found necessary by VIERORDT for the jugular-femoral path, as is the case with the time obtained by our calculation ; certainly not less, as would follow if VOLKMANN'S ratio were correct.

Another consideration which influences us in supposing that the maximum outflow at each systole from the isolated heart is about the normal outflow during life, is based on the pressure in the left auricle.

As far as we know, no one has ever determined the pressure in the left auricle during life without opening the thorax, but it is fair to suppose that it is as great as that in the right auricle. The maximum pressure in the right auricle of the Dog, according to an experiment of GOLTZ and GAULE (5), is about 19·6 millims. of mercury. We determined in one of our isolated hearts the pressure in the left auricle (in a way described in Section III.) for each different venous pressure used on the right side. According to this experiment the mean pressure in the left auricle when the feeding-flask stood at a height of 60 centims. above the right auricle, was only 16 millims. of mercury, the maximum pressure 20 millims., and consequently no greater than that which it is probable exists during life. Our maximum outflow was obtained in all cases either at or below a venous pressure on the right side of 60 centims.

The mean ratio of the weight of blood thrown out at each systole to the body weight, obtained by VOLKMANN for the Dog, is ·0027, or about twice the ratio obtained from our experiments. That there could have been an error of such magnitude in the method employed by us does not seem at all possible, and when we consider, on the one hand, the very uncertain data upon which VOLKMANN'S results are based and the complexity of the disturbing conditions, and, on the other hand, the comparative simplicity and directness of the method we have used, its freedom from sources of error, and the agreement of different experiments, we are

justified, we think, in claiming that our results, at least as far as the Dog is concerned, come nearer to the truth than do those of VOLKMANN or of VIERORDT.

Any inference from the results obtained for the Dog to the heart of Man will in the present condition of our knowledge be more or less uncertain. VIERORDT concludes, though upon no very firm grounds, that the weight of blood sent out from the ventricle at each systole in different animals is nearly proportional to the body weight. VOLKMANN also takes the same ratio $\frac{1}{400}$ as holding good for different Mammals.

If the ratio found by us for the Dog can be applied directly to Man it will support the results of FICK's experiments.

There is one fact, however, which it seems to us makes such an inference inadmissible, and that is the difference in pulse-rate between the heart of the Dog and the heart of Man: the average pulse-rate for Man is about 72 per minute, the average pulse-rate for the Dog about 120 per minute. Ought not this to make a difference in the amount of blood thrown out at each systole?

If the supposition is true that the ventricle in the Dog, and presumably in Man also, is distended during each diastole to about its maximum capacity, it would seem that variations in pulse-rate should have but little influence upon the quantity of blood ejected at each systole. From our experiments on the influence of pulse-rate on the outflow from the Dog's heart (see Section IV.), even when a pressure was used that gave at the ordinary temperature at which the blood was kept the maximum outflow, or very nearly the maximum outflow at each beat, it was found that slowing the pulse-rate increases very considerably the outflow at each beat. It is true that this may have been owing to the fact that the method used for slowing the pulse-rate, viz., by running cold blood through the heart, may have caused more or less change in the elastic properties of the ventricular walls. The whole question is one that rests as yet, as far as our experiments are concerned at least, upon suppositions that lack experimental confirmation. We hope ultimately, after carrying out similar experiments upon other animals, to be in a better condition to speak on the subject.

II.

Influence of variations of arterial pressure upon the work done by the heart.

In the experiments made upon this point the venous pressure was kept constant at 15 or 20 centims. of defibrinated Calf's blood (11·7 to 15·6 millims. of mercury); the arterial pressure was varied by simply raising or lowering the end S of the outflow tube *t*, (Plate 7). That variations of arterial pressure have no direct effect on the pulse-rate of the isolated Dog's heart has been clearly proved by Professor MARTIN's experiments. It would have been preferable, perhaps, to have kept the venous

pressure at the height at which the maximum outflow from the left ventricle was obtained, since in this case we believe that the left ventricle works under conditions most closely resembling those to which it is subject during life; but owing to the rapidity with which our flasks were emptied at these high pressures, and other mechanical difficulties, this was not attempted.

The heart was allowed to work for a short time, from one to two minutes, at each given arterial pressure before an observation was made.

We give below two examples of the results obtained; several other experiments were made, but the two given were those in which the heart and lungs were in the best condition, and the pulse-rate remained very nearly constant.

The work done in gramme-meters at each systole of the left ventricle at various arterial pressures is given in the last column, and was calculated by multiplying the amount of blood thrown out at each systole into the height to which it was raised, making of course the necessary corrections for the specific gravity of the blood used. The formula which we employed is $W = A.s.\frac{H}{1000} \cdot \frac{S}{s} = \frac{A.H.S.}{1000}$, in which W represents the work done, A the quantity in cubic centimetres thrown out from the ventricle at each systole, H the arterial pressure in millimetres of mercury, S the specific gravity of mercury, and s the specific gravity of the blood.

The arterial pressure was measured in the carotid, and is no doubt a little less than that at the root of the aorta.

In both the experiments given the observations upon arterial pressure were made after a series of observations at different venous pressures had been taken, the results of which are given in detail in Section III. under the same dates.

TABLE showing the effect of variations of arterial pressure on the work done by the heart.

April 13, 1882.—Weight of Dog, 8125 grms. Weight of heart, 92 grms.

Observations.	Time, P.M.	Temp. C. in superior cava.	Beats in 30 seconds.	Arterial pressure in carotid. Millims. of mercury.	Venous pressure in superior cava. Centims. of blood.	Outflow in 30 seconds. Cub. centims.	Outflow in 1 beat. Cub. centims.	Work done at each systole of the left ventricle in gramme-metres.
	h. m.	°						
1	2 14	35·75	83·5	112	20	375	4·49	6·78
2	2 16	35·25	78·5	88	20	360	4·58	5·45
3	2 19	36+	80	62	20	379	4·74	3·97
4	2 21	36+	81·5	92	20	367	4·50	5·59
5	2 24	36+	82	120	20	367	4·48	7·25
6	2 26	36·5	82	142	20	355	4·33	8·30
7	2 28	36·5	82	108	20	357	4·35	6·34

May 2, 1882.—Weight of Dog, 8610 grms. Weight of heart, 76 grms.

Observations.	Time, P.M.	Temp. C. in superior cava.	Beats in 30 seconds.	Arterial pressure in carotid. Millims. of mercury.	Venous pressure in superior cava. Centims. of blood.	Outflow in 30 seconds. Cub. centims.	Outflow in 1 beat. Cub. centims.	Work done at each systole of the left ventricle in grammemetres.
	h. m.	°						
1	3 46	34·5	82	120	20	385	4·69	7·59
2	3 48	35·25	83	84	20	375	4·52	5·13
3	3 49	35+	83·5	58	20	360	4·31	3·40
4	3 51	34·5	82	107	20	372	4·54	6·56
5	3 53	35	79·5	138	20	345	4·34	8·09
6	3 55	35	82	147	20	365	4·45	8·82
7	3 56	35−	81·5	136	20	361	4·43	8·13
8	3 57	34·75	81·5	62	20	350	4·29	3·59

It is seen from the tables that *variations of arterial pressure from* 58 *to* 147 *millims. of mercury have practically no effect whatever on the quantity of blood sent out from the ventricle at each systole.* The small differences that appear easily come within the limits of error. The methods of catching the blood is one that will unavoidably introduce small errors. Another source of error is connected apparently with the use of Calf's blood. After the heart had been working in the case for some time, it was almost always found that the pericardium was tightly filled with exuded serum, preventing complete distension of the ventricle during diastole, and consequently diminishing the outflow—in such cases by cutting a slit in the pericardium the outflow would immediately increase.

The observations given in the table were all taken before this filling of the pericardium had become sufficiently advanced to cause any important error; in later observations in the course of the same experiments its influence was very marked. The figures as they stand, however, show clearly, as we have said, that, within the limits given, variations of arterial pressure have no direct effect on the amount of blood thrown out from the left ventricle. For how much wider limits than those indicated this statement may be true, we cannot yet say. It is also clear that, as Professor MARTIN had already proved (10), the pulse-rate remains unchanged. Since now the work done by the contraction of the ventricle depends on two factors, viz., the amount pumped out at each systole, and the height to which this amount is raised, and one of these factors remains practically constant, it follows that *the work done by the left ventricle of the Dog's heart varies directly as the arterial pressure against which it works within the limits named above.*

BLASIUS (6), in his experiments on the isolated Frog's heart, found that the work of each single beat increased for a time with increase in arterial pressure, but that, nevertheless, "die Intensität des Wachsthums mit nur wenigen Ausnahmen allmählich abnimmt."

By the investigations of WEBER, HEIDENHAIN, FICK, and others, it has been proved for ordinary skeletal muscles that the work done in contracting, measured by the product of the load into the lift, increases up to a certain limit with the load to be raised. The increase in work, however, is not proportional to the increase of load, since as a general rule the lift is diminished as the load is increased, except, perhaps, for minimal weights. If, now, aortic pressure is taken as the equivalent of the load which an ordinary muscle raises when it contracts, the law given above for the work of the left ventricle may be expressed in the terms of muscle physiology in this way. The work done by the heart muscle when it contracts, measured by the product of the weight of blood ejected at each contraction into the height of aortic pressure, not only increases with the load against which it contracts, but increases in direct proportion to the load, within the limits given. It is not probable that this proportional increase of work by the heart muscle is owing to any nervous mechanism co-ordinating the discharge of energy with the resistance to be overcome. Considering it as a muscle phenomenon alone, two explanations suggest themselves. It might be conceived that within the limits given, the total energy liberated at each contraction remains constant, and that that portion of it which is not used up in external work disappears as heat liberated in the heart itself. So that as the arterial pressure increases, making the resistance to be overcome greater, a correspondingly greater portion of the energy appears as external work, and *vice versâ*. Outside of the waste of energy which such a supposition involves, the study of the development of heat in a contracting skeletal muscle teaches us that for it, at least, there is no such inverse proportion between the amount of heat liberated and of external work done in a muscle contracting under different loads. The curves of heat development and of mechanical work, on the contrary, follow a somewhat similar course. An explanation more in accordance with what is known of the physiology of ordinary muscle is found in the supposition that as the load increases a greater amount of energy is liberated, in consequence of some change in the molecular state of the ventricular muscle associated with increased tension at the commencement and during the early stages of its contraction. This is the supposition adopted for ordinary muscle; as FOSTER states it, "the tension of the muscular fibre increases the facility with which the explosive changes resulting in a contraction take place" (Physiol., 1883, p. 88). The difference between the two muscles is that for the heart muscle the energy liberated as external work bears a direct proportion to the tension exerted by the load, while for ordinary muscle this is not true. The lift of an ordinary muscle when contracting is represented, in the case of the heart muscle, by the extent of the contraction, measured by the volume of blood ejected. Since within the range of arterial pressures given the volume of blood thrown out at each systole remains the same, it follows that the extent of the contraction is unchanged. The most probable interpretation of this fact is that the contraction in each case is maximal, and completely empties the ventricular cavity; a conclusion which is in accordance with the work of BOWDITCH, KRONECKER, and others on the isolated Frog's heart.

A curve of work constructed upon the arterial pressures as abscissas, and the work done at each beat of the ventricle under these pressures as ordinates, would, within the limits for which we have investigated it, be a straight line. Owing to the sources of error which we have enumerated above, the results were not sufficiently accurate to construct such a curve.

III.

Influence of venous pressure on the work done by the heart.

In our experiments on the maximum outflow from the ventricles at each systole the venous pressure was varied, as stated, from 10 centims. to 60 or 70 centims. The complete tables of four of the experiments are given below.

In accordance with the results of Professor MARTIN'S previous work, it was found that variations of venous pressure from 10 centims. to even as high as 70 centims. have no direct effect on the pulse-rate.

It was not possible to keep the arterial pressure constant, since at the higher venous pressures the left ventricle pumped out much more blood at each systole, and the increased outflow caused an increased tension in the outflow tubes and the roots of the great arteries still connected with the left ventricle. Since, however, as we have seen, the amount of outflow is not affected by the arterial pressure within the limits occurring in the experiments, we have sought to eliminate the influence of the variations in arterial pressure on the work done, by calculating the work for 100 millims. arterial pressure in all cases, and placing the results in the column to the right of those calculated from the arterial pressures recorded; in this way a clearer idea of the influence of venous pressure alone on the work done by the heart is obtained—the same thing is shown, of course, in the column giving the outflows at each beat.

TABLE showing the effect of variations of venous pressure on the work done by the heart.

April 4, 1882.—Weight of Dog, 5891 grms. Weight of heart, 51·5 grms.

Observations.	Time.	Temp. C. in superior cava.	Beats in 30 seconds.	Arterial pressure in carotid. Millims. of mercury.	Venous pressure in superior cava. Centims. of blood.	Outflow in 30 seconds. Cub. centims.	Outflow in 1 beat. Cub. centims.	Work done at each systole of the left ventricle, in gramme metres.	
								With arterial pressure actually observed.	With arterial pressure = 100 millims.
	h. m.								
1	2 56	38°	116·5	88	10	255	2·19	2·60	2·96
2	2 58	38+	120	92	15	333	2·77	3·44	3·74
3	2 59	38	120	96	20	404	3·37	4·37	4·55
4	3 00	38+	120	100	25	489	4·08	5·50	5·50
5	3 01	38	116	106	30	582	5·02	7·17	6·78
6	3 03	37	116	110	36·8	629	5·42	8·04	7·32
7	3 06	..	113	107	35	610	5·40	7·80	7·29
8	3 09	36·5	106	88	10	230	2·17	2·58	2·93
9	3 11	36·5	105·5	92	15	299	2·83	3·51	3·82
10	3 12	36	104	100	20	381	3·66	4·93	4·94
11	3 13	36	103·5	100	25	460	4·44	5·99	5·99
12	3 15	36	101·5	102	30	506	4·99	6·87	6·74
13	3 16	36−	101·5	107	35	572	5·63	8·13	7·60
14	3 17	36−	99·5	109	36·8	576	5·79	8·52	7·82
15	3 20	35·5	99·75	88	10	175	1·75	2·08	2·36

April 13, 1882.—Weight of Dog, 8125 grms. Weight of heart, 92 grms.

Observations.	Time.	Temp. C. in superior cava.	Beats in 30 seconds.	Arterial pressure in carotid. Millims. of mercury.	Venous pressure in superior cava. Centims. of blood.	Outflow in 30 seconds. Cub. centims.	Outflow in 1 beat. Cub. centims.	Work done at each systole of the left ventricle, in gramme-metres.	
								With arterial pressure actually observed.	With arterial pressure = 100 millims.
	h. m.								
1	1 40	36°−	94·5	105	10	220	2·33	3·30	3·15
2	1 41	36−	89	112	20	400	4·49	6·78	6·06
3	1 43	36	92	121	30	535	5·81	9·48	7·84
4	1 44	36−	92	125	35	618	6·71	11·30	9·06
5	..	34·5	89·5	129	40	680	7·59	13·21	10·25
6	..	34·5	89·5	131	45	690	7·71	13·63	10·41
7	..	34·25	86·5	135	50	750	8·67	15·79	11·70
8	..	35·75	93	131·5	58	700	7·53	13·35	10·17
9	..	35−	93	105	10	202	2·17	3·08	2·93

New series: on same animal.

Observations.	Time.	Temp. C. in superior cava.	Beats in 30 seconds.	Arterial pressure in carotid. Millims. of mercury.	Venous pressure in superior cava. Centims. of blood.	Outflow in 30 seconds. Cub. centims.	Outflow in 1 beat. Cub. centims.	Work done at each systole of the left ventricle, in gramme-metres.	
								With arterial pressure actually observed.	With arterial pressure = 100 millims.
	h. m.	°							
10	1 57	35	87·5	105	10	220	2·51	3·55	3·39
11	1 58	35	88·25	113	20	375	4·25	6·48	5·74
12	2 00	34·5	88	120	30	510	5·79	9·36	7·82
13	2 03	35—	88	126·5	35 (?)	605	6·88	11·74	9·29
14	2 07	35	85	132·5	50	705	8·29	14·81	11·19
15	2 10	35·25	84	134	58	715	8·51	15·40	11·49

April 18, 1882.—Weight of Dog, 7725 grms. Weight of heart, 63 grms.

Observations.	Time.	Temp. C. in superior cava.	Beats in 30 seconds.	Arterial pressure in carotid. Millims. of mercury.	Venous pressure in superior cava. Centims. of blood.	Outflow in 30 seconds. Cub. centims.	Outflow in 1 beat. Cub. centims.	Work done at each systole of the left ventricle, in gramme-metres.	
								With arterial pressure actually observed.	With arterial pressure = 100 millims.
	h. m.	°							
1	2 18	37·5	103	99	10	204	1·98	2·65	2·67
2	2 19	37+	97·5	106	20	390	4·00	5·72	5·40
3	2 21	36	95	116	30	545	5·74	8·99	7·75
4	2 23	36·25	98·5	126	40	667	6·77	11·51	9·14
5	2 25	36—	97·5	131	45	720	7·38	13·05	9·96
6	2 27	37—	103	133	50	755	7·33	13·16	9·90
7	2 29	36+	102·5	136	55	781	7·62	13·98	10·29
8	2 32	36+	103	143	60	841	8·16	15·75	11·02
9	2 34	36	103·5	142	65	850	8·21	15·73	11·08
10	2 38	37	103	143	70	850	8·25	15·91	11·14
11	2 41	36·5	99	99	10	150	1·51	2·01	2·04

May 2, 1882.—Weight of Dog, 8610 grms. Weight of heart, 76 grms.

Observations.	Time.	Temp. C. in superior cava.	Beats in 30 seconds.	Arterial pressure in carotid. Millims. of mercury.	Venous pressure in superior cava. Centims. of blood.	Outflow in 30 seconds. Cub. centims.	Outflow in 1 beat. Cub. centims.	Work done at each systole of the left ventricle, in grammemetres.	
								With arterial pressure actually observed.	With arterial pressure = 100 millims.
	h. m.	°							
1	3 19	37·75	95·5	113	10	240	2·51	3·82	3·39
2	3 21	37	93	121	20	473	5·08	8·29	6·86
3	3 23	37·5	96	126	30	617	6·42	10·91	8·67
4	3 27	37−	93·5	134	40	780	8·34	15·08	11·26
5	3 30	36+	91·5	135	50	861	9·40	17·12	12·69
6	3 33	35·5	89·5	138	60	982	10·97	20·43	14·81
7	3 36	36−	87	140	65	940	10·80	20·40	14·58
8	3 40	35+	87	140	70	940	10·80	20·40	14·58
9	3 43	35	84	112	10	200	2·38	3·60	3·21

The great effect which increase of venous pressure and the attendant increase in the blood-flow have upon the total outflow from the left side of the heart, is shown in a striking manner by the preceding tables. From the numbers in the last column it is seen that *the work done by the left ventricle at each systole increases with the venous pressure, but not proportionally, up to the point of maximum work.*

One must be certain in such experiments that at each venous pressure used the quantity of blood sent into the heart is greater than that which is pumped out; in all cases the possible quantity of blood which could flow into the right side of the heart from the flasks at the different pressures was determined beforehand, and found to exceed the actual quantities thrown out from the left side in any given time.

The maximum pressure in the right auricle of the Dog during life is, according to GOLTZ and GAULE, 19·6 millims. of mercury. The pressure to which the right auricle was exposed in our experiments before the maximum outflow was obtained, varied from 27·2 millims. to about 46 millims. of mercury. Such pressures place the right side of the heart under conditions different from those which exist during life. However the right side of the heart was affected by these high pressures, the left side, with which we are more immediately concerned was, as will be shown below, under pressures which in all probability kept within the limits of pressures to which it is exposed during life.

It is certain that the most direct factor influencing the quantity of blood sent out from the ventricle, and hence the work done by the ventricle, is the intra-ventricular pressure by which the ventricle is distended during diastole. Leaving out the aspiratory action of the thorax, the intra-ventricular pressure during life must be mainly owing to the action of the auricle, since the pressure in the great veins

emptying into the auricle probably never rises to any important positive value; indeed, according to the experiments of LUDWIG, VOLKMANN, WEYRICH, and others, has always a mean negative value. The contraction of the auricles, then, must have the most important and direct effect upon the work done by the ventricles, and we are able to confirm for the Mammalian heart the statement made by ROY (8) for the Frog's heart, viz., "The work of the heart in the living animal is governed chiefly by the auricles, the ventricle influencing the amount of work done only indirectly."

In calculating the work done by the left ventricle, we have made no allowance for the venous pressure by which the left ventricle was distended.

A part only, though the much greater part, of the work done at each beat of the ventricle, as given in the tables, is owing to the active contraction of the ventricle itself, the other part is proportional to the pressure by which the ventricle is distended, and is owing to the elastic reaction of its walls.

With a desire to learn how great a correction must be made for this factor, as well as to find out the maximum pressure in the left auricle with the highest venous pressure used, we endeavoured to use upon our hearts the device employed by WALLER (9) upon Rabbits for ascertaining pressure in the left auricle.

After the heart was in the warm chamber and going well, the pericardium was slit open, the tip of the left auricular appendage seized with a pair of forceps, and gently pulled into view; the appendage was then clamped lower down with a weak clamp made especially for the purpose, a slit cut in its walls, and a cannula filled with salt solution 0·6 per cent. and connected by a piece of lead tubing, also filled with 0·6 per cent. salt solution, with a mercury manometer, was introduced into its cavity and firmly tied; the clamp was then removed.

Of three such experiments tried, two failed, the third was so far successful that the results obtained from it can be accepted as approximately correct. The beats of the auricle were plainly recorded upon the kymograph paper.

The outflow from the left ventricle at each venous pressure was estimated in the usual way, and was about equal to that obtained from Dogs of the same size in the experiments which have been given.

The results obtained are given in the following table.

June 2, 1882.—Weight of Dog, 6085 grms.

Observations.	Venous pressure in right auricle.			Pressure in left auricle in millims. of mercury.	
	Centims. of blood.		Millims. of mercury.	Mean.	Max.
1	10	=	7·8	8	..
2	20	=	15·6	10	..
3	30	=	23·3	12	..
4	40	=	31·1	13·5	15·5
5	50	=	38·9	15	19
6	60	=	46·7	16	20
7	50	=	38·9	15	20
8	40	=	31·1	14	18
9	30	=	23·3	13	16
10	20	=	15·6	10	12·5
11	10	=	7·8	8	10

It is seen that the mean intra-auricular pressure of the left heart at the highest venous pressure used was only 16 millims. of mercury, which, as we have before said, is probably inside the limits which auricular pressure may reach during life, that is, if the left auricle is exposed to as great a pressure as the right, and there is no reason for supposing that it is not. We did not think it necessary to apply the corrections to the tables given of the effect of venous pressure upon the work done by the left ventricle, since except for the purpose of constructing a curve of work the absolute value of the work done by the contracting ventricle is of no especial importance.

IV.

Influence of rate of beat on the work done by the heart.

In endeavouring to arrive at a conclusion as to the average quantity of blood thrown out from the ventricle of Man's heart at each systole upon the basis of the results obtained from the Dog, we were led, as has been said, to consider the influence of pulse-rate upon this quantity.

Looking over the literature of the subject as far as it has been accessible to us, we find that no definite knowledge has been gained upon this point. VOLKMANN, in discussing the effect of changes of pulse-rate upon the mean velocity, says that he can make no definite statement of the relations between the two. By bleeding his animals he got a diminished velocity together with an increased pulse-rate, but it is evident that from such a method of operating no causal relation can be assumed to exist between these two factors.

By cutting the vagi and thus producing a much more rapid pulse, he found that in some

cases this increased pulse-rate was accompanied by a greater velocity, in other cases by a smaller velocity of the blood stream.

VIERORDT found with reference to the relation between pulse-rate and what he calls the "greatness of the systole," *i.e.*, the quantity of blood thrown out at each systole, that sometimes, perhaps in most cases, a diminished pulse was accompanied by an increase in the greatness of the systole, while on the other hand cases were observed in which the reverse happened. The relative greatness of the systole was determined from the number of beats which occurred in the time necessary for the completion of a circulation.

In our experiments we increased or diminished the pulse-rate by raising or lowering the temperature of the blood flowing into the heart, using for this purpose the same method as that described by Professor MARTIN (Phil. Trans., 1883, p. 663).

We give below the records of two experiments.

In the experiment of May 18 a low venous pressure, 20 centims., was used, and besides the thermometer in the inflow tube, another was placed in the left subclavian artery, the bulb projecting into the aorta. The temperatures were read from both of these thermometers at the end of each observation; in the table the temperature on the arterial side is marked A, that on the venous side V. After observation 18 of this experiment, one of the supply flasks unfortunately ran completely empty, allowing some air to get into the heart, so that the succeeding observations could not be trusted. In the experiment of May 30 a venous pressure was used of such a height that, judging from our other experiments, the maximum outflow at each systole for the given pulse-rate was obtained.

In this case a thermometer was not placed in the subclavian artery, since it would have interfered to some extent with the flow from the left ventricle, and owing to the rapidity with which the flasks were emptied, we were not always able to get the temperature of the inflowing blood, nor the times of the observations; these latter however were made at intervals of from one to three minutes.

TABLE showing the influence of the rate of beat on the work done by the heart.

May 18, 1882.—Weight of Dog, 7005 grms. Weight of heart, 59·5 grms. Thermometer in the inflow tube and in the left subclavian artery.

Observations.	Time.	Temp. C.	Beats in 30 seconds.	Arterial pressure. Millims. of mercury.	Venous pressure. Centims. of blood.	Outflow in 30 seconds. Cub. centims.	Outflow in 1 beat. Cub. centims.
	h. m.	°					
1	1 30	A. 38 + V. 38 +	114	100	20	325	2·85
2	1 32	A. 38·25 V. 38·25	113·5	100	20	315	2·78
3	1 36	A. 36 + V. 36 −	101	100	20	283	2·80
4	1 39	A. 34 + V. 34 −	90·5	100	20	275	3·04
5	1 42	A. 33 V. 32·25	79	98	20	260	3·29
6	1 44	A. 31·75 V. 31 −	70	98	20	250	3·57
7	1 46	A. 31 + V. 30	66·5	97	20	240	3·61
8	1 47	A. 30·5 V. 29·75	62·25	97	20	220	3·53
9	1 48	A. 30·25 V. 29·75	60·5	97	20	208	3·44
10	1 51	A. 29·5 V. 27·75	55·75	96	20	208	3·73
11	1 52	A. 28·5 V. 27·5	49·5	96	20	200	4·04
12	1 54	A. 28·5 V. 27·5	49·75	96	20	193	3·88
13	1 57	A. 27·75 V. 27	43·5	94 (?)	20	185	4·25
14	1 59	A. 26·5 V. 25 −	38·5	90 (?)	20	180	4·68
15	2 02	A. 28 V. 28·5	44	95	20	184	4·18
16	2 04	A. 28·75 V. 29	47	96	20	192	4·09
17	2 07	A. 31·75 V. 33	64·5	96	20	230	3·56
18	2 08	A. 32·5 V. 33·5	70·75	97	20	255	3·60

May 30, 1882.—Weight of Dog, 9555 grms. Weight of heart, 81·5 grms.
Thermometer in the inflow tube.

Observations.	Temp. C.	Beats in 30 seconds.	Arterial pressure. Millims. of mercury.	Venous pressure. Centims. of blood.	Outflow in 30 seconds. Cub. centims.	Outflow in 1 beat. Cub. centims.
1	37°	98	121	59	905	9·23
2	37–	102	124	59	950	9·31
3	34·25	88	123	59	875	9·94
4	34–	80·5	122	59	830	10·31
5	30+	65	123·5	59	780	12·00
6	..	49·5	121	59	670	13·53
7	..	45·75	122	59	670	14·64
8	26·25	35·75	116·5	59	583	16·30
9	..	32·75	116·5	59	525	16·03
10	..	36·5	116·5	59	590	16·16
11	..	46	118	59	643	13·98
12	..	56·25	119	59	663	11·79
13	35–	68	120·5	59	710	10·44
14	37	78·5	120	59	730	9·30

From a consideration of these tables there can be no doubt of the general fact that *a diminution of pulse-rate, brought about by lowering the temperature of the blood flowing into the heart, causes an increase in the quantity of blood thrown out from the ventricle at each systole, and consequently an increase in the work done at each systole; and vice versâ.*

The changes in the outflow from the ventricle at each systole are not, however, inversely proportional to the changes in the pulse-rate, so that the total outflow, and, therefore, the total work during any given period of time, decreases with a diminished pulse-rate, and increases with an increased pulse-rate.

Whether any definite relation, beyond the general one given above, can be established between the pulse-rate and the outflow at each systole, we are not as yet prepared to say; a consideration of this and of some other interesting points which suggest themselves in this connexion must be left for a future paper.

In conclusion, we desire to express our most earnest thanks to Professor MARTIN for the aid and encouragement which he has given us during the progress of this work. We are indebted to him not only for many valuable suggestions in the earlier part of the investigation, when success seemed doubtful, but also for personal assistance which he has sometimes kindly given. The Plate representing the apparatus employed is that which was published with his paper, and it is here reproduced for the convenience of the reader.

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VII. *On the Steady Motion and Small Vibrations of a Hollow Vortex.*

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Received May 31,—Read June 21, 1883.

CONTENTS.

THE following pages form a continuation of some researches commenced about three years ago, but which the author was compelled by other engagements to lay aside until the beginning of the present year. The general theory of the functions employed was published in the Transactions of this Society (Part III., 1881), under the title of "Toroidal Functions." These and analogous functions are employed in the present communication, and references in square brackets, with the letters T.F., refer to this paper. Since it was written I have found that CARL NEUMANN had already given the general transformation [T.F. §1] by means of conjugate functions, in a

pamphlet published at Halle in 1864, with the title 'Theorie der Elektricitäts- und Wärme-Vertheilung in einem Ringe.'

The theory of the motion of vortices is interesting, not only from the mathematical difficulties encountered in its treatment, but also from its connexion with Sir W. THOMSON'S theory of the vortex atom constitution of matter. In an abstract of the present paper intended for the Proceedings of this Society, I have given some physical speculations which induced me to take up the question of the motion of a hollow vortex—that is, where cyclic motion exists in a fluid without the presence of any actual rotational filaments—in which case there must be a ring-shaped hollow in the fluid, however great the pressure may be, so long as it is finite. The essential quality of all vortex motion is the *cyclic* motion existing in the fluid outside the filament, and not the rotational motion of the filament itself. Whether the filament be present or not, it is often possible to get some general idea of the motion that ensues in many cases without recourse to actual calculation. Thus, for instance, the treatment by Sir W. THOMSON of the action of two vortices on one another,* and of the form of the axis of a ring, along which waves of displacement are running,† may be cited. The same course of general reasoning, which was applied in a paper on the steady motion of two cylinders in a fluid,‡ will also apply to illustrate the *mechanism*, so to speak, which causes a single vortex ring to move with a motion of translation. Thus suppose a single vortex ring, which is for a moment at rest. It is clear that the velocity of the fluid just inside the aperture is greater than outside, and therefore the pressure less inside than outside, whilst the pressure is the same at corresponding points in the front and hinder portions. The consequence of this is that the ring begins to contract without a general motion of translation. But the effect of this contraction of aperture itself produces velocities in the surrounding fluid, which, combined with the cyclic motion, increase the velocities in front of the ring, and decrease them behind. The consequence of this is a difference of pressures, which urges the ring in the direction of the cyclic motion through the ring, and it begins to move forward with increasing velocity. After a time this translatory motion would increase so much as to make the velocity within the aperture approach to that without; the state of motion will therefore be one in which the translatory velocity tends continually to a limit.

The present communication is divided into three sections. In the first, new functions are introduced to give the stream lines. These functions are connected with, and have analogous properties to, the Toroidal Functions; are, in fact, given by $R = SdP/du$ and $T = -SdQ/du$. They have the property of being single-valued, even when they represent cyclic motion—a motion which the single-valued Toroidal Functions cannot by themselves represent. At the end of the section the values of

* "Vortex Motion," Trans. Roy. Soc. Edin., xxv.

† "Vortex Statics," Proc. Roy. Soc. Edin., ix.

‡ Quart. Jour. Math., xvii.

the first few terms in the expansions of the first four orders of P, Q, R, T, are given. Section II. is devoted to the consideration of the motion of a rigid ring in fluid, when it moves parallel to its straight axis. The functions for the motion apply directly to the case considered afterwards of the vortex. The points of division of the stream, the quantity of fluid carried forward, and the energy of the motion are considered.

In Section III. the problem of the steady motion of a hollow vortex is treated, together with the small vibrations when the hollow is fluted, and when it pulsates. The section of a ring is throughout considered as small compared with the aperture, and the expressions giving the form of the hollow, the surface velocity, velocity of translation and energy, are carried to a second approximation, the quantity by which the approximation proceeds being the ratio $r/\{R+\sqrt{(R^2-r^2)}\}=k$ where r, $R-r$ denote the radii of the mean section and aperture respectively; when the ring is very small, this is very approximately $r/2R$. The condition that the hollow must be a free surface over which the pressure is constant gives a relation which R, r must always satisfy, which for very small rings reduces to the constancy of the radius of the hollow. For a solid ring the corresponding condition is, of course, the constancy of volume. This makes an essential difference between the two theories. To a second approximation the velocity of translation is unaltered, and is given by*

$$V=\frac{\mu}{4\pi a}\left(\log\frac{4}{k}-\frac{1}{2}\right)$$

whilst to the second approximation the surface velocity, relative to the hollow itself, is

$$U=\frac{\mu}{4\pi ak}\{1-\tfrac{1}{2}(\log\frac{4}{k}+5)k^2\}$$

where a is the radius of the "critical" circle—or the length of a tangent from the centre to the ring, and is therefore equal to R for small rings—and μ is the cyclic constant.

In the steady motion considered, the fluid carried forward with the ring forms a single mass, without aperture even for extremely small tores, though not for infinitely small ones. For values of $R/r>10^3$ there will be no aperture, whilst for less values the fluid carried forward will be ring-shaped. To a first approximation the energy due to the cyclic motion is the most important, and is the same as for a rigid ring at rest of the same size. It does not depend on the velocity of translation, except in so far as this determines the size of the aperture; as entering in this way the principal term varies inversely as the velocity of translation, and thus increases with diminished

* [April, 1884.—Owing to an error in § 3, the values given in the Proceedings require correction.]

translatory motion, a result obtained by Sir W. THOMSON* from general reasoning. The terms obtained by the second approximation arise from the translatory motion.

In Art. 13 the time of vibration of the steady form is obtained, when the cross section is crimped, or the whole hollow surface fluted. For this mode the time of vibration is, for small rings, given very approximately by $\mu d/(2p\sqrt{n})$, d being the density, and p the pressure of the fluid at a great distance, whilst n is the number of crimpings in a section. This, it is to be noted, is independent of the energy, and depends only on constants of the ring, and the fluid, and the mode of vibration. If the hollow pulsates, or changes its volume periodically, the time of pulsation is $(\mu d/2p)\sqrt{(\log 4/k)}$. As k depends on the size of the ring, and therefore on the energy, this time is not independent of the latter, but it varies extremely slowly with it. The times here given must be understood to apply to the steady motion; when the ring is changing its size they must be modified. The investigation of this case, and of that in which there is a core of denser matter than the surrounding fluid, I hope shortly to take up.

Section I.—*The functions.*

1. The functions whose properties were investigated in my paper on Toroidal Functions are only suitable for expressing fluid motions about circular tores when there is no cyclic motion through the aperture. It will be necessary therefore to investigate some method by which this can be taken into consideration. If we consider only motions symmetrical about an axis, and in planes through that axis, it is well known that the motion can be represented by STOKES' stream function. This function is only multiple valued when there are sources or sinks in the fluid, the cyclic constants in this case being the normal flows outwards through surfaces completely enclosing the various sources or sinks. If ψ denote the stream function, the velocities at any point are given by $-\frac{1}{\rho}\frac{\delta\psi}{\delta z}, \frac{1}{\rho}\frac{\delta\psi}{\delta\rho}$, and, when the motion is irrotational, ψ satisfies the equation

$$\frac{\delta^2\psi}{\delta\rho^2}+\frac{\delta^2\psi}{\delta z^2}-\frac{1}{\rho}\frac{\delta\psi}{\delta\rho}=0$$

To transform this to the independent variables $(u.v)$, where $u+vi=f(\rho+zi)$, we notice that the kinetic energy of fluid motion within any space, with given normal motions over this surface, is a minimum when the motion is irrotational, or the above differential equation is satisfied. The condition is therefore found by making

$$\int\frac{1}{\rho^2}\left\{\left(\frac{\delta\psi}{\delta u}\right)^2+\left(\frac{\delta\psi}{\delta v}\right)^2\right\}\left(\frac{du}{dn}\right)^2 2\pi\rho dz d\rho$$

a minimum. Now

* "Vortex Atoms," Proc. Roy. Soc. Edin., vi., and Phil. Mag. (4), 34.

$$dzd\rho=\frac{d(z.\rho)}{d(u.v)}dudv=\frac{1}{\left(\frac{du}{dn}\right)^2}dudv$$

Therefore the expression to be made a minimum is

$$\int\frac{1}{\rho}\left\{\left(\frac{\delta\psi}{\delta u}\right)^2+\left(\frac{\delta\psi}{\delta v}\right)^2\right\}dudv$$

whence

$$\frac{\delta}{\delta u}\left(\frac{1}{\rho}\frac{\delta\psi}{\delta u}\right)+\frac{\delta}{\delta v}\left(\frac{1}{\rho}\frac{\delta\psi}{\delta v}\right)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this put

$$\psi=\chi\sqrt{\rho}$$

and the equation in χ becomes, remembering that

$$\frac{\delta^2\rho}{\delta u^2}+\frac{\delta^2\rho}{\delta v^2}=0$$

$$\frac{\delta^2\chi}{\delta u^2}+\frac{\delta^2\chi}{\delta v^2}-\frac{3}{4}\frac{\chi}{\rho^2}\left\{\left(\frac{\delta\rho}{\delta u}\right)^2+\left(\frac{\delta\rho}{\delta v}\right)^2\right\}=0$$

The particular transformation employed for the Toroidal Functions makes

$$\frac{1}{\rho^2}\left\{\left(\frac{\delta\rho}{\delta u}\right)^2+\left(\frac{\delta\rho}{\delta v}\right)^2\right\}=\sinh^{-2}u=\mathrm{S}^{-2}$$

whence

$$\frac{\delta^2\chi}{\delta u^2}+\frac{\delta^2\chi}{\delta v^2}-\frac{3}{4\mathrm{S}^2}\chi=0$$

Put $\chi=\mathrm{S}^{-\frac{1}{2}}\mathrm{R}_n\cos(nv+\alpha)$, where R_n is a function of u only; then R_n must satisfy

$$\frac{d^2\mathrm{R}}{du^2}-\frac{\mathrm{C}}{\mathrm{S}}\frac{d\mathrm{R}}{du}-(n^2-\tfrac{1}{4})\mathrm{R}=0$$

which may be compared with the equation for Toroidal Functions, viz.,

$$\frac{d^2\mathrm{P}}{du^2}+\frac{\mathrm{C}}{\mathrm{S}}\frac{d\mathrm{P}}{du}-(n^2-\tfrac{1}{4})\mathrm{P}=0$$

It is easy to see that the equation in R is satisfied by

$$\mathrm{R}=\mathrm{AS}\frac{d\mathrm{P}}{du}$$

We will choose the constants so that the two integrals are

$$
\left.\begin{aligned}
R_n &= S\frac{dP}{du}=\frac{4n^2-1}{8n}(P_{n+1}-P_{n-1})\\
T_n &= -S\frac{dQ}{du}=\frac{4n^2-1}{8n}(Q_{n-1}-Q_{n+1})
\end{aligned}\right\} \qquad (2)
$$

also

$$\frac{dR_n}{du}=(n^2-\tfrac{1}{4})SP_n,\quad \frac{dT_n}{du}=-(n^2-\tfrac{1}{4})SQ_n$$

The value of ψ is now, putting in the value of ρ, viz., $\rho=aS/(C-c)$

$$\psi=\frac{1}{\sqrt{(C-c)}}\Sigma_0^\infty(A_nR_n+B_nT_n)\cos(nv+\alpha)$$

and clearly R, T belong to the same spaces as P, Q respectively, that is, R to space outside, and T to space inside a tore.

It is easy now to prove from the value of Q_n, viz.,

$$Q_n\sqrt{2}=\int_0^\pi\frac{\cos nv}{\sqrt{(C-c)}}dv$$

that

$$T_n=-\frac{4n^2-1}{2\sqrt{2}}\int_0^\pi\cos nv\sqrt{(C-c)}dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

The R, T are all positive, except R_0.

2. *Cyclic constant.*—The cyclic constant of ψ is the flow along any closed curve threading the tore once. We know that this must be independent of the form of the curve. To find it, choose the curve to be $u=u'$ a constant; the flow along this is then

$$\int_0^{2\pi}\frac{1}{\rho}\frac{\delta\psi}{\delta u}\frac{du}{dn}\cdot\frac{dn}{dv}dv=\int_0^{2\pi}\frac{1}{\rho}\frac{\delta\psi}{\delta u}dv$$

the velocity in the aperture being in the positive direction. Consider first the general term in R_n; the flow due to this is

$$
\begin{aligned}
&\frac{A_n}{aS}\int_0^{2\pi}\left\{\sqrt{(C-c)}\frac{dR_n}{du}-\frac{S}{2\sqrt{(C-c)}}R_n\right\}\cos nv\,dv\\
&=\frac{2A_n}{aS}\int_0^\pi\left\{(n^2-\tfrac{1}{4})SP_n\sqrt{(C-\cos v)}\cos nv-\tfrac{1}{2}SR_n\frac{\cos nv}{\sqrt{(C-c)}}\right\}dv\\
&=\frac{2A_n}{a}\left(-\frac{\sqrt{2}}{2}T_nP_n-\frac{\sqrt{2}}{2}R_nQ_n\right)\\
&=\frac{A_n\sqrt{2}}{a}S\left(P_n\frac{dQ_n}{du}-Q_n\frac{dP_n}{du}\right)=-\frac{\pi}{a}\sqrt{2}A_n \quad [\text{T.F. } 24.\beta],
\end{aligned}
$$

which is independent of u as it ought to be. The corresponding terms in $\sin nv$ evidently disappear. Similarly the terms in T_n would produce

$$-\frac{B_n\sqrt{2}}{a}S\left(Q_n\frac{dQ_n}{du}-Q_n\frac{dQ_n}{du}\right)=0$$

Hence the cyclic constant is

$$\mu=-\frac{\pi\sqrt{2}}{a}\Sigma_0^{\infty}A_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

3. In the paper on Toroidal Functions several examples were given of the determination of the potential function ϕ when ϕ is given over a tore; but when the variation of ϕ along the normal to the surface is given, the determination of the co-efficients becomes more difficult, and one case only, for the motion of a tore perpendicular to its plane, was given. It will be well, therefore, to consider here the general theory for this class of surface conditions. The co-efficients are to be determined from the fact that ϕ is (1) finite in the space considered, and at infinity, and (2) $d\phi/dn$ has a given value over the surface of a tore u'. Here I consider only the case where the motion is symmetrical about the axis, and therefore the normal velocity given by a function of v only, say $f(v)$. Condition (1) is satisfied by space outside the tore by taking only functions P_n. We put then

$$\phi=\sqrt{(C-c)}\Sigma_0(A_n\cos nv+B_n\sin nv)P_n$$

and determine A_n, B_n from the equation

$$f(v)=-\frac{\delta\phi}{\delta u}\cdot\frac{du}{dn}$$

when $u=u'$, for all values of v.

Consider separately the terms in $\cos nv$ and $\sin nv$. For the cosines we have

$$\frac{\delta\phi}{\delta u}=\frac{1}{2\sqrt{(C-c)}}\Sigma_0A_n\left\{SP_n+2(C-c)\frac{dP_n}{du}\right\}\cos nv$$

For shortness write $\frac{dP}{du}=P'$. Then

$$\frac{\delta\phi}{\delta u}=\frac{1}{2\sqrt{(C-c)}}[(SP_0+2CP'_0)A_0-A_1P'_1-A_0P'_0\cos v^*+\Sigma_1\{(SP_n+2CP'_n)A_n$$
$$-A_{n+1}P'_{n+1}-A_{n-1}P'_{n-1}\}\cos nv]$$

But

$$SP_n+2CP'_n=P'_{n+1}+P'_{n-1} \quad \text{[T.F., p. 646]}$$
$$SP_0+2CP'_0=2P'_1$$

Therefore

$$\frac{\delta\phi}{\delta u}=\frac{1}{2\sqrt{(C-c)}}[(2A_0-A_1)P'_1-A_0P'_0\cos v+\Sigma_1\{(A_n-A_{n+1})P'_{n+1}-(A_{n-1}-A_n)P'_{n-1}\}\cos nv]$$

* [April, 1884.—This term was omitted in the paper as read. It has necessitated slight alterations in some of the results then given.]

and

$$\frac{du}{dn}=\frac{C-c}{a}$$

Hence the A have to be determined from

$$\frac{2a}{(C-c)^{\frac{1}{2}}}f(v)=-(2A_0-A_1)P'_1+A_0P'_0\cos v-\Sigma_1\{(A_n-A_{n+1})P'_{n+1}-(A_{n-1}-A_n)P'_{n-1}\}\cos nv$$

Suppose now

$$f(v)=\left(\frac{C-c}{S}\right)^{\frac{1}{2}}\alpha_m\cos mv$$

Then, writing for the present $(A_n-A_{n-1})P'_{n-1}P'_n=x_n$

$$\left.\begin{array}{r}x_{n+1}-x_n=0\\ \ldots\ldots=0\end{array}\right\}\quad n>m$$

$$x_{m+1}-x_m=\frac{2a\alpha_m}{\sqrt{S}}P'_m$$

$$\begin{array}{rl}x_{n+1}-x_n=0 & \quad n<m\\ \ldots\ldots=0 & \\ x_2-x_1+A_0P'_0P'_1=0 & \\ x_1-x_0=0 & \quad\text{where } x_0=A_0P'_0P'_1\end{array}$$

Hence for

$$\begin{array}{ll}n>m+1 & x_n=x_{m+1}\\ n<m & x_m=x_n=0\end{array}$$

and

$$x_{m+1}=\frac{2a\alpha_m}{\sqrt{S}}P'_m$$

or

$$x_n=\frac{2a\alpha_m}{\sqrt{S}}P'_m\ .\ (n>m)$$

$$x_n=0\qquad (n\leqq m)$$

Therefore

$$A_n-A_{n-1}=\frac{2a\alpha_m}{\sqrt{S}}\cdot\frac{P'_m}{P'_nP'_{n-1}}\quad (n>m)$$

$$A_n-A_{n-1}=0\qquad (n\leqq m>1)$$

whence

$$A_n=A_m+\frac{2a\alpha_m}{\sqrt{S}}P'_m\sum_{r=m+1}^{r=n}\frac{1}{P'_rP'_{r-1}}\quad (n>m)$$

$$A_n=A_1=2A_0\qquad (n\leqq m)$$

or

$$A_n = 2A_0 + \frac{2a\alpha_m}{\sqrt{S}} P'_m \sum_{r=m+1}^{r=n} \frac{1}{P'_r P'_{r-1}} \qquad (n > m)$$

$$A_n = 2A_0 \qquad (n \overline{\gtrless} m)$$

The co-efficients are now determined to the extent of one arbitrary constant. This appears because ϕ is also indeterminate to the extent of an additive constant. As this constant is expansible in a series of the form $\sqrt{(C-c)}\Sigma A_n \cos nv$, it introduces the undetermined constant A_0 above, which must be determined by the condition that the series must be convergent. This cannot be unless $A_\infty = 0$, which requires

$$A_0 = -\frac{a\alpha_m P'_m}{\sqrt{S}} \Sigma_{m+1}^{\infty} \frac{1}{P'_r P'_{r-1}}$$

whence

$$\left.\begin{aligned} A_n &= -\frac{2a\alpha_m P'_m}{\sqrt{S}} \Sigma_{n+1}^{\infty} \frac{1}{P'_r P'_{r-1}} \qquad (n > m) \\ A_n &= -\frac{2a\alpha_m P'_m}{\sqrt{S}} \Sigma_{m+1}^{\infty} \frac{1}{P'_r P'_{r-1}} \qquad (n \overline{<} m) \end{aligned}\right\} \qquad (5)$$

So also the terms in sin nv will give

$$\frac{2a}{(C-c)^{\frac{1}{2}}} f(v) = -\Sigma_1 \{ (B_n - B_{n+1}) P'_{n+1} - (B_{n-1} - B_n) P'_{n-1} \} \sin nv$$

and the particular case $f(v) = \left(\frac{C-c}{S}\right)^{\frac{1}{2}} \beta_m \sin mv$ produces the same equations as before, except that the last is

$$x_2 - x_1 = 0 \text{ where } x_1 = B_1 P'_0 P'_1$$

whence

$$B_n = B_1 P'_0 P'_1 \Sigma_1^n \frac{1}{P'_r P'_{r-1}} + \frac{2a\beta_m}{\sqrt{S}} P'_m \Sigma_{m+1}^n \frac{1}{P'_r P'_{r-1}} \qquad (n > m)$$

$$B_n = B_1 P'_0 P'_1 \Sigma_1^n \frac{1}{P'_r P'_{r-1}} \qquad (n \overline{<} m)$$

and the condition of convergency determines B_1, so that

$$\left.\begin{aligned}
B_n &= -\frac{2a\beta_n P'_m}{\sqrt{S}} \frac{\Sigma_1^m \frac{1}{P'_r P'_{r-1}}}{\Sigma_1^\infty \frac{1}{P'_r P'_{r-1}}} . \Sigma_{n+1}^\infty \frac{1}{P'_r P'_{r-1}} \qquad (n>m) \\
B_n &= -\frac{2a\beta_n P'_m}{\sqrt{S}} . \frac{\Sigma_{m+1}^\infty \frac{1}{P'_r P'_{r-1}}}{\Sigma_1^\infty \frac{1}{P'_r P'_{r-1}}} \Sigma_1^n \frac{1}{P'_r P'_{r-1}} \qquad (n \overline{\gtrless} m)
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

It remains to show that with these values of A_n, B_n the series $\Sigma A_n P_n$ and $\Sigma B_n P_n$ are convergent. The parts of A_n, B_n depending on n, when n is large,

$$\propto \Sigma_{n+1}^\infty \frac{1}{P'_r P'_{r-1}} = \lambda \Sigma_{n+1}^\infty \frac{1}{P'_r P'_{r-1}}.$$

Now

$$\frac{P'_{n-1}}{P'_n} = \frac{P_n - CP_{n-1}}{CP_n - P_{n-1}} = \frac{1}{C} . \frac{P_n - CP_{n-1}}{P_n - P_{n-1}/C} \qquad [\text{T.F., 12, 13}]$$

$$< \frac{1}{C} \text{ since } P_n > CP_{n-1}$$

Hence

$$\Sigma_{n+1}^\infty \frac{1}{P'_r P'_{r-1}} < \frac{1}{P'_{n+1} P'_{n-1}} \left\{ 1 + \frac{1}{C} + \frac{1}{C^2} + \dots \right\}$$

$$< \frac{C}{C-1} \frac{1}{P'_{n+1} P'_n}.$$

Therefore

$$\left.\begin{matrix} A_n \\ B_n \end{matrix}\right| P_n \text{ is ultimately} < \frac{\lambda C}{C-1\, P'_{n+1} P'_n}$$

or the series are convergent.

We are now in a position to determine ϕ for any normal motion. All we have to do is to expand $\{S/(C-c)\}^{\frac{1}{2}} f(v)$ in a series of sines and cosines of multiples of v and consider each term as giving rise to a value of ϕ, whose form we have just determined, and take the sum of the various values.

A form for $\delta\psi/\delta u$ analogous to that for $\delta\phi/\delta u$ can easily be found. If

$$\psi = \frac{1}{\sqrt{(C-c)}} \Sigma A_n R_n \cos nv$$

$$\frac{\delta\psi}{\delta u} = \frac{1}{(C-c)^{\frac{3}{2}}} \Sigma \left\{ (C-c) \frac{dR_n}{du} - \tfrac{1}{2} S R_n \right\} A_n \cos nv$$

$$= \tfrac{1}{2} \frac{1}{(C-c)^{\frac{3}{2}}} \Sigma \left\{ \left(2C \frac{dR_n}{du} - SR_n\right) \cos nv - (\cos \overline{n+1}v + \cos \overline{n-1}v) \frac{dR_n}{du} \right\} A_n$$

$$= \frac{S}{2(C-c)^{\frac{3}{2}}} \left[-\tfrac{1}{2} A_0 P_1 - \tfrac{3}{4} A_1 P_1 + \frac{1}{S} \Sigma_1 \left\{ \left(2C \frac{dR_n}{du} - SR_n\right) A_n - A_{n+1} \frac{dR_{n+1}}{du} - A_{n-1} \frac{dR_{n-1}}{du} \right\} \cos nv \right]$$

But

$$2C\frac{dR_n}{du}-SR_n=(n^2-\tfrac{1}{4})\left\{2CSP_n-\frac{S}{2n}(P_{n+1}-P_{n-1})\right\}=(n^2-\tfrac{1}{4})S(P_{n+1}+P_{n-1})$$

Hence

$$\frac{\delta\psi}{\delta u}=\frac{S}{2(C-c)^{\frac{1}{2}}}\left\{-\tfrac{1}{4}A_0P_1-\tfrac{3}{4}A_1P_1+\tfrac{1}{4}A_0P_0\cos v+\Sigma_1 B_n\cos nv\right\} \quad . \quad . \quad . \quad (7)$$

where

$$B_n=\{(n^2-\tfrac{1}{4})A_n-(\overline{n+1}|^2-\tfrac{1}{4})A_{n+1}\}P_{n+1}-\{(\overline{n-1}|^2-\tfrac{1}{4})A_{n-1}-(n^2-\tfrac{1}{4})A_n\}P_{n-1}$$

4. For reference I here insert the values of the first five orders of the functions, expressed (α) exact in terms of the elliptic integrals, and (β) approximate in a series of ascending powers of the modulus. Throughout this paper the moduli k, k' are used instead of the k', k of the paper on Toroidal Functions. It has been thought advisable to do this as all the approximations go according to powers of k (the old k'). Hence, of course, E, F appear in place of E′, F′, and *vice versâ*.

We know that $P_n=\alpha_n E'+\beta_n F'$, $Q_n=\alpha_n(F-E)+\beta_n F$.

Hence for the first set of formulæ we require only to tabulate α_n, β_n. For the first five they are

$$\left.\begin{array}{ll}
\alpha_0=0 & \beta_0=2k^{\frac{1}{2}} \\
\alpha_1=2k^{-\frac{1}{2}} & \beta_1=0 \\
\alpha_2=\frac{4}{3}(1+k^2)k^{-\frac{1}{2}} & \beta_2=-\frac{2}{3}k^{\frac{1}{2}} \\
\alpha_3=\frac{2}{15}(8+7k^2+8k^4)k^{-\frac{1}{2}} & \beta_3=-\frac{8}{15}(k^2+k^4)k^{-\frac{1}{2}} \\
\alpha_4=\frac{16}{3.5.7}(6+5k^2+5k^4+6k^6)k^{-\frac{1}{2}}, & \beta_4=-\frac{2}{3.5.7}(24k^2+23k^4+24k^6)k^{-\frac{1}{2}} \\
\alpha_5=\frac{2}{5.7.9}(128+104k^2+99k^4+104k^6+128k^8)k^{-\frac{1}{2}}, & \beta_5=-\frac{8}{5.7.9}(16k^2+15k^4+15k^6 \\
 & \qquad +16k^8)k^{-\frac{1}{2}}
\end{array}\right\} \quad (8)$$

These are exact. In the applications which follow k will nearly always be a very small quantity, so that a few terms of the series will give the values very approximately. By substituting their values for E, F, E′, F′ in terms of k, the expressions become, writing L for $\log\frac{4}{k}$

$$\begin{aligned}
P_0&=2\{L+\tfrac{1}{4}(L-1)k^2+\tfrac{9}{64}(L-\tfrac{7}{6})k^4+\tfrac{25}{256}(L-\tfrac{37}{30})k^6+\ldots\}k^{\frac{1}{2}} \\
P_1&=2\{1+\tfrac{1}{2}(L-\tfrac{1}{2})k^2+\tfrac{3}{16}(L-\tfrac{13}{12})k^4+\tfrac{15}{128}(L-\tfrac{6}{5})k^6+\ldots\}k^{-\frac{1}{2}} \\
P_2&=\tfrac{4}{3}\{1+\tfrac{3}{4}k^2+\tfrac{9}{16}(L-\tfrac{7}{12})k^4+\tfrac{15}{64}(L-\tfrac{67}{20})k^6+\ldots\}k^{-\frac{1}{2}} \\
P_3&=\tfrac{16}{15}\{1+\tfrac{5}{8}k^2+\tfrac{45}{64}k^4+\tfrac{75}{128}(L-\tfrac{37}{60})k^6+\ldots\}k^{-\frac{1}{2}} \\
P_4&=\tfrac{32}{35}\{1+\tfrac{7}{12}k^2+\tfrac{85}{64}k^4+\tfrac{175}{256}k^6+\ldots\}k^{-\frac{1}{2}} \\
P_5&=\frac{256}{5.7.9}\{1+\tfrac{9}{16}k^2+\tfrac{63}{128}k^4+\tfrac{525}{1024}k^6+\ldots\}k^{-\frac{1}{2}}
\end{aligned} \quad (9)$$

$$\left.\begin{aligned}
Q_0&=\pi\{1+\tfrac{1}{4}k^2+\tfrac{9}{64}k^4+\tfrac{25}{256}k^6+\ldots\}k^{\frac{1}{2}}\\
Q_1&=\tfrac{1}{2}\pi\{1+\tfrac{3}{8}k^2+\tfrac{15}{64}k^4+\ldots\}k^{\frac{1}{2}}\\
Q_2&=\frac{3\pi}{8}(1+\tfrac{5}{12}k^2+\ldots)k^{\frac{1}{2}}\\
Q_3&=\frac{5\pi}{16}k^{\frac{1}{2}}+\ldots\\
Q_4&=0+\ldots\\
Q_5&=0+\ldots
\end{aligned}\right\}\quad\ldots\ldots\quad(10)$$

$$\left.\begin{aligned}
R_0&=-\{\tfrac{1}{2}L-1+\tfrac{1}{8}(L+1)k^2+\tfrac{1}{128}(L-\tfrac{1}{2})k^4+\tfrac{1}{512}(L-\tfrac{5}{6})k^6+\ldots\}k^{-\frac{1}{2}}\\
R_1&=\tfrac{1}{2}\{1-\tfrac{3}{2}(L-\tfrac{1}{2})k^2+\tfrac{3}{16}(L+\tfrac{1}{4})k^4+\tfrac{3}{128}(L-23)k^6+\ldots\}k^{-\frac{1}{2}}\\
R_2&=\{1-\tfrac{5}{4}k^2-\tfrac{15}{16}(L-\tfrac{5}{4})k^4+\tfrac{15}{64}(L+\tfrac{1}{12})k^6+\ldots\}k^{-\frac{1}{2}}\\
R_3&=\tfrac{4}{3}\{1-\tfrac{7}{8}k^2-\tfrac{35}{64}k^4-\tfrac{35}{32}(3L-\tfrac{17}{4})k^6+\ldots\}k^{-\frac{1}{2}}\\
R_4&=\tfrac{8}{5}\{1-\tfrac{3}{4}k^2-\tfrac{21}{64}k^4-\tfrac{105}{256}k^6+\ldots\}k^{-\frac{1}{2}}
\end{aligned}\right]\quad.\quad(11)$$

$$\left.\begin{aligned}
T_0&=\tfrac{1}{4}\pi(1+\tfrac{1}{4}k^2+\tfrac{1}{64}k^4+\tfrac{1}{256}k^6+\ldots)k^{-}\\
T_1&=\frac{3\pi}{8}(1-\tfrac{1}{8}k^2-\tfrac{1}{64}k^4+\ldots)k^{\frac{1}{2}}\\
T_2&=\frac{15\pi}{32}(1-\tfrac{1}{4}k^2+\ldots)k^{\frac{1}{2}}\\
T_3&=\tfrac{35}{64}\pi k^{\frac{1}{2}}+\ldots
\end{aligned}\right.\quad(12)$$

Section II.—*Motion about a rigid tore which moves perpendicularly to its plane.*

As the motion of a tore throws some light on the analogous problem of the uniform translation of a vortex ring, and as the functions required in its discussion will be needed in investigating the latter, it will be useful to give a short treatment of the question, especially as the motion can be determined for any size of tore, whereas our methods, in the case of hollow vortices, will only apply when the cross section of the hollow is not large compared with the aperture. The stream function is necessary for the cyclic motion, and it will therefore be convenient to take the stream function also for the motion of translation.

5. *Stream function for cyclic motion.*—If the tore be given by $u=u'$ the conditions which ψ must satisfy are that it must be finite for space outside the tore, and be constant for all values of v when $u=u'$. Hence ψ must be expansible in the form

$$\psi=\frac{1}{\sqrt{(C-c)}}\Sigma A_n R_n \cos nv$$

Let ψ_0 be the constant value over the surface of the tore. Then, dashed letters denoting the values of the functions on the tore,

$$\frac{\pi}{2}A_nR'_n=\psi_0\int_0^\pi\sqrt{(C'-\cos\theta)}\cos n\theta d\theta$$

$$=-\frac{2\sqrt{2}\psi_0}{4n^2-1}T'_n \qquad \text{(by Eq. 3)}$$

but

$$\pi A_0R'_0=2\psi_0\sqrt{2}T'_0$$

Hence

$$\psi=\frac{2\psi_0\sqrt{2}}{\pi\sqrt{(C-c)}}\left\{T'_0\frac{R_0}{R'_0}-2\Sigma_1^\infty\frac{T'_n}{4n^2-1}\frac{R_n}{R'_n}\cos nv\right\} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

This is more convergent than ΣT_n, and is therefore convergent.

Let μ denote the cyclic constant, then by (4)

$$\mu=\frac{4\psi_0}{a}\left\{-\frac{T'_0}{R'_0}+2\Sigma_1^\infty\frac{1}{4n^2-1}\frac{T'_n}{R'_n}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

When the section of the tore is small compared with the aperture, the value of μ, correct to the fourth power of k, is

$$\mu=\frac{2\pi\psi_0}{a}\left\{\frac{1}{L-2}+\left(1-\frac{3}{4}\frac{1}{(L-2)^2}\right)k^2+\frac{3}{4}\left(2L-1+\frac{1}{32}\frac{7L+10}{(L-2)^3}\right)k^4\right\} \quad . \quad . \quad . \quad (15)$$

6. *Stream function for translation.*—In the preceding case the conditions were that ψ must be constant over the tore and finite at an infinite distance from it. In the present case ψ must be finite at an infinite distance and $=\frac{1}{2}V\rho^2$ over the surface, V being the velocity of translation, and ψ the stream function for the tore moving in the fluid, at rest at infinity and referred to its instantaneous position. But if this condition be applied, we shall also, on account of the cyclosis, obtain besides an added cyclic motion through the aperture determined by the surface condition $\psi_0=0$. It will be necessary to subtract this cyclic motion therefore from the result obtained by applying the condition above. This condition gives

$$\Sigma A_nR'_n\cos nv=\frac{1}{2}a^2V\frac{S'^2}{(C'-c)^2}$$

for all values of v.

Therefore

$$\tfrac{1}{2}\pi A_n R'_n = \tfrac{1}{2}a^2 V S'^2 \int_0^\pi \frac{\cos n\theta}{(C'-\cos\theta)^{\frac{3}{2}}}\, d\theta$$

$$= -a^2 V S' \frac{d}{du}\int_0^\pi \frac{\cos n\theta}{\sqrt{C'-\cos\theta}}\, d\theta$$

$$= -a^2 V \sqrt{2} S' \frac{dQ'}{du} = a^2 V \sqrt{2} T'_n$$

but

$$\pi A_0 R'_0 = a^2 V \sqrt{2} T'_0$$

Hence

$$\psi = \frac{a^2\sqrt{2}V}{\pi\sqrt{(C-c)}}\left\{T'_0 \frac{R_0}{R'_0} + 2\Sigma_1^\infty T'_n \frac{R_n}{R'_n}\cos nv\right\}$$

a convergent series.

The circulation of this is by (4)

$$2aV\left(-\frac{T'_0}{R'_0} - 2\Sigma_1^\infty \frac{T'_n}{R'_n}\right)$$

Let the stream function for this be

$$\frac{2\psi_0\sqrt{2}}{\pi\sqrt{(C-c)}}\left\{T'_0\frac{R_0}{R'_0} - 2\Sigma_1^\infty \frac{T'_n}{4n^2-1}\frac{R_n}{R'_n}\cos nv\right\}$$

Then

$$\frac{4\psi_0}{a}\left\{-\frac{T'_0}{R'_0} + 2\Sigma_1^\infty \frac{1}{4n^2-1}\frac{T'_n}{R'_n}\right\} = 2aV\left(-\frac{T'_0}{R'_0} - 2\Sigma_1^\infty \frac{T'_n}{R'_n}\right)$$

If then

$$\lambda = \frac{-\dfrac{T'_0}{R'_0} - 2\Sigma_1^\infty \dfrac{T'_n}{R'_n}}{-\dfrac{T'_0}{R'_0} + 2\Sigma_1^\infty \dfrac{1}{4n^2-1}\dfrac{T'_n}{R'_n}}$$

The stream function for translation alone is

$$\psi = \frac{a^2\sqrt{2}V}{\pi\sqrt{(C-c)}}\left\{(1-\lambda)T'_0\frac{R_0}{R'_0} + 2\Sigma_1^\infty\left(1+\frac{\lambda}{4n^2-1}\right)T'_n\frac{R_n}{R'_n}\cos nv\right\} \quad . \quad . \quad . \quad (16)$$

The principal term here is the second, in R_1. To k^4 the value of λ is

$$\lambda = 1 - 4(L-2)k^2 - \left(2L^2 - \frac{21}{2}L + 16\right)k^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The value of ψ along the tore is

$$\tfrac{1}{2}V(\rho^2 - \lambda a^2)$$

The stream lines will of course in general be closed curves, having their extremities on the surface of the tore; one set going through the aperture, and the other outside.

To find the point where the two sets meet on the tore, we notice that the stream line there goes to infinity, and its value is the same as for a point on the axis, it is, in fact, a part of the same stream line. For this $\psi=0$; hence the point on the tore, where this stream line meets it, is given by the value of v, which satisfies the equation

$$(1-\lambda)\mathrm{T}_0+2\Sigma_1^\infty\left(1+\frac{\lambda}{4n^2-1}\right)\mathrm{T}_n\cos nv=0$$

where T_n are the values of T_n when $u=u'$.

It is clear that when k is very small, $\cos v$ must be negative, that is $v>\frac{1}{2}\pi$, or that the point of division must lie inside a tangent from the centre to the tore.

7. *Combined translation and cyclic motion.*—The expressions just obtained enable us to determine the amount of fluid carried forward bodily with the ring. Let x denote the ratio $a^2\mathrm{V}/2\psi_0$; then the stream function for the combined motion is

$$\psi=\frac{2\psi_0\sqrt{2}}{\pi\sqrt{(\mathrm{C}-c)}}\Sigma\mathrm{A}_n\frac{\mathrm{R}_n}{\mathrm{R}'_n}\cos nv$$

where

$$\mathrm{A}_0=\{1+(1-\lambda)x\}\mathrm{T}'_0$$

$$\mathrm{A}_n=2\left\{\left(1+\frac{\lambda}{4n^2-1}\right)x-\frac{1}{4n^2-1}\right\}\mathrm{T}'_n.$$

This is the stream function when the fluid is at rest at an infinite distance. To find the portion carried forward, impress on the whole system a velocity equal and opposite to V; the problem then is to determine the portion of fluid which remains circulating round the ring at rest, without streaming away. The stream function for the new motion is

$$\chi=\psi-\tfrac{1}{2}\mathrm{V}\rho^2$$

The portion remaining with the tore lies inside the surface given by putting χ equal to a certain constant, which we proceed to determine.

This portion may either be ring-shaped or not. The limiting case between the two is when the velocity at the centre of the tore is zero. The value of x for this case we shall call the critical value of x. It is given by

$$\mathrm{V}=\left[\frac{1}{\rho}\frac{\delta\psi}{\delta u}\frac{du}{dn}\right]_{u=0,v=\pi}$$

$$=\frac{8\psi_0\sqrt{2}}{\pi a^2}\mathrm{L}_{u=0}\frac{1}{\mathrm{S}}\left\{-\frac{1}{2}\frac{\mathrm{S}}{2^{\frac{1}{2}}}\Sigma(-)^n\mathrm{A}_n\frac{\mathrm{R}_n}{\mathrm{R}'_n}+\frac{1}{\sqrt{2}}\Sigma(-)^n\mathrm{A}_n(n^2-\tfrac{1}{4})\frac{\mathrm{SP}_n}{\mathrm{R}'_n}\right\}$$

$$=\frac{8\psi_0}{a^2}\Sigma(-)^n(n^2-\tfrac{1}{4})\frac{\mathrm{A}_n}{\mathrm{R}'_n}$$

or

$$x = -\{1+(1-\lambda)x\}\frac{T'_0}{R'_0}+2\Sigma_1(-)^n\{(4n^2-1+\lambda)x-1\}\frac{T'_n}{R'_n}$$

$$x\left\{1+(1-\lambda)\frac{T'_0}{R'_0}+2\Sigma_1(-)^{n-1}(4n^2-1+\lambda)\frac{T'_n}{R'_n}\right\} = -\frac{T'_0}{R'_0}+2\Sigma_1(-)^{n-1}\frac{T'_n}{R'_n}.$$

The right hand member of this equation is the velocity at the centre due to the cyclic motion alone, divided by $2\psi_0/a^2$. Call this velocity V_1, and denote the critical value of V by V_0, then

$$\frac{V_1}{V_0}=1+(1-\lambda)\frac{T'_0}{R'_0}+2\Sigma_1(-)^{n-1}(4n^2-1+\lambda)\frac{T'_n}{R'_n}$$

The most important terms in these expressions are

$$\left.\begin{aligned} x_0 &= \frac{\pi}{2(L-2)}\left\{1+\left(3L-6-4\pi-\tfrac{3}{4}\frac{1}{L-2}\right)k^2+\dots\right\} \\ \frac{V_1}{V_0} &= 1+4\pi k^2+\dots \end{aligned}\right\} \quad . \quad . \quad (18)$$

The stream lines will be given by

$$\psi-\tfrac{1}{2}V\rho^2 = \text{const}$$

and by choosing the constant properly, we may make this represent the surface of the fluid carried forward. To determine the constant we need only find one point on the surface by the above method. If the value of x is less than the critical value, the surface will extend to the axis; in this case the best way will be to put $u=0$ and find v from the equation

$$V=\left[\frac{1}{\rho}\frac{d\psi}{du}\frac{du}{dn}\right]_{u=0}$$

If on the contrary x is greater than x_0, the surface is ring-shaped, and it will be best to find u from the equation

$$V=\left[\frac{1}{\rho}\frac{d\psi}{du}\frac{du}{dn}\right]_{v=\pi}$$

If x be negative, or the velocity of translation and the cyclic motion within the aperture be in the opposite direction, the corresponding equation will be

$$V=\left[\frac{1}{\rho}\frac{d\psi}{du}\cdot\frac{du}{dn}\right]_{v=0}$$

In tabulating corresponding values of u, v and V/V_0 the best way would be to insert values of u, v and determine V/V_0. The following numbers in the case of $k=\sin 1°$ were obtained in this way. For the case of x less than the critical value, the surface cuts the straight axis at points given in the several cases by v,

v	60°	90°	120°	180°
$\frac{V}{V_0}$	·125	·355	·652	

For a ring-shaped surface

$$u=2{\cdot}9662,\ v=180,\ \text{and}\ V/V_0=1{\cdot}699$$

whilst for a negative translation

$$u=2{\cdot}9662,\ v=0,\ \text{and}\ V/V_0=-{\cdot}3708$$

are sets of corresponding values.

8. *The energy of the fluid motion.*—The energy is given by

$$E=\tfrac{1}{2}\iint\frac{1}{\rho^2}\left\{\left(\frac{\delta\psi}{\delta\rho}\right)^2+\left(\frac{\delta\psi}{\delta z}\right)^2\right\}2\pi\rho d\rho dz=\pi\iint\frac{1}{\rho}\left\{\left(\frac{\delta\psi}{\delta\rho}\right)^2+\left(\frac{\delta\psi}{\delta z}\right)^2\right\}d\rho dz$$

supposing the density of the fluid to be unity. Treating this in a similar way to the analogous expression in terms of the velocity potential, and remembering that whenever the volume of the surfaces immersed remains constant, as here, ψ is single valued, we shall find (by means of equation 1) that

$$E=\pi\int\frac{1}{\rho}\psi\frac{\delta\psi}{\delta n}ds$$

the integration being extended over any meridian curve of the solid, and dn being measured inwards along the normal (*i.e.*, from the fluid).

In the case, therefore, of circular tores

$$E=\pi\int_0^{2\pi}\frac{1}{\rho}\psi\frac{\delta\psi}{\delta u}\cdot\frac{\delta u}{\delta n}\cdot\frac{dn'}{dv}.dv=\pi\int_0^{2\pi}\frac{\psi}{\rho}\frac{\delta\psi}{\delta u}dv$$

Now we know that for the cyclic motion the energy is $\frac{1}{2}\times$ cyclic constant $\times$ flow through the aperture, and, therefore, with our notation is $\mu\times\pi\psi_0$. But it will be interesting to see how this is also arrived at from the preceding expression. The whole energy can be put in the form

$$E=(\alpha\mu^2+\beta x^2+2\gamma\mu x)\psi_0^2$$

we proceed to determine α, β, γ by means of the above formula.

α. Here along the surface $\psi=\psi_0$ a constant, and,

$$\frac{1}{\rho}\frac{\delta\psi}{\delta u}=\frac{1}{a}\Sigma\left\{-\frac{R_n}{2\sqrt{(C-c)}}+\frac{4n^2-1}{4}P_n\sqrt{(C-c)}\right\}A_n\cos nv$$

Therefore

$$\int_0^{2\pi}\frac{1}{\rho}\frac{\delta\psi}{\delta u}dv=\frac{2}{a}\Sigma\left\{-\tfrac{1}{2}R_nQ_n\sqrt{2}-\frac{1}{\sqrt{2}}P_nT_n\right\}A_n$$

$$=-\frac{\sqrt{2}}{a}\Sigma S\left(Q_n\frac{dP_n}{du}-P_n\frac{dQ_n}{du}\right)A_n$$

$$\frac{\pi\sqrt{2}}{a}\Sigma A_n \qquad [\text{T.F., } 24]$$

(by 4)

Hence energy $=\pi\mu\psi_0$, as is right, or

$$\alpha=\frac{\pi}{\mu\psi_0}=\frac{\pi a}{4\psi_0^2}\frac{1}{-\frac{T_0}{R_0}+2\Sigma_1^\infty\frac{1}{4n^2-1}\frac{T_n}{R_n}}$$

β. Here ψ along the tore is $\frac{1}{2}V(\rho^2-\lambda a^2)$, and denoting the general value of ψ by $\mu\psi_1+x\psi_2$

$$\beta\psi_0^2x^2=\pi x\int_0^{2\pi}\tfrac{1}{2}V(\rho^2-\lambda a^2)\frac{1}{\rho}\frac{\delta\psi_2}{\delta u}dv$$

Now $\int_0^{2\pi}\frac{1}{\rho}\frac{\delta\psi_2}{du}dv$ is proportional to the flow round the ring due to translation alone, and is therefore zero.

Hence

$$\beta=\frac{\pi}{a^2\psi_0}\int_0^{2\pi}\rho\frac{\delta\psi_2}{\delta u}dv$$

But

$$\psi_2=\frac{2\psi_0\sqrt{2}}{\pi\sqrt{(C-c)}}\left\{(1-\lambda)T'_0\cdot\frac{R_0}{R'_0}+2\Sigma\left(1+\frac{\lambda}{4n^2-1}\right)T'_n\frac{R_n}{R'_n}\cos nv\right\}$$

$$=\frac{2\psi_0\sqrt{2}}{\pi\sqrt{(C-c)}}\Sigma A_nR_n\cos nv \qquad \text{(say)}$$

Therefore

$$\beta=\frac{2S'\sqrt{2}}{a}\int_0^{2\pi}\frac{1}{C'-c}\cdot\frac{\delta\psi_2}{\delta u}dv$$

Hence by 7

$$\beta=\frac{4S'\sqrt{2}}{a}\int_0^{\pi}\frac{1}{2}\frac{S'}{(C-c)^{\frac{3}{2}}}\left[-\tfrac{1}{2}A_0P_1-\tfrac{3}{4}A_1P_1+\tfrac{1}{4}A_0P_0\cos v+\Sigma_1B_n\cos nv\right]dv$$

Now

$$Q_n\sqrt{2}=\int_0^{\pi}\frac{\cos nvdv}{(C-c)^{\frac{1}{2}}}.$$

Therefore

$$\int_0^{\pi} \frac{\cos nv}{(\mathrm{C}-c)^{\frac{3}{2}}} dv = -\frac{2\sqrt{2}}{\mathrm{S}} \frac{d\mathrm{Q}_n}{du}$$

$$\int_0^{\pi} \frac{\cos nv}{(\mathrm{C}-c)^{\frac{5}{2}}} = \frac{4\sqrt{2}}{3\mathrm{S}} \frac{d}{du}\left(\frac{1}{\mathrm{S}} \frac{d\mathrm{Q}_n}{du}\right)$$

$$\mathrm{S}^3\int_0^{\pi} \frac{\cos nv}{(\mathrm{C}-c)^{\frac{5}{2}}} = \frac{4\sqrt{2}}{3}\left(\frac{d^2\mathrm{Q}_n}{du^2} - \frac{\mathrm{C}}{\mathrm{S}} \frac{d\mathrm{Q}_n}{du}\right)$$

$$= \frac{\sqrt{2}}{3}\left\{(4n^2-1)\mathrm{Q}_n - \frac{8\mathrm{C}}{\mathrm{S}} \frac{d\mathrm{Q}_n}{du}\right\}$$

Hence dropping the dashes, and u denoting the value of u along the tore

$$\beta = \frac{4}{3a}\left[\tfrac{1}{2}(\mathrm{A}_0 + \tfrac{3}{2}\mathrm{A}_1)\mathrm{P}_1\left(\mathrm{Q}_0 - \frac{8\mathrm{C}}{\mathrm{S}^2}\mathrm{T}_0\right) + \tfrac{1}{4}\mathrm{A}_0\mathrm{P}_0\left(3\mathrm{Q}_1 + \frac{8\mathrm{C}}{\mathrm{S}^2}\mathrm{T}_1\right) + \Sigma_1\mathrm{B}_n\left\{(4n^2-1)\mathrm{Q}_n + \frac{8\mathrm{C}}{\mathrm{S}^2}\mathrm{T}_n\right\}\right]$$

A_n having the values given above, and B_n the values given in 7.

γ. The value of γ is given by

$$2\gamma\mu x\psi_0^2 = \pi\int_0^{2\pi} \frac{\mu x}{\rho}\left(\psi_1\frac{\delta\psi_2}{\delta u} + \psi_2\frac{\delta\psi_1}{\delta u}\right)dv$$

Here ψ_1 is constant and $\int_0^{2\pi}\frac{1}{\rho}\frac{\delta\psi_2}{\delta u} = 0$, also

$$x\psi_2 = \tfrac{1}{2}\mathrm{V}(\rho^2 - \lambda a^2) = \frac{x\psi_0}{a^2}(\rho^2 - \lambda a^2)$$

Hence

$$\gamma = \frac{\pi}{a^2\psi_0}\int_0^{\pi}(\rho^2 - \lambda a^2)\,\frac{1}{\rho}\frac{\delta\psi_1}{du}dv$$

Further $\int_0^{\pi}\frac{\mu}{\rho}\frac{\delta\psi_1}{du}dv$ is the flow along a closed curve threading the aperture and is therefore the cyclic constant μ. Therefore

$$\gamma = -\frac{\pi\lambda}{\psi_0} + \frac{\pi}{a^2\psi_0}\int_0^{\pi}\rho\frac{\delta\psi_1}{du}dv$$

The last integral may be expressed as in the analogous case for β.

Section III.—*Steady motion of hollow vortex.*

9. The form of a hollow vortex and its motion are conditioned by the fact that the velocity of the fluid relatively to the hollow, when the motion is steady, must be constant over the whole surface of the tore. When the section is small compared

with the aperture, the section will clearly be very approximately circular, and to a first approximation the motion will be represented by the stream function found in the previous section, the value of x therein being chosen so as to make the coefficient of $\cos v$ in the expression for the velocity disappear. This will give the first term in the expression for the velocity of translation of the vortex, when it moves forward without change of form. In order to arrive at closer approximations it will be necessary to take account of the form of the section, and this is done in the following investigation, so far as to get a second approximation, although the method employed is capable of being carried further, of course, with more and more complexity in the calculations.

By impressing on the whole fluid a velocity equal and opposite to that of the hollow, the hollow is brought to rest, with the fluid streaming past it. The stream function in this case becomes

$$\psi=-\tfrac{1}{2}a^2\mathrm{V}\frac{\mathrm{S}^2}{(\mathrm{C}-c)^2}+\frac{2\psi_0\sqrt{2}}{\pi\sqrt{(\mathrm{C}-c)}}\Sigma\mathrm{A}_n\frac{\mathrm{R}_n}{\mathrm{R}'_n}\cos nv$$

where

$$\mathrm{A}_0=\{1+(1-\lambda)x\}\mathrm{T}'_0$$

$$\mathrm{A}_n=2\left\{\left(1+\frac{\lambda}{4n^2-1}\right)x-\frac{1}{4n^2-1}\right\}\mathrm{T}'_n$$

The values of the first three are

$$\mathrm{A}_0=\frac{\pi}{4}[1+\tfrac{1}{4}k^2+\tfrac{1}{64}k^4+\{4(\mathrm{L}-2)k^2+(2\mathrm{L}^2-\tfrac{19}{2}\mathrm{L}+14)k^4\}x]k^{-1}$$

$$\mathrm{A}_1=\frac{\pi}{4}[\{4-\tfrac{1}{2}(8\mathrm{L}-15)k^2-(2\mathrm{L}^2-11\mathrm{L}+17\tfrac{1}{16})k^4\}xk-(1-\tfrac{1}{8}k^2-\tfrac{1}{64}k^4)k]k^{-} \qquad (19)$$

$$\mathrm{A}_2=\frac{\pi}{16}[\{16k^2-4(\mathrm{L}-1)k^4\}x-(k^2-\tfrac{1}{4}k^4)]k^{-\frac{1}{2}}$$

The approximation proceeding according to powers of k, each coefficient is one order higher than the preceding.

Let U be the velocity at any point of the hollow. Then, to the first order of small quantities, where the section is circular

$$\mathrm{U}=\left[\frac{1}{\rho}\frac{\delta\psi}{\delta u}\frac{du}{dn}\right]_{u=u'}=\left[\frac{(\mathrm{C}-c)^2}{a^2\mathrm{S}}\frac{\delta\psi}{\delta u}\right]_{u=u'}$$

where

$$\frac{\psi}{2\psi_0}=-\tfrac{1}{2}\frac{x\mathrm{S}^2}{(\mathrm{C}-c)^2}+\frac{\sqrt{2}}{\pi\sqrt{(\mathrm{C}-c)}}\left(\mathrm{A}_0\frac{\mathrm{R}_0}{\mathrm{R}'_0}+\mathrm{A}_1\frac{\mathrm{R}_1}{\mathrm{R}'_1}\cos v\right)$$

The part of U due to the first term is

$$\frac{\mathrm{U}_1}{2\psi_0}=-\frac{x}{a^2}\cdot\frac{1-\mathrm{C}c}{\mathrm{C}-c}$$

In finding the second part it will be well for the later approximations to carry ψ a term further to include A_2. Then from (7), if U_2 be the part of U due to this

$$\frac{a^2 U_2}{2\psi_0}=\frac{\sqrt{2}}{2\pi}\sqrt{(C-c)}\{B_0+\Sigma B_n \cos nv\}$$

where

$$B_0=-\tfrac{1}{2}A_0\frac{P_1}{R'_0}-\tfrac{3}{4}A_1\frac{P_1}{R'_1}$$

$$B_1=\tfrac{1}{2}A_0\frac{P_0}{R'_0}+\tfrac{3}{4}A_1\frac{(P_0+P_2)}{R'_1}-\tfrac{15}{4}A_2\frac{P_2}{R'_2}$$

$$B_n=\left\{(n^2-\tfrac{1}{4})\frac{A_n}{R'_n}-(\overline{n+1}|^2-\tfrac{1}{4})\frac{A_{n+1}}{R'_{n+1}}\right\}P_{n+1}-\left\{(\overline{n-1}|^2-\tfrac{1}{4})\frac{A_{n-1}}{R'_{n-1}}-(n^2-\tfrac{1}{4})\frac{A_n}{R'_n}\right\}P_{n-1}$$

These values of U_1, U_2 are to be expanded in a series of cosines of multiple angles. But here it is only needful to keep terms of the same order as A_2, or compared with A_0 of order k^2. Now

$$2C=k+\frac{1}{k}$$

Hence

$$\frac{1-Cc}{C-c}=\left(\frac{1}{C}-c\right)\left(1+\frac{1}{2C^2}+\frac{\cos v}{C}+\frac{\cos 2v}{2C^2}\right)$$

$$=\frac{1}{2C}-\left(1-\frac{1}{4C^2}\right)\cos v-\frac{\cos 2v}{2C}-\frac{1}{4C^2}\cos 3v$$

$$=k(1-k^2)-(1-k^2)\cos v-k(1-k^2)\cos 2v-k^2\cos 3v$$

Also

$$\sqrt{2}(C-c)^{\frac{1}{2}}\{B_0+\Sigma B_n\cos nv\}$$

$$=\sqrt{2C}\left\{1-\frac{1}{16C^2}-\frac{\cos v}{2C}-\frac{\cos 2v}{16C^2}\right\}\{B_0+B_1\cos v+B_2\cos 2v+B_3\cos 3v\}$$

$$=\frac{1+\frac{1}{2}k^2}{\sqrt{k}}\{1-\tfrac{1}{4}k^2-k(1-k^2)\cos v-\tfrac{1}{4}k^2\cos 2v\}\{B_0+\ldots\}$$

$$\therefore\{1+\tfrac{1}{4}k^2-k\cos v-\tfrac{1}{4}k^2\cos 2v\}\{B_0+B_1\cos v+B_2\cos 2v\}k^{-\frac{1}{2}}$$

$$=\{1+\tfrac{1}{4}k^2-k\cos v-\tfrac{1}{4}k^2\cos 2v\}(B_0+B_1\cos v)k^{-\frac{1}{2}}+(1-k\cos v)B_2k^{-\frac{1}{2}}\cos 2v$$

considering at present B_0 and B_1 to be of the same order. From this it is easy to show that if

$$\left.\begin{aligned}
\frac{a^2\mathrm{U}}{2\psi_0}&=\alpha+\beta\cos v+\gamma\cos 2v+\delta\cos 3v\\
\alpha&=-k(1-k^2)x+\frac{1}{2\pi}(1+\tfrac{1}{4}k^2)\mathrm{B}_0k^{-\frac{1}{2}}-\frac{1}{4\pi}\mathrm{B}_1k^{\frac{1}{2}}\\
\beta&=(1-k^2)x-\frac{1}{2\pi}\mathrm{B}_0k^{\frac{1}{2}}+\frac{1}{2\pi}(1+\tfrac{1}{8}k^2)\mathrm{B}_1k^{-\frac{1}{2}}-\frac{1}{4\pi}\mathrm{B}_2k^{\frac{1}{2}}\\
\gamma&=k(1-k^2)x-\frac{1}{8\pi}\mathrm{B}_0k^{\frac{3}{2}}-\frac{1}{4\pi}\mathrm{B}_1k^{\frac{1}{2}}+\frac{1}{2\pi}\mathrm{B}_2k^{-\frac{1}{2}}\\
\delta&=k^2x-\frac{1}{16\pi}\mathrm{B}_1k^{\frac{3}{2}}-\frac{1}{4\pi}\mathrm{B}_2k^{\frac{1}{2}}+\mathrm{B}_3k^{-\frac{1}{2}}
\end{aligned}\right\}\quad . \quad . \quad . \quad (20)$$

For the first approximation the lowest term in k in β must vanish. Hence

$$x-\frac{1}{2\pi}\mathrm{B}_0k^{\frac{1}{2}}+\frac{1}{2\pi}\mathrm{B}_1k^{-\frac{1}{2}}=0$$

Now

$$\mathrm{B}_0=-\tfrac{1}{2}\mathrm{A}_0\frac{\mathrm{P}_1}{\mathrm{R}_0}-\tfrac{3}{4}\frac{\mathrm{A}_1\mathrm{P}_1}{\mathrm{R}_1}$$

and the lowest terms in B_1 are

$$\mathrm{B}_1=\tfrac{3}{4}\mathrm{A}_1\frac{\mathrm{P}_2}{\mathrm{R}_1}+\tfrac{1}{2}\mathrm{A}_0\frac{\mathrm{P}_0}{\mathrm{R}_0}$$

Substituting the values of A_0, &c., from (19)

$$\frac{1}{2\pi}\mathrm{B}_0=\tfrac{1}{16}k^{-\frac{1}{2}}\frac{2}{\frac{1}{2}\mathrm{L}-1}-\tfrac{3}{4}\frac{4x-1}{8}k^{\frac{1}{2}}4k$$

$$=\tfrac{1}{4}\frac{k^{-\frac{1}{2}}}{\mathrm{L}-2}-\tfrac{3}{8}(4x-1)k^{\frac{3}{2}}$$

$$\frac{1}{2\pi}\mathrm{B}_1=\tfrac{3}{32}k^{\frac{1}{2}}(4x-1)\frac{\frac{4}{3}k^{-\frac{1}{2}}}{\frac{1}{2}k^{-\frac{1}{2}}}-\tfrac{1}{16}k^{-\frac{1}{2}}\frac{2\mathrm{L}k^{\frac{1}{2}}}{(\frac{1}{2}\mathrm{L}-1)k^{-\frac{1}{2}}}$$

$$=\left\{\tfrac{1}{4}(4x-1)-\tfrac{1}{4}\frac{\mathrm{L}}{\mathrm{L}-2}\right\}k^{\frac{1}{2}}$$

Hence

$$x-\frac{1}{4(\mathrm{L}-2)}+x-\tfrac{1}{4}-\tfrac{1}{4}\frac{\mathrm{L}}{\mathrm{L}-2}=0$$

$$2x=\tfrac{1}{4}\frac{2\mathrm{L}-1}{\mathrm{L}-2}$$

$$x=\tfrac{1}{8}\frac{2\mathrm{L}-1}{\mathrm{L}-2}$$

To the same order

$$\frac{2\psi_0}{a^2}=\frac{\mu}{\pi a}(\mathrm{L}-2) \qquad \text{by (15)}$$

Therefore

$$\mathrm{V}=\frac{\mu}{4\pi a}(\mathrm{L}-\tfrac{1}{2}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The principal term in U is found by equating it to the principal terms in α, *i.e.*,

$$\frac{a^2\mathrm{U}_0}{2\psi_0}=\frac{1}{2\pi}\,\mathrm{B}_0k^{-1}=\tfrac{1}{4}\,\frac{k^{-1}}{\mathrm{L}-2}$$

and is therefore independent of the velocity of translation, as ought to be the case, since the latter depends on the difference of the cyclic tangential velocities inside and outside the tore. Substituting for ψ_0

$$\mathrm{U}_0=\frac{\mu}{4\pi ak}$$

Now, for steady motion, the equation of pressure gives at the surface of the hollow, if Π and ρ be the pressure at an infinite distance, and the density of the fluid respectively

$$\frac{2\Pi}{\rho}=\mathrm{U}_0^2$$

Hence U must be the same for hollows of all sizes, and consequently ak constant for all the steady motions of the same vortex. When the hollow is small this is approximately the same as saying that the radius of the cross section is constant. The corresponding theorem for a solid ring is of course that the volume is constant.

10. For the second approximation we need to determine the stream function when the cross section of the ring is not an exact circle. The following investigation is slightly more general than is necessary for our present purposes.

Let k be the value of k for the mean section, and let the section be given by

$$\kappa=k+\Sigma(\mathrm{M}_n\cos nv+\mathrm{N}_n\sin nv)=k+\xi,\ \text{say}$$

where M_n, N_n are small quantities with respect to k. When the tore is at rest with fluid streaming past it, the stream function is

$$\psi=-x\psi_0\,\frac{\mathrm{S}^2}{(\mathrm{C}-c)^2}+\frac{2\psi_0\sqrt{2}}{\pi\sqrt{(\mathrm{C}-c)}}\,\Sigma\mathrm{A}_n\,\frac{\mathrm{R}_n}{\mathrm{R}'_n}\cos nv$$

where the A_n have the values given in (19).

Let the stream function for the non-circular section be $\psi+\chi$ where

$$\chi=\frac{1}{\sqrt{(C-c)}}\Sigma(X_n\cos nv+Y_n\sin nv)\frac{R_n}{R'_n}$$

and X_n, Y_n are also small. The necessary condition is, that when κ has the value given above, $\psi+\chi$ must be constant, say $\psi'_0+\epsilon$. Then neglecting squares of ξ

$$\psi'_0+\epsilon=\psi'_0+\left.\frac{\delta\psi}{\delta\kappa}\right|\xi+\frac{1}{\sqrt{(C_0-c)}}\Sigma(X_n\cos nv+Y_n\sin nv)$$

The value of ϵ is arbitrary, since with any given surface conditions the circulation remains undetermined. We shall choose it so as to make the circulation zero. It would be impossible to determine X_n, Y_n in the general case where both ξ and $\delta\psi/\delta\kappa$ are infinite series; but in the case required in the present paper A_n/A_{n-1} is of the order k, k being small, and the terms in A_n are neglected after A_2. This simplifies the calculation, and it is easy to determine the terms X_n, Y_n in terms of M_n, N_n. But it is further greatly simplified by the fact that in the case to which we have to apply it the velocity along the surface is already uniform to the first order—in other words

$$U=\frac{(C_0-c)^2}{a^2S_0}\left(\frac{\delta\psi}{\delta\kappa}\right)_0\frac{d\kappa}{du}$$

whence

$$\left(\frac{\delta\psi}{\delta\kappa}\right)_0=-\frac{a^2S_0}{k(C_0-c)^2}U$$

Hence the equation determining the X_n, Y_n is

$$\epsilon=-\frac{a^2S_0}{k(C_0-c)^2}U\xi+\frac{1}{\sqrt{(C_0-c}}\Sigma(X_n\cos nv+Y_n\sin nv)$$

But since in our applications k is itself so small that k^3 has been neglected compared with unity, the above becomes

$$\begin{aligned}\epsilon=&-2a^2(1+3k^2+4k\cos v+6k^2\cos 2v)U\xi\\&+\sqrt{(2k)}(1+\tfrac{1}{4}k^2+k\cos v+\tfrac{3}{4}k^2\cos 2v)\Sigma(X_n\cos nv+Y_n\sin nv)\end{aligned}$$

The various normal functions will therefore be composed of a set of principal terms in cos nv, &c., each corrected by an infinite convergent series of small terms of the others. The principal will be given by

$$\epsilon=-2a^2U\Sigma_1(M_n\cos nv+N_n\sin nv)+\sqrt{(2k)}\Sigma(X_n\cos nv+Y_n\sin nv)$$

Therefore

$$X_0=\frac{\epsilon}{\sqrt{(2k)}},\quad Y_0=0$$

$$X_n=\frac{2a^2U}{\sqrt{(2k)}}M_n,\quad Y_n=\frac{2a^2U}{\sqrt{(2k)}}N_n$$

The series connected with X_n, Y_n to complete the solution for a given M_n, N_n are found from

$$\begin{aligned}0= & -2a^2Uk(3k+4\cos v+6k\cos 2v)M_n\cos nv \\ & +k(\tfrac{1}{4}k+\cos v+\tfrac{3}{4}k\cos 2v)(\epsilon+2a^2UM_n\cos nv) \\ & +\sqrt{(2k)}(1+k\cos v)\Sigma X_n\cos nv\end{aligned}$$

with a corresponding equation for Y_n in which $\epsilon=0$.

We need only consider for the first approximation the principal terms, which give

$$\chi\sqrt{(C-c)}=\frac{\epsilon}{\sqrt{(2k)}}\frac{R_0}{R'_0}+\frac{2a^2U}{\sqrt{(2k)}}(M_n\cos nv+N_n\sin nv)\frac{R_n}{R'_n}$$

Since the circulation is to vanish,

$$\epsilon+2a^2M_nU=0$$

$$\chi=\frac{2a^2U}{\sqrt{2k}}\frac{1}{\sqrt{(C-c)}}\left\{-M_n\frac{R_0}{R'_0}+(M_n\cos nv+N_n\sin nv)\frac{R_n}{R'_n}\right\}$$

11. We are now in a position to determine the first term in the expression, giving the form of the hollow, viz., that part which will destroy the term in $\cos 2v$ in the value of the surface velocity. $\overline{U}$ denoting this velocity we have

$$\overline{U}^2=\frac{(C-c)^4}{a^4S^2}\left\{\left(\frac{\delta\psi}{\delta u}+\frac{\delta\chi}{\delta u}\right)^2+\left(\frac{\delta\psi}{\delta v}+\frac{\delta\chi}{\delta v}\right)^2\right\}$$

Now at the mean section $\delta\psi/\delta v=0$, and is therefore at least of the first order of small quantities near the circle $u=u_0$. Hence

$$\overline{U}=\frac{(C-c)^2}{a^2S}\left\{\frac{\delta\psi}{\delta u}+\frac{\delta\chi}{\delta u}\right\}$$

$$=U_1+\frac{(C_0-c)^2}{a^2S_0}\left(\frac{d\chi}{\delta u}\right)_0$$

where

$$\frac{a^2U_1}{2\psi_0}=\alpha+\beta\cos v+\gamma\cos 2v$$

and α, β, γ are the values given in (20) when $k+\xi$ is substituted for k *in the functions in* u.

Now ξ is of the form $M\cos 2v$, hence

$$\chi=\frac{2a^2U_0}{\sqrt{(2k)}}\cdot\frac{M}{\sqrt{(C-c)}}\left\{-\frac{R_0}{R'_0}+\frac{R_2}{R'_2}\cos 2v\right\}$$

Therefore

$$\left(\frac{\delta\chi}{\delta u}\right)_0=\frac{2a^2\mathrm{MU}_0}{\sqrt{(2k)}}\left\{-\tfrac{1}{2}\frac{\mathrm{S}}{(\mathrm{C}-c)^{\frac{3}{2}}}(-1+\cos 2v)+\frac{\mathrm{S}}{(\mathrm{C}-c)^{\frac{1}{2}}}\left(\tfrac{1}{4}\frac{\mathrm{P}_0}{\mathrm{R}_0}+\tfrac{15}{4}\frac{\mathrm{P}_2}{\mathrm{R}_2}\cos 2v\right)\right\}$$

and

$$\frac{(\mathrm{C}-c)^2}{a^2\mathrm{S}}\left(\frac{\delta\chi}{\delta u}\right)_0=\frac{(\mathrm{C}-c)^{\frac{1}{2}}\mathrm{MU}_0}{2\sqrt{(2k)}}\left\{2-2\cos 2v+(\mathrm{C}-c)\left(\frac{\mathrm{P}_0}{\mathrm{R}_0}+15\frac{\mathrm{P}_2}{\mathrm{R}_2}\cos 2v\right)\right\}$$

The principal term here is

$$\frac{\mathrm{MU}_0}{4k}\left(2-2\cos v+\frac{\mathrm{CP}_0}{\mathrm{R}_0}+15\frac{\mathrm{CP}_2}{\mathrm{R}_2}\cos 2v\right)$$

$$=\frac{\mathrm{MU}_0}{4k}\left(2-\frac{2\mathrm{L}}{\mathrm{L}-2}-2\cos 2v+10\cos 2v\right)$$

$$=-\frac{\mathrm{MU}_0}{k}\left(\frac{1}{\mathrm{L}-2}-2\cos 2v\right).$$

It remains now to determine α, β, γ to the same order of approximation, that is so far as the first power of k,

$$\alpha=-\kappa x+\frac{1}{2\pi}\mathrm{B}_0\kappa^{-\frac{1}{2}}-\frac{1}{4\pi}\mathrm{B}_1\kappa^{\frac{1}{2}}$$

$$=-kx+\frac{1}{2\pi}\left[\mathrm{B}_0k^{-\frac{1}{2}}-\tfrac{1}{2}\mathrm{B}_1k^{\frac{1}{2}}\right]_0-\xi x_0-\frac{1}{2\pi}\left[\frac{d}{du}(\mathrm{B}_0\kappa^{-\frac{1}{2}})\right]_0\frac{\xi}{k}$$

$$=-k(x_0+\delta x)+\tfrac{1}{4}\frac{k^{-1}}{\mathrm{L}-2}-\tfrac{3}{8}(4x_0-1)k-\tfrac{1}{8}\left(4x_0-1-\frac{\mathrm{L}}{\mathrm{L}-2}\right)k-\left\{x_0k+\frac{1}{2\pi}\frac{d}{du}(\mathrm{B}_0\kappa^{-\frac{1}{2}})\right\}_0\frac{\xi}{k}$$

$$=-kx_0+\tfrac{1}{4}\frac{k^{-1}}{\mathrm{L}-2}-\tfrac{1}{2}\left(4x_0-1-\tfrac{1}{4}\frac{\mathrm{L}}{\mathrm{L}-2}\right)k-\left\{x_0k+\frac{1}{2\pi}\frac{d}{du}(\mathrm{B}_0\kappa^{-\frac{1}{2}})\right\}\frac{\xi}{k}$$

Now

$$\mathrm{B}_0=-\tfrac{1}{2}\mathrm{A}_0\frac{\mathrm{P}_1}{\mathrm{R}'_0}-\tfrac{3}{4}\mathrm{A}_1\frac{\mathrm{P}_1}{\mathrm{R}'_1}$$

therefore

$$\frac{d}{du}(\mathrm{B}_0\kappa^{-\frac{1}{2}})=\tfrac{1}{2}\mathrm{B}_0k^{-\frac{1}{2}}-k^{-\frac{1}{2}}\left\{\tfrac{1}{2}\mathrm{A}_0\frac{\frac{d\mathrm{P}_1}{du}}{\mathrm{R}'_0}+\tfrac{3}{4}\mathrm{A}_1\frac{\frac{d\mathrm{P}_1}{du}}{\mathrm{R}'_1}\right\}.$$

therefore

$$\frac{1}{2\pi}\frac{d}{du}(\mathrm{B}_0\kappa^{-\frac{1}{2}})_0=\left\{\tfrac{1}{4}\frac{k^{-1}}{\mathrm{L}-2}-\tfrac{3}{8}(4x_0-1)k\right\}\left\{\tfrac{1}{2}+\frac{\frac{d\mathrm{P}_1}{du}}{\mathrm{P}_1}\right\}$$

$$=\tfrac{1}{4}\left\{\frac{k^{-1}}{\mathrm{L}-2}-\tfrac{3}{2}(4x_0-1)k\right\}$$

Therefore, substituting the value of x_0 already found

$$\alpha=-\tfrac{1}{2}\left(6x_0-1-\tfrac{1}{4}\frac{L}{L-2}\right)k+\tfrac{1}{4}\frac{k^{-1}}{L-2}-\tfrac{1}{4}\left\{\frac{k^{-1}}{L-2}-(2x_0-\tfrac{3}{2})k\right\}\frac{\xi}{\kappa}$$

$$=-\tfrac{1}{8}\frac{L+5}{L-2}k+\tfrac{1}{4}\frac{k^{-1}}{L-2}-\tfrac{1}{4}\left(\frac{k^{-1}}{L-2}\right)\frac{\xi}{k}$$

for, since ξ is at least of order k^2, the last term in the factor of ξ, can be neglected, and

$$\alpha=-\tfrac{1}{8}\frac{L+5}{L-2}k+\tfrac{1}{4}\frac{k^{-1}}{L-2}-\tfrac{1}{4}\frac{k^{-2}}{L-2}\xi$$

again

$$\beta=\beta_0+\delta x_0+\frac{1}{2\pi}\left[\frac{d}{du}(B_0\kappa^{\frac{1}{2}}-B_1\kappa^{-\frac{1}{2}})\right]_0\frac{\xi}{k}-\frac{1}{2\pi}\frac{d}{dx}(B_0k^{\frac{1}{2}}-B_1k^{-\frac{1}{2}})\delta x_0$$

Now

$$\frac{1}{2\pi}\frac{d}{du}(B_0\kappa^{\frac{1}{2}})=\frac{1}{2\pi}\left\{\kappa\frac{d}{du}(B_0\kappa^{-\frac{1}{2}})+B_0\kappa^{-\frac{1}{2}}\frac{d\kappa}{du}\right\}$$

$$=-\tfrac{1}{4}\frac{1}{L-2}+\tfrac{1}{4}\frac{1}{L-2}=0$$

Also since

$$B_1=\tfrac{3}{4}A_1\frac{P_2}{R'_1}+\tfrac{1}{2}A_0\frac{P_0}{R'_0}$$

$$\frac{dB_1}{du}=\tfrac{3}{4}A_1\frac{R_2}{SR_1}+\tfrac{1}{2}A_0\frac{1}{S}$$

$$\frac{1}{2\pi}\frac{dB_1}{du}=\tfrac{3}{8}(4x-1)k^{\frac{1}{2}}+\tfrac{1}{8}k^{\frac{1}{2}}$$

$$\frac{1}{2\pi}\frac{d}{du}(B_1\kappa^{-\frac{1}{2}})=\tfrac{3}{8}(4x-1)+\tfrac{1}{8}+\tfrac{1}{2}\left\{\tfrac{1}{4}(4x-1)-\tfrac{1}{4}\cdot\frac{L}{L-2}\right\}$$

$$=\tfrac{1}{2}(4x_0-1)-\tfrac{1}{4}\frac{1}{L-2}=\tfrac{1}{2}\frac{1}{L-2}$$

also

$$\frac{1}{2\pi}\frac{d}{dx}(B_0k^{\frac{1}{2}}-B_1k^{-\frac{1}{2}})=-1$$

therefore

$$\beta=2\delta x_0-\tfrac{1}{2}\frac{1}{L-2}\frac{\xi}{k}$$

$$\gamma=x_0k-\frac{1}{8\pi}B_0k^{\frac{3}{2}}-\frac{1}{4\pi}B_1k^{\frac{1}{2}}+\frac{1}{2\pi}B_2k^{-\frac{1}{2}}$$

The principal term in B_2 is

$$B_2=\tfrac{15}{4}A_2\frac{P_3}{R_2}-\tfrac{3}{4}A_1\frac{P_1}{R_1}$$

therefore

$$\frac{1}{2\pi}B_2=\tfrac{15}{4}\cdot\tfrac{1}{32}(16x-1)k^4\frac{\frac{16}{5}k^{-4}}{k^{-2}}-\tfrac{3}{4}\cdot\tfrac{1}{8}(4x-1)k^4\frac{2k^{-4}}{\frac{1}{2}k^{-4}}$$

$$=\tfrac{1}{4}(2x+1)k^2$$

therefore

$$\gamma=\left\{x_0-\tfrac{1}{16}\cdot\frac{1}{L-2}-\tfrac{1}{8}\left(4x_0-1-\frac{L}{L-2}\right)+\tfrac{1}{4}(2x_0+1)\right\}k$$

$$=\left(x_0+\tfrac{3}{8}+\tfrac{1}{16}\frac{2L-1}{L-2}\right)k=\tfrac{1}{16}\frac{12L-15}{L-2}k$$

Hence, substituting their values for α, β, γ

$$\overline{U}=\left\{\tfrac{1}{4}\frac{k^{-1}}{L-2}-\tfrac{1}{8}\frac{L+5}{L-2}k-\tfrac{1}{4}\cdot\frac{k^{-2}}{L-2}M\cos 2v\right\}\frac{2\psi_0}{a^2}$$

$$+\left\{\left(2\delta x_0-\tfrac{1}{2}\frac{1}{L-2}\,\frac{M}{k}\cos 2v\right)\frac{2\psi_0}{a^2}\right.$$

$$\left.+\frac{2\psi_0}{a^2}\cdot\tfrac{1}{16}\,\frac{12L-15}{L-2}k\cos 2v-\frac{MU_0}{k}\left(\frac{1}{L-2}-2\cos 2v\right)\right.$$

The condition that the coefficient of $\cos 2v$ vanishes gives

$$M\left\{\frac{2U_0}{k}-\tfrac{1}{4}\frac{2\psi_0}{a^2k^2(L-2)}\right\}+\tfrac{1}{16}\cdot\frac{12L-15}{L-2}k\cdot\frac{2\psi_0}{a^2}=0$$

or since

$$U_0=\tfrac{1}{4}\frac{k^{-1}}{L-2}\cdot\frac{2\psi_0}{a^2}$$

therefore

$$\tfrac{1}{4}\frac{Mk^{-2}}{L-2}=-\tfrac{1}{16}\frac{12L-15}{L-2}k$$

or

$$M=-\tfrac{1}{4}(12L-15)k^3$$

Also, since $\beta=0$,

$$\delta x_0=0$$

Therefore

$$\overline{U}=\frac{2\psi_0}{a^2}\left\{\tfrac{1}{4}\,\frac{k^{-1}}{L-2}-\tfrac{1}{8}\,\frac{L+5}{L-2}k\right\}$$

$$=\frac{\mu}{4\pi ak}\{1-\tfrac{1}{2}(L+5)k^2\}$$

Hence,

(1.) The velocity of translation remains unaltered to this order.

(2.) The form of the hollow is given by

$$\kappa=k\{1-\tfrac{1}{4}(12L-15)k^2\cos 2v\}$$

(3.) The surface velocity is

$$\frac{\mu}{4\pi ak}\{1-\tfrac{1}{2}(L+5)k^2\}$$

The effect of the correction to the form of the hollow is to make the section slightly elliptic with the major axis perpendicular to the plane of the ring, and with the inner side slightly flatter than the other.

The value of x obtained above is, when k is infinitely small, larger than the critical value given in (18). The fluid carried forward will therefore be ring-shaped. If for a rough approximation we take the two first terms of the expressions, the value of k for a hollow vortex in steady motion, and carrying forward a simply connected mass of fluid, will be found from

$$L=\frac{4\pi+1}{2}$$

or

$$k=4e^{-\frac{4\pi+1}{2}}=4e^{-6\cdot78}$$
$$=\cdot00522 \text{ very nearly}$$

this would make $R/r=10^2$, about. Since k is so small our approximations are very close, and it follows that for even extremely small cores of hollow vortices, the fluid carried forward is a mass without aperture. For infinitely small ones this is not the case.

The form of the hollow has been determined above by the value of k. If the normal variation from the circle be denoted by δn

$$\delta n=\frac{dn}{du}\frac{du}{dk}\delta k=-\tfrac{1}{4}\frac{a}{C-c}(12L-15)k^2\cos 2v$$

$$=-\tfrac{1}{16}\frac{r^3}{R^3}\left(12\log_e\frac{8R}{r}-15\right)\cos 2v$$

The figure below represents the form given by this expression to a circle in which $r/R=\cdot2$. Though this value is large, it shows the nature of the change of form better than a smaller value.

12. On account of the constant surface velocity along a hollow vortex, with fluid streaming past it, it is very easy to determine the energy of the motion, when the ring moves through a fluid otherwise at rest. For § 8 the energy is given by

$$\pi\int_0^{2\pi}\frac{\psi}{\rho}\frac{\delta\psi}{\delta u}dv=\pi\int_0^l U'\psi ds$$

where ds is an element of the arc of the cross-section, l its length, and U' the velocity along the surface regarded as for the moment fixed in space. It is therefore the component of V along the surface and U, that is

$$U'=U+V\frac{R-\rho}{r}$$

Also

$$\psi=\tfrac{1}{2}V\rho^2+\text{const}$$
$$=\tfrac{1}{2}V(\rho^2-\lambda a^2)+\psi_0$$

Hence, to the first order, where the section is a circle, the energy is

$$\pi\int_0^{2\pi}\left(U+V\frac{R-\rho}{r}\right)\left(\psi_0-\frac{V}{2}\lambda a^2+\tfrac{1}{2}V\rho^2\right)rd\theta$$

where

$$\rho=R+r\cos\theta$$

Therefore the energy is

$$\pi r\int_0^{2\pi}[U\{\psi_0+\tfrac{1}{2}V(R^2-\lambda a^2)\}+(\tfrac{1}{2}UVr^2+V^2Rr)\cos^2\theta]d\theta$$
$$=2\pi^2 r[U\{\psi_0+\tfrac{1}{2}V(R^2-\lambda a^2)\}+\tfrac{1}{2}Vr(\tfrac{1}{2}Ur+VR)]$$

But

$$U=\frac{\mu}{4\pi ak}$$

$$V=\frac{\mu}{4\pi a}(L-\tfrac{1}{2})$$

$$\psi_0=\frac{\mu a}{2\pi}(L-2)$$

Substituting these values, the energy is

$$\tfrac{1}{4}\mu^2 rk^{-1}\left\{L-2+\frac{1}{4a^2}(R^2+\tfrac{1}{2}r^2-\lambda a^2)(L-\tfrac{1}{2})+\frac{Rr}{4a^2}(L-\tfrac{1}{2})^2 k\right\}$$

Now to the order of approximation of circular section $\lambda=1$, $r=2ak$, $R=a$, and the energy is $\frac{1}{2}\mu^2 a(L-2)$, which is the same as for a rigid tore at rest. If the shape be

regarded, then since here the variation from the circle depends on k^3, we may treat it as circular in the integration, provided we do not carry our approximation beyond k^2. In this case

$$\lambda=1-4(L-2)k^2, \quad r=2ak, \quad R=a(1-2k^2)$$

and the energy is

$$\tfrac{1}{2}\mu^2a\{L-2+(L-\tfrac{3}{2})(L-\tfrac{1}{2})k^2+\tfrac{1}{2}(L-\tfrac{1}{2})^2k^2\}$$

$$=\tfrac{1}{2}\mu^2a\{(L-2)+\tfrac{1}{4}(2L-1)(3L-11)k^2\}$$

To the lowest order this is

$$=\frac{\mu^3}{4\pi V}(L-2)(L-\tfrac{1}{2})$$

13. If the steady shape as just found receive a slight disturbance symmetrical about the straight axis, a series of waves will be propagated round the hollow. To prove this, and to find the time of oscillation for different modes, will be the aim of the remainder of this paper; and firstly I consider the case where the cross-section is crimped into a form given by $\xi=\delta k=M\cos nv+N\sin nv$, where M.N are small compared with k, and functions of the time. Since they are functions of the time, the volume of the hollow will change, and consequently the stream function will be cyclic. The rate of change of volume is

$$\int_0^{2\pi}\dot{\xi}\frac{dn}{dk}\delta v\frac{dn'}{dv}=\frac{a^2}{k}\int_0^{2\pi}\frac{\dot{M}\cos nv+\dot{N}\sin nv}{(C-c)^2}dv$$

$$=\frac{a^2\dot{M}}{k}\int_0^{2\pi}\frac{\cos nv}{(C-c)^2}dv=-\frac{a^2\dot{M}}{kS}\frac{d}{du}\int_0^{2\pi}\frac{\cos nv}{C-}dv$$

$$=-\frac{\pi a^2\dot{M}}{kS}\frac{dB_n}{du}$$

where B_n is the coefficient of $\cos nv$ in the expansion of $(C-c)^{-1}$

Hence

$$B_n=\frac{2e^{-nu}}{S}$$

and the rate of change of volume

$$=\frac{2\pi a^2\dot{M}}{kS^2}\left(n+\frac{C}{S}\right)e^{-nu}$$

which is of the order $8\pi a^2\dot{M}(n+1)k^{n+1}$, a quantity beyond that which we neglect. Hence we may employ the stream function. Let then χ denote this function for the small motion given by ξ. The condition to find it is that for all values of the time t

$$\dot{\xi}\frac{dn}{dk}=-\frac{1}{\rho}\frac{d\chi}{dv}\frac{dv}{dn'}$$

Considering first the term $\xi = \mathrm{M} \cos nv$, the corresponding form for χ will be

$$\chi = \frac{1}{\sqrt{(\mathrm{C}-c)}} \Sigma \mathrm{Y}_m \frac{\mathrm{R}_m}{\mathrm{R}'_m} \sin mv$$

the coefficient Y being determined from the condition

$$\dot{\mathrm{M}} \cos nv = -\frac{k(\mathrm{C}-c)^3}{a^3\mathrm{S}} \frac{d}{dv} \left\{ \frac{1}{\sqrt{(\mathrm{C}-c)}} \Sigma \mathrm{Y}_m \frac{\mathrm{R}_m}{\mathrm{R}'_m} \sin mv \right\}$$

for all values of v when $u = u'$. Therefore

$$\dot{\mathrm{M}} \cos nv = -\frac{k(\mathrm{C}-c)^3}{a^3\mathrm{S}} \Sigma \left\{ \frac{m \cos mv}{\sqrt{(\mathrm{C}-c)}} - \frac{1}{2} \frac{\sin v \sin mv}{(\mathrm{C}-c)^{\frac{3}{2}}} \right\} \mathrm{Y}_m$$

or

$$2 \frac{\dot{\mathrm{M}} a^3 \mathrm{S}}{k} \cdot \frac{\cos nv}{(\mathrm{C}-c)^{\frac{5}{2}}} = -\Sigma \{ 2m\mathrm{C} \cos mv - 2m \cos mv \cos v - \sin mv \sin v \} \mathrm{Y}_m$$

$$\frac{2\dot{\mathrm{M}} a^3 \mathrm{S}}{k\mathrm{C}} \cdot \frac{\cos nv}{(\mathrm{C}-c)^{\frac{5}{2}}} = -\Sigma \left\{ 2m \cos mv - \frac{2m-1}{2\mathrm{C}} \cos \overline{(m+1)}v - \frac{2m+1}{2\mathrm{C}} \cos (m-1)v \right\}$$

From this we may obtain sequence equations to determine the Y_n; but we require only the most important terms, hence

$$2n\mathrm{Y}_n = -\frac{2\dot{\mathrm{M}} a^3 \mathrm{S}}{k\mathrm{C}^{\frac{7}{2}}}$$

$$\mathrm{Y}_n = -\frac{2\sqrt{2}}{n} a^3 \dot{\mathrm{M}} k^{\frac{1}{2}}$$

and

$$\chi = -\frac{2\sqrt{2} a^3 \dot{\mathrm{M}} k^{\frac{1}{2}}}{n\sqrt{(\mathrm{C}-c)}} \cdot \frac{\mathrm{R}_n}{\mathrm{R}'_n} \sin nv$$

Since the cyclic motion due to this is zero, there is no correction to be introduced for it as in former cases.

If ϕ be the velocity potential, the condition for a free surface gives

$$0 = \frac{\Pi}{\rho} - \dot{\phi} - \tfrac{1}{2}(\mathrm{vel})^2 + f(t)$$

$f(t)$ being an arbitrary function of the time. The velocity normal to the surface is of the first order of small quantities, and its square is to be neglected.

The velocity along the surface is

$$\mathrm{U}_0 + \left(\frac{d\mathrm{U}}{dk}\right)_0 \xi$$

where U_0 is the velocity determined in § 10 and

$$\frac{\Pi}{\rho} - \tfrac{1}{2}\mathrm{U}_0^2 = 0$$

Hence

$$\dot{\phi}+\mathrm{U}_0\left(\frac{d\mathrm{U}}{dk}\right)_0\xi+f(t)=0$$

Now ϕ is the flow along any curve from a fixed point up to the point in question. Let us take the curve to be formed by a straight line from the centre in the plane of the ring ($v=\pi$) up to the surface ($u=u'$), and then along the ring to the point (u', v). The first part will be a function of the time alone, and will therefore disappear with $f(t)$; of the part along the ring, that due to the cyclic motion will be constant, and therefore the corresponding part in $\dot{\phi}$ will disappear. The part depending on the velocity of translation will be proportional to x, which will introduce a quantity proportional to $\dot{x}$ in $\dot{\phi}$. This will contain terms in $\cos v$, which will not enter again. Hence $\dot{x}$ must be equated to zero, or the velocity of translation will not be affected. There remains only the part depending on the flow along the surface due to the motion χ. This we proceed to find. Denoting it by $\phi_{\prime}$

$$\phi_{\prime}=\int_v^\pi\left[\frac{1}{\rho}\frac{d\chi}{du}\frac{du}{dn}\cdot\frac{dn}{dv}\right]_{u'}dv$$

$$=-\frac{2a^2\dot{\mathrm{M}}\sqrt{(2k)}}{n\mathrm{SR}'_n}\int_v^\pi(\mathrm{C}-c)\left[\frac{d}{du}\frac{\mathrm{R}_n}{\sqrt{(\mathrm{C}-c)}}\right]_{u'}\sin nvdv$$

$$=-\frac{2a^2\dot{\mathrm{M}}\sqrt{(2k)}}{n\mathrm{S}}\int_v^\pi\left\{-\tfrac{1}{2}\frac{\mathrm{S}}{\sqrt{(\mathrm{C}-c)}}+(n^2-\tfrac{1}{4})\sqrt{(\mathrm{C}-c)}\frac{\mathrm{SP}_n}{\mathrm{R}_n}\right\}\sin nvdv$$

the principal part of which is

$$\phi_{\prime}=-\frac{4a^2\dot{\mathrm{M}}k}{n^2}\left(-\tfrac{1}{2}+(n^2-\tfrac{1}{4})\frac{\mathrm{CP}_n}{\mathrm{R}_n}\right)\{\cos nv-(-1)^n\}$$

The part of this, independent of $\cos nv$, will disappear with $f(t)$.

Further, since $\mathrm{U}\dfrac{d\mathrm{U}}{dk}$ is multiplied by ξ, we must only take their lowest terms, which are independent of v. Finally then equating to zero the coefficient of $\cos nv$

$$-\frac{4a^2k}{n^2}\left\{-\tfrac{1}{2}+(n^2-\tfrac{1}{4})\frac{\mathrm{CP}_n}{\mathrm{R}_n}\right\}\ddot{\mathrm{M}}+\mathrm{U}_0\frac{d\mathrm{U}_0}{dk}\cdot\mathrm{M}=0$$

Now

$$\mathrm{U}=\frac{2\psi_0}{a^2}\alpha$$

therefore

$$\frac{d\mathrm{U}}{dk}=\frac{2\psi_0}{a^2}\frac{d\alpha}{dk}$$

$$=\frac{\mathrm{U}}{\alpha}\frac{d\alpha}{dk}$$

To the order here reached

$$\alpha=\frac{1}{2\pi}\left(-\tfrac{1}{2}A_0\frac{P_1k^{-1}}{R'_0}\right)$$

$$\frac{d\alpha}{dk}=\frac{1}{2\pi}\frac{A_0}{R_0}k^{-2}=-\alpha k^{-1}$$

therefore

$$\frac{dU}{dk}=-Uk^{-1}$$

Hence

$$\frac{a^2k^2}{n^2}\left\{(4n^2-1)\frac{CP_n}{R_n}-2\right\}\ddot{M}+U^2M=0$$

Now

$$(4n^2-1)\frac{CP_n}{R_n}-2=2\left(\frac{4nCP_n}{P_{n+1}-P_{n-1}}-1\right)$$

$$=4n\cdot\frac{P_{n+1}+P_{n-1}}{P_{n+1}-P_{n-1}}$$

Therefore

$$\ddot{M}+\frac{nU^2}{4a^2k^2}\cdot\frac{P_{n+1}-P_{n-1}}{P_{n+1}+P_{n-1}}M=0$$

The coefficient of M is always positive; hence the hollow is stable for displacements of this kind, and the time of vibration for displacement of order n is

$$\frac{4\pi ak}{U}\sqrt{\left\{\frac{1}{n}\cdot\frac{P_{n+1}+P_{n-1}}{P_{n+1}-P_{n-1}}\right\}}$$

Since throughout our approximations we have neglected k^2 compared with unity, we may simplify this further by obtaining the value of the expression under the square root to the same order,

Now

$$\frac{(2n+1)(P_{n+1}+P_{n-1})}{(2n+1)(P_{n+1}-P_{n-1})}=\frac{4nCP_n-2P_{n-1}}{4nCP_n-4nP_{n-1}}$$

$$=\frac{1-\dfrac{1}{2nC}\cdot\dfrac{(2n-1)P_{n-1}}{(2n-1)P_n}}{1-\dfrac{1}{C}\cdot\dfrac{(2n-1)P_{n-1}}{(2n-1)P_n}}$$

$$=\frac{1-\dfrac{2n-1}{8n(n-1)C^2}}{1-\dfrac{2n-1}{4(n-1)C^2}}$$

$$=\frac{1-\dfrac{2n-1}{2n(n-1)}k^2}{1-\dfrac{2n-1}{n-1}k^2}$$

$$=1+\frac{(2n-1)^2}{2n(n-1)}k^2$$

The time of vibration may also be written in the forms

$$\frac{\mu\rho}{2\Pi}\sqrt{\frac{1}{n}\left(\frac{P_{n+1}+P_{n-1}}{P_{n+1}-P_{n-1}}\right)} \text{ or } \frac{\mu\rho}{2\Pi\sqrt{n}}$$

which shows that the time is independent of the velocity of translation, a result which has important bearing on the theory that atoms of matter are hollow vortices. For the different orders of vibration, the time of vibration varies inversely as the square root of the number of crests running round the hollow.

14. *Pulsation of hollow.*—In the preceding case, $n=0$ would correspond to pulsations of the hollow, in which therefore the whole motion is a change of volume, and the use of the stream function is not allowable. But as it happens, the application of the velocity potential is here very easy. Let, as in Art. 13, the displacement be given by

$$\delta k=\xi=\left(\frac{C-c}{S}\right)^{\frac{1}{2}}M$$

Then the velocity potential is

$$\phi=\sqrt{C-c}\,\Sigma A_n\frac{P_n}{P'_n}\cos nv$$

with

$$\frac{\dot{\xi}}{k}\cdot\frac{a}{C-c}=-\frac{C-c}{a}\frac{\delta\phi}{\delta u} \text{ when } u=u'$$

Therefore

$$\frac{a}{k}\frac{\sqrt{(C-c)}}{S^{\frac{1}{2}}}\dot{M}=-\frac{\sqrt{(C-c)}}{a}\Sigma\frac{A_n}{P_n}\left\{\tfrac{1}{2}SP_n+(C-c)\frac{dP_n}{du}\right\}\cos nv$$

whence the principal term is

$$A_0=-\frac{2a^2}{kS^{\frac{1}{2}}}\frac{P_0}{SP_0+2C\frac{dP_0}{du}}\dot{M}$$

$$=-2a^2\sqrt{2}k^{-\frac{1}{2}}\frac{4P_0}{k^{-2}P_0+4k^{-1}R_0}\dot{M}$$

$$=-4a^2\sqrt{2}k^{\frac{1}{2}}L\dot{M}$$

Hence

$$\phi=-4a^2\sqrt{2}k^{\frac{1}{2}}L\dot{M}\sqrt{(C-c)}\frac{P_0}{P'_0}$$

as before

$$\dot{\phi}+U\frac{dU}{dk}\xi=0$$

Therefore

$$4a^2\sqrt{(2C)}k^{\frac{1}{2}}L\ddot{M}+U^2k^{-1}M=0$$

$$\ddot{M}+\frac{U^2}{4a^2k^2L}M=0$$

Therefore time of pulsation

$$=\frac{4\pi ak}{U}\sqrt{L}=\frac{\mu\rho}{2\Pi}\left(\log\frac{4}{k}\right)^{\frac{1}{2}}$$

and therefore varies slowly with the energy.

VIII. *Contributions to our knowledge of the connexion between Chemical Constitution, Physiological Action, and Antagonism.**

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[PLATES 8–10.]

THE great object of Pharmacology is to obtain such a knowledge of the relation between the chemical constitution and physiological action of bodies as to be able to predict with certainty what the action of any substance will be. One of the most important steps towards this object was made by CRUM-BROWN and FRASER, who showed that the introduction of methyl into the molecule of strychnia or thebaia changed the tetanising action of those poisons on the spinal cord into a paralyzing one on the ends of the motor nerves.

As the organic alkaloids are compound ammonias, it seemed probable that a similar change in the chemical constitution of ammonia itself might produce a corresponding change in physiological action. This was tested by CRUM-BROWN and FRASER, who found that trimethyl-ammonium iodide possessed a paralyzing action similar to that of methyl strychnia or methyl thebaia, while ammonia itself has been shown by FUNKE and DEAHNA to have a tetanising action very much like that of strychnia. A number of other ammonium compounds have been shown to have a similar paralyzing action; but there is no complete investigation of the whole series, nor has the relation of the acid with which the base is combined been determined.

In the present research we have attempted—

1st. To ascertain how the general action of ammonia is modified by its combination with an acid radical. Under this heading we have investigated: (*a*) the alteration in its general effects upon the organism; and (*b*) the alterations in muscle and nerve by which the general effects are to a great extent determined.

2nd. To investigate the general action of the compound ammonias containing the more common radicals of the alcohol series in the same way as the ammonium salts in the first part of the paper.

* The present research forms part of an investigation into the action of certain drugs on muscle and nerve, for which a grant was given to one of us (BRUNTON) in 1877, but the prosecution of which was much delayed by various circumstances, amongst others, the rebuilding of the laboratory in which the experiments were made.

3rd. To compare the action of ammonia on muscle and nerve with that of other substances nearly allied to it in chemical properties, and belonging to the group of alkalies.

4th. To examine the action of acid and alkali upon muscle independently of the chemical composition of the acids or alkalies employed.

5th. To extend the research on muscle and nerve to the elements belonging to the group of alkaline earths.

General action of Ammonium Salts.

From experiments with ammonium chloride, sulphate, phosphate, tartrate, benzoate, and hippurate, Feltz and Ritter concluded that ammoniacal salts all had a similar action, producing convulsions and coma, slowing of the pulse and lowering of the temperature. They considered the action to be the same in kind, but differing in intensity. The convulsions produced by ammoniacal salts were shown by Funke and Deahna to be similar to the tetanus produced by strychnia, differing from it only in the fact that a single convulsion instead of a series of convulsions was produced by the poison. The cause of this result they believed to be the rapid production of paralysis of the motor nerves by the ammoniacal salt, which prevented the occurrence of more than one tetanic convulsion.

As the action of chloride of ammonium has already been pretty thoroughly investigated, it seemed to us unnecessary to make any more experiments upon its general action. We have therefore restricted our researches to the action of the bromide, iodide, sulphate and phosphate, and have experimented only on Frogs with the bromide. The result of these experiments seems to be that ammonium chloride, bromide and iodide form a series. At one end of it is ammonium chloride having a stimulant action on the spinal cord, and, at the other, the iodide having a paralyzing action upon motor nerves. Ammonia and ammonium chloride produce tetanus; the bromide, hyperæsthesia, with some clonic spasm, passing into tetanus, which, however, comes on very late in the course of the poisoning. The iodide produces rapid failure of higher reflexes, such as that from the conjunctiva, and caused in our experiments progressive paralysis, but no tetanus. At an early stage of poisoning by it the Frog responded with a croak when stroked on the back, and as this has been shown by Goltz to occur after removal of the cerebral hemispheres, its occurrence in poisoning by ammonium iodide may be looked upon as a proof that the higher centres are poisoned first. After injection of ammonium phosphate also, there is throughout an absence of true spasm. The usual movements become sprawling, and when taken up and gently set down again, the animal remains plastic, with the limbs extended. Before the cessation of reflex in the hind limbs, slight twitchings are observed to accompany induced movement. After the injection of sulphate of ammonium a slight degree of hyperæsthesia is developed. In a variable length of time

twitchings occur. They appear first in the anterior extremities, and then spread all over the body to the hind limbs. This spasm increases in intensity, and often manifests itself by a number of clonic convulsions occurring at tolerably regular intervals. These seldom pass into a rigid tetanus. They are, however, provoked by touching the animal, by the application of cold to the surface of its body, or by a blow upon the table upon which it is resting. When the sciatic nerve was divided on one side before the injection of the poison, twitchings did not occur upon that side. The action of the salts of ammonia upon the circulation was also found to be various. Thus, in poisoning by the bromide, it was unusual to find the heart materially influenced in its activity, even when the most marked motor symptoms had been developed. With the iodide, however, an early arrest of the heart in diastole, with the auricles and ventricle distended by dark blood, was very usual. A larger dose of the phosphate, and not unfrequently an equal dose of the sulphate, had a somewhat similar effect. An examination of the blood showed that after poisoning by bromide of ammonium, a marked change had taken place in the red blood-corpuscles. These exhibited numerous coagulations in their stroma; an increase of free nuclei was likewise observed in the blood; where the blood from the corresponding limb to which the poison had not had access was examined, no such changes were observed. A similar result is occasionally noticed after poisoning by the sulphate; it is much more unusual where the iodide and phosphate have been employed.

Examination of the reaction of the muscle to direct and indirect stimulation was made as rapidly as possible, when it was desired to examine their reaction at any stage which the poisoning had reached. The ligatured limb was used for a contrast; and as it has been shown by KÜHNE* that in cold-blooded animals the irritability of the muscle declines when containing blood in a condition of stasis, allowance must be made for this decrease in irritability when contrasting its reaction with that of the poisoned muscle. The irritability was tested by means of approximating the secondary coil of a DU BOIS REYMOND's inductorium to the primary, the greatest distance at which a minimal contraction was produced being registered both for direct and indirect stimulation. This figure was controlled by removing the secondary coil from the primary, in which case contraction often persisted at a more distant position than it was observed at when the coil was approximated.† The muscle poisoned by bromide showed an increase of irritability in the early stages, and before the action of the poison was complete. There was a slight but less marked increase occasionally in the case of iodide, but usually the irritability in cases of slight poisoning is diminished. There is usually no marked increase of irritability in muscles poisoned by the phosphate and sulphate, though in exceptional cases it has been observed as a temporary condition in both The muscle responds to direct and indirect stimulation (opening shock) by a long, at first equally high, but then rapidly falling curve, in comparison with the normal. The

* Archiv. f. Anat. u. Physiol., 1859.

† The excitability of the muscle appearing to be increased by its contraction.

response to indirect stimulation is, however, much feebler than to direct. The tetanus of both is impaired, but especially that of indirect stimulation. The total failure of reaction upon stimulation of the nerve frequently occurs whilst the muscle yields a moderate tetanus. If the heart has not been arrested by the injection of too large a dose of ammonium iodide before the circulation has distributed the poison sufficiently, it is often found that stimulation of the nerve does not produce any contraction, or it may be only a few faint twitches of the muscle. In poisoning by the phosphate of ammonium direct stimulation produces, as a rule, a tolerably good, though prolonged contraction, but the failure of reaction to direct and indirect stimulation is more parallel than in poisoning by the iodide, and if the irritability of the nerve is entirely lost, it is usually found that the muscle when stimulated directly contracts but very feebly even to the strongest tetanising current. Ammonium sulphate paralyses both muscle and nerve. The reactions given by the former are, however, longer, and outlast those of the latter. The tetanus curve of both is feeble, even in cases of rapid poisoning.

Action of Compound Ammonias.

Our experiments with these bodies were made upon frogs, rats, and rabbits. The substances employed, twenty-six in number, were:—Ethylamine, trimethylamine, triethylamine; the chlorides of methyl-ammonium, ethyl-ammonium, amyl-ammonium, dimethyl-ammonium, diethyl-ammonium, trimethyl-ammonium, and triethyl-ammonium; the iodides of methyl-ammonium, ethyl-ammonium, amyl-ammonium, dimethyl-ammonium, diethyl-ammonium, trimethyl-ammonium, triethyl-ammonium, tetramethyl-ammonium, and tetraethyl-ammonium; the sulphates of methyl-ammonium, ethyl-ammonium, amyl-ammonium, dimethyl-ammonium, diethyl-ammonium, trimethyl-ammonium, and triethyl-ammonium. The action of all these bodies was tested in Frogs, but the whole of the series was not investigated in Rats and Rabbits. All the salts of the compound ammonias which we used were obtained from Messrs. Hopkins and Williams, who prepared them expressly for us, and guaranteed their purity. The poison was in all cases administered by subcutaneous injection.

We have compared first the action of the compound ammonias, uncombined with an acid radical, with the action of ammonia itself. We have then compared the actions of the chlorides, iodides, and sulphates, of the compound ammonias with each other, and with the corresponding salts of ammonium. It will be noticed that there is a considerable difference between the action of the compound ammonias and of ammonia. The tendency to produce tetanus resembling that of ammonia was noticed in ethylamine, which was the only one of the compound ammonias containing only one atom of hydrogen, replaced by a radical, that we investigated in a free state, uncombined with acid. When used as a chloride, the convulsive action was less marked. The substitution of even a single atom of hydrogen by an alcohol radical appears to

lessen the tetanising action of ammonia, and this diminution is increased by the substitution of two or three atoms, then a change takes place, and when the ammonia is combined with four atoms of an alcohol radical, a convulsant action again becomes more marked, though it is not so great as in the case of ammonia itself.

With these exceptions, the symptoms were those of gradual motor paralysis. This motor paralysis appeared to us to be due, in a great measure, to a paralyzing action of the substance on the spinal cord, as motion ceased in the animal at a time when the muscles and motor nerves were still capable of vigorous action.

The tetramethyl- and tetraethyl-ammonias appear to have a particular tendency to paralyse the higher reflexes before the lower, so that reflex from the cornea disappears sooner than from the foot. They appear also to affect the heart more than the other compound ammonias, so that in poisoning by them the heart was generally found motionless, in complete diastole, and distended with dark blood.

We did not observe the same marked difference between the action of the different salts of the compound ammonias that we did in the case of ammonia itself. The iodides, however, appear to affect the heart more powerfully than other salts, and to cause its arrest in diastole.

The chlorides and sulphates also appear to have a greater tendency to produce muscular tremor than other salts.

We have drawn up, in a tabular form, an epitome of the symptoms of poisoning produced by salts of the compound ammonias in Frogs, Rabbits, and Rats. The tables may appear bulky, but the number of salts experimented upon was great, and as they were difficult to prepare, and expensive to procure, we have thought it advisable to give an example of the general action of each drug, as well as a summary of the results which we have obtained. We have, however, put them as shortly as possible, and restricted ourselves to one experiment with each substance on each kind of animal.

ACTION of Simple and Compound Ammonias on Frogs.

Substance.	Dose.	Symptoms.	Post-Mortem Appearances and Reactions.
(*a*) Ammonium bromide . . .	·025 to ·05 grm.	In some cases hyperæsthesia, followed by spasmodic twitchings, succeeded by well marked tetanic spasm. The last symptom is late in appearing, $1\frac{1}{2}^{h}$ to 2^{h} 20^{m}. Heart usually active throughout.	Heart is usually beating. There are many coagulations in red corpuscles. In early stage of poisoning there is exalted irritability of poisoned as compared with non-poisoned muscle. Later, the M.* and N. curve of the poisoned limb are both longer, their response to tetanic current more feeble. Eventually this condition increases to total failure of reaction of the nerve. This occurs when the muscle still reacts to direct stimulation, but its performance is weaker, especially its tetanic curve, and a stronger shock is needed to excite its activity than that needed for the normal muscle. The paralysis is not due to exhaustion from tetanus, as it occurs when tetanus has not been a prominent symptom. The change of corpuscles and the continued action of the heart are constant.
(*b*) Ammonium iodide	·025 to ·05 grm.	Higher reflexes quickly disappear. Limb reflexes persist longer, and require stronger stimulation to elicit them. There is increasing sluggishness, but no spasm; in fact, a striking absence of nervous symptoms.	Heart is usually at rest in diastole, containing dark blood. It is unusual to see coagulations in red corpuscles. It seems doubtful whether muscular irritability is increased as a temporary condition. If the heart has not been arrested too soon by the extent of the dose, the nerve soon becomes deeply affected by the poison, and only yields two or three faint responses to the strongest stimulation. Muscle-curve, though prolonged, is not, as a rule, extensively impaired. Frequently the muscle contracts well when irritated directly, either by a single shock or tetanic irritation, when the nerve refuses to react to the strongest stimulation.
(*c*) Ammonium phosphate . . .	·04 to ·075 grm.	Increasing sluggishness, with gradual failure of reflex. Sprawling movements on stimulation, but the animal tends to remain plastic. There is an entire absence of active nervous symptoms, except occasionally a slight twitching in drawing up leg before reflex manifestation has ceased.	In any but cases of very slow poisoning, heart is found in diastolic arrest and full of dark blood. The red corpuscles do not usually show coagulations. There may be a slight and transitory increase of irritability in early stage of poisoning. Nerve and muscle both tend to fail on poisoned side, and that pretty equally. In two cases the nerve did not respond at all, and the muscle only by very feeble curve to strongest tetanus. Heart still contracted feebly in answer to stimulation.
(*d*) Ammonium sulphate . . .	·025 to ·05 grm.	Movements soon accompanied by twitchings, and in 30^{m} to 40^{m} clonic spasm becomes established. A lasting tetanus is decidedly rare, but yet occurs. Clonic spasm commences in arms and chest. There may be 5 to 7 of these short spasms in the minute.	Heart frequently in diastolic still-stand, with very dark blood. Red corpuscles of blood only very occasionally show slight coagulations. The muscle and nerve appear both to be paralysed, the nerve, however, giving way first. The tetanus curve is feeble in both, even in cases of rapid poisoning. If the sciatic be cut so as to prevent exhaustion from spasm, the reaction of muscles from the two limbs is equal.

* M. stands for direct stimulation of the muscle itself, N. for indirect stimulation of the muscle through its nerve.

ACTION of Simple and Compound Ammonias on Frogs (continued).

Substance.	Dose.	Symptoms.	Post-Mortem Appearances and Reactions.
1. Ethylamine	About ·06 grm.	Circulation becomes slow. Reflex slow apparently, then spasm chiefly in pectorals and abdominal muscles is noticed, and this develops into a well marked and general tetanus.	Heart beating. Irritability of N.* and M. increased as compared with ligatured limb. Thus strong tetanus of poisoned at 6 centims. induction coil, while that of ligatured limb was only partially developed. Contracture tends to increase rapidly on repeated stimulation. In another case curve of N. and M. prolonged. N. soon gives way in tetanus.
2. Trimethylamine	About ·05 grm.	Gradual failure of circulation and of reflex. Pigment cells dilated. No spasm of a tetanic character, though some clonic contractions towards the end of the poisoning.	Heart in diastolic still-stand, and containing dark blood. Both M. and N. yield strong contraction and tetanus. The tetanus of the N., however, soon tends to clonus.
3. Triethylamine	About ·10 grm.	Slowing of circulation; dilatation of pigment cells of web. Gradual failure of reflex, unaccompanied by spasm.	Heart beating feebly, with tendency to diastolic arrest. Blood smells strongly of triethylamine. Both M. and N. cause contraction freely. Poisoned muscle and nerve are rather more irritable than the ligatured. Tetanus is more extensive at first, but soon tends to give way. There is increased viscosity from M. on repeatedly stimulating by induction shocks.
4. Methyl-ammonium chloride .	2·3 gtt. of a saturated solution.	Gradual failure of circulation and of reflex, generally unaccompanied by spasm. Reflex of lower limbs is longer preserved.	Heart tending to diastole, or in diastolic arrest, and containing dark blood. M. yields a slightly prolonged curve. Nerve, as a rule, reacts but faintly to strongest stimulation. The tetanus of N. is imperfect and very feeble.
5. Ethyl-ammonium chloride .	About ·03 grm.	Reflex becomes gradual, slow, and spasmodic, though there is no genuine tetanus. Just before its abolition pinching of toe causes a general though faint reflex of all body, without withdrawal of foot. Circulation continues moderately good.	Heart beating. Muscle yields a good curve, which shows, however, rapidly increasing contracture on repeatedly stimulating. The tetanic (M.) curve is also good, though followed by very slow relaxation. On strong stimulation there was no N., though strong tetanising current caused a very feeble and transient contraction.
6. Amyl-ammonium chloride ..	3 gtt. sat. sol.	Excitement at first, with acceleration of heart. Lower reflexes persist. There is no spasm. Higher reflexes soon lost.	Heart beating in vermicular manner, does not empty itself. Curve from M. tends to shorten rapidly on repeated stimulation. Tetanus curve strong, though not equal to normal. N. yields only a few contractions to strongest stimulation. Its tetanus is feeble, and rapidly diminishes.
7. Dimethyl-ammonium chloride	·1 grm.	Becomes tremulous and weak. Gradual failure of reflex.	Heart beating. Curve of M. and N. lengthened and higher, and tends to become elongated more rapidly than normal.
8. Diethyl-ammonium chloride .	·05 grm.	Increasing lethargy with failure of reflex. At last slight tremor of foot and of all body occurs on irritating foot.	Heart in diastolic still-stand. Beats with vermicular movement if irritated. Blood dark. Red corpuscles show considerable coagulations, whilst none are to be seen in ligatured limb. Curves of both M. and N. prolonged, and especially N. soon fails. Tetanus of N. soon collapses.

* M. stands for direct stimulation of the muscle itself, N. for indirect stimulation of the muscle through its nerve.

ACTION of Simple and Compound Ammonias on Frogs (continued).

Substance.	Dose.	Symptoms.	Post-Mortem Appearances and Reactions.
9. Trimethyl-ammonium chloride	·03 grm.	Circulation good. No spasm. Increasing lethargy.	Heart beating. Curve of both M.* and N. more extensive and longer than normal.
10. Triethyl-ammonium chloride .	·125 grm.	Circulation continues good. No spasm. Attempt to spring, but no force in the movement. Gradual loss of power and reflex manifestations.	Heart beating. No abnormality. Muscle and nerve more irritable to minimal stimulation. The curve, though strong at first, rapidly falls on repetition, and lengthens considerably.
11. Methyl-ammonium iodide . .	·15 grm.	Increasing torpor. Heart slowed.	Heart beating slowly. N. and M. more irritable. Give ordinary curve.
12. Ethyl-ammonium iodide . .	·15 grm.	Increasing sluggishness. Croaking reflex. Respiration and heart good.	Heart beating. N. and M. more irritable. Give ordinary curve. In another case total paralysis of N. with prolonged and humped curve.
13. Amyl-ammonium iodide . .	·15 to ·2 grm.	Circulation tends to slow. Leg drawn up with jerking (staccato). Becomes difficult to excite reflex.	Heart beating slowly. Some twitching of leg still. Muscle and nerve irritability is increased in relation to the other limb. N. gives a feeble contraction, but with double hump. Muscle gives an extensive contraction, much longer than normal, and with distinct double hump. It is of normal altitude.
14. Dimethyl-ammonium iodide .	·1 grm.	Increasing apathy. No spasm.	N. soon fails in tetanus. Both M. and N. curve prolonged.
15. Diethyl-ammonium iodide .	·2 grm.	Circulation good. Croaking reflex. Tends to lethargy. No spasm. Colour dark.	Heart beating slowly. Minimal irritability of N. and M. distinctly increased. Single curve of each like normal, but on stimulation being repeated a distinct double hump occurs in both cases, with rapid lengthening of the curve.
16. Trimethyl-ammonium iodide .	·15 grm.	Becomes gradually sluggish. Movements more short, feeble, and staccato.	Heart full of dark blood—still-stand. Red corpuscles show coagulation to some extent, the blood of ligatured leg being normal. Nerve gives curve which rapidly falls to insignificant proportions, becoming humped. M. gives a strong contraction, with a most distinct hump. On repeated stimulation the muscle shortens rapidly.
17. Triethyl-ammonium iodide .	·1 grm.	Increasing sluggishness and gradual loss of reflex.	Heart beating slowly, with vermicular movement. Red corpuscles show slight coagulations. Irritability of M. and N. diminished. Both yield a longer lower curve, rapidly tending to fall. Tetanus of N. is very weak.
18. Tetramethyl-ammonium iodide	·007 to ·028 grm.	Spasmodic twitchings of trunk and leg muscles. Limbs drawn up in very tremulous manner. Whole body twitches in reponse to pinching foot, even when the foot is no longer withdrawn. Death within a minute with larger dose.	When all reflex has ceased but faint tremor, the N. may still respond well.

* M. stands for direct stimulation of the muscle itself, N. for indirect stimulation of the muscle through its nerve.

ACTION of Simple and Compound Ammonias on Frogs (continued).

Substance.	Dose.	Symptoms.	Post-Mortem Appearances and Reactions.
19. Tetraethyl-ammonium iodide .	·007 to ·028 grm.	As in tetramethyl-ammonium iodide.	Heart in diastolic still-stand, engorged with dark blood. The M.* yields usually a good curve to *powerful* stimulation. This tends to lengthen rapidly on repeating stimulation. In one case the muscle was almost completely paralysed. The nerve, when heart has not been too soon arrested, is profoundly paralysed.
20. Methyl-ammonium sulphate .	·3 grm.	Slowing of circulation. Increasing lethargy. Reflex falls to faint tremble without withdrawal of leg.	Heart beating. Red corpuscles almost all disappeared. Nuclei free. N. gives no response to strongest tetanus. M. still irritable, gives a low prolonged curve. It gives a tetanus, at first firm, but soon collapsing.
21. Ethyl-ammonium sulphate . .	·2 grm.	Slowing of circulation. Increasing sluggishness. Slow answer to stimulation by reflex movement. Reflex gradually reduced to tremor.	Heart beating slowly (12 per min.). Both systole and diastole slow. Minimal irritability of M. and N. greater. Curves tend to lengthen rapidly. Both give strong continuous tracing.
22. Amyl-ammonium sulphate . .	·1 grm.	Slowing of circulation. Increasing sluggishness. Leg drawn up tremblingly and incompletely. Leg reflex ceases.	Heart beating 24, feebly. Diastole long, and systole slow and stiff. Minimal irritability of nerve much diminished. N. yields good curve to powerful stimulation, but it tends to fall rapidly on repeating stimulation. Muscle gives a strong curve, with rapidly increasing tonus. Continuous curve of M. is much shorter than that of normal muscle.
23. Dimethyl-ammonium sulphate .	·25 grm.	Heart's activity impaired. Tremulousness and weakness increase. All reflex eventually ceases in leg.	Heart beating slowly. Coagulation in all red corpuscles, and some free nuclei. No response of N. to strongest tetanus. Muscular irritability impaired. Curve is long and low, and its tetanus is very feeble, whilst that of ligatured limb is very extensive.
24. Diethyl-ammonium sulphate .	·25 grm.	Increasing torpor, but if sufficiently excited gives strong extension with long latency. Slightly tremulous.	Heart beating slowly. Red corpuscles full of coagulations. Poisoned N. and M. more irritable. On continuous tetanus N. tends to pass into clonus.
25. Trimethyl-ammonium sulphate	·25 grm.	Tremulousness develops. All movements become rapid, tremulous, and uncertain. Leg at last drawn up by a sharp and unsustained contraction, or by a series of twitches.	Heart beating well. Red corpuscles all show coagulations, many free nuclei. Poisoned leg gives good curves, though minimal irritability is much impaired. In tetanus, however, nerve soon gives way and passes into clonus.
26. Triethyl-ammonium sulphate .	·2 grm.	Irritability appears at first to be increased. Spring becomes tremulous, staccato and weaker. Spasmodic movements continue some time after stimulation. At last stimulation of foot causes slight trembling of foot and whole body, but no withdrawal of foot.	Heart beating slowly, much engorged, scarcely empties itself at all. Red cells all contain coagulations. There is no change in minimal irritability, and none in curve, except that there is tendency for altitude to fall in case of poisoned. N. tetanus is also weaker. In another case total paralysis of nerve.

* M. stands for direct stimulation of the muscle itself, N. for indirect stimulation of the muscle through its nerve.

Action of Compound Ammonias on Rats.

Solution Injected.	Quantity gradually injected into Abdominal Cavity.	Symptoms in Brief.	Ultimate Result.	Post-Mortem Appearances and Reactions.
1. Diethyl-ammonium chloride .	·5 grm.	Walk becomes straddling and ataxic, movements become jerking and tremulous. Attacks of shaking provoked by movement. Raps with lower jaw on table, and extensors jerk back head in a clonic manner. Dyspnœa, laboured breathing. Runs backwards. Facial muscles spasmodically contracted.	Death.	Heart beating. Right side of heart full.
2. Triethyl-ammonium chloride .	·6 grm.	Listless. Occasionally raises itself in a spasmodic manner. Slight tremulousness. Rapid flexure of head, causing rapping on table. Tremor becomes general. Springs upwards and backwards. Spasm more violent. Some dyspnœa. No symptom of pain.	Death.	Heart beating. Right side of heart full. Left empty.
3. Methyl-ammonium iodide . .	·2 grm.	Quiet. Waddles in walk. Movements sprawling. Somewhat anæsthetic. Progress slow and wavering. Hind limbs much paralysed. Tail sometimes involuntarity thrown up. Occasional twitching of tail and limbs, but no general convulsion. General body reflex from stimulation of hind limbs, but hardly in them.	Death.	Heart contains some blood. Mesenteric vessels somewhat dilated. Stimulation of nerve gives hardly any contraction, but direct stimulation gives free contraction.
4. Ethyl-ammonium iodide . .	·5 grm.	Apathetic, increasing weakness. Power soon begins to fail in hind limbs; tends to fall over on side when walking. Respiration slow. No spasm. Withdrawal of fore feet if hind touched, but not of latter. Reflex only in fore part of body.	Death.	Heart and lungs normal. Mesenteric vessels not dilated.
5. Amyl-ammonium iodide . .	m.	Twitchings of hind legs, with occasional active extension. Head rapidly flexed and extended, so that jaw raps on table. Movements sprawling. Dyspnœa. Rests on belly. Falls on side. Spasm of legs and body, and death. There was no true tetanus.	Death.	Heart contains little blood in left side. Right somewhat congested. Rigor early and strong. No mesenteric inflammation.
6. Dimethyl-ammonium iodide .	·5 grm.	Torpidity. Sways after movement. Falls over on side repeatedly when walking. No spasm. No anæsthesia. Gradual failure and death.	Death.	Right side of heart full. Left empty. No mesenteric inflammation.

Action of Compound Ammonias on Rats (continued).

Solution Injected.	Quantity gradually injected into Abdominal Cavity.	Symptoms in Brief.	Ultimate Result.	Post-Mortem Appearances and Reactions.
7. Diethyl-ammonium iodide . .	·4 grm.	Gait becomes feeble and wavering. Sways when trying to sit up. Scrambles with feet in order to preserve balance. An occasional instantaneous twitch in back and fore limbs observed, which gives the impression of a hiccough. Lies prone on belly. Spasmodic twitchings, 8 or 10 per minute, occur.	Killed.	Heart normal. There is no great change in excitability of muscle to direct or indirect stimulation.
8. Tetramethyl-ammonium iodide	·2 grm.	At once powerful convulsions (no tetanus). Dyspnœa. Falls on side and dies (action very rapid). A much smaller dose fatal in other cases.	Death.	Brain not markedly congested. Abdominal vessels dilated.
9. Ethyl-ammonium sulphate . .	·5 grm.	Torpid. An occasional heaving of body (like hiccough) and throwing up of head. Lachrymation. Movements gradually become tremulous. Rocks when walking. Breathing accelerated. No spasm. No anæsthesia. Ceases to move at all, except when disturbed.	Killed.	Right heart full. Left empty.
10. Amyl-ammonium sulphate . .	·15 grm.	Walking slightly tremulous. Soon violent trembling of head and fore part of body, increased on movement. Dyspnœa. Lies on belly. Anæsthesia. Scrambling with feet to maintain equilibrium. Runs forward rapidly, and stops short. Gait like paralysis agitans. Powerful general convulsion, extension of legs, and jumping from side to side. Runs backwards. Falls on side.	Death.	Heart beating 60 per 1′. Both muscle and nerve respond well and equally to induction shock. Some congestion of brain, of membranes of cord, and of cord itself.
11. Diethyl-ammonium sulphate .	·5 grm.	Movements straggling, tremulous, like paralysis agitans. Rocks from side to side. Flexion and extension of head; "rapping" develops and become frequent, often accompanied by starting of all body. Rises with difficulty if laid on side. Rapping ceases. Respiration becomes very feeble. Reflex gradually lost.	Death.	Muscle and nerve are both irritable. Heart normal. No peritonitis.

ACTION of Simple and Compound Ammonias on Rabbits.

Solution Injected.	Quantity.	Symptoms, with Notes of Time of their Occurrence in Brief.	Ultimate Result.	Post-Mortem Appearances.
1. Dimethyl-ammonium chloride.	1 grm.	In 35^m lies on belly. Breathing slow. 1^h is approaching normal, but is weak, and rocks if moved. Found dead in morning.	Died.	Kidneys are much congested. There is a good deal of yellowish viscid fluid in the intestines. Bladder is full of a grumous, deeply-coloured urine, containing much white floculent matter. Brain and cord are congested. Liver is much congested. Death may perhaps be attributed primarily to the ailing condition of the animal. The symptoms due to the poison were very slight, the chief one being increasing weakness.
2. Diethyl-ammonium chloride.	1 grm.	In 25^m movements of legs rather staccato, but soon assume normal character.	Recovered.	
3. Trimethyl-ammonium chloride	1 grm.	In 25^m lies with fore legs extended. Head rocks if touched. Much salivation, which continues all through. Pupil contracted. 1^h head tends to fall on table. $2^h\ 5^m$ lies with fore legs fully extended on either side. $3^h\ 35^m$ lies on side with legs in all directions. Tries to escape if approached. No anæsthesia. Reflex almost lost in hind limbs. $5^h\ 35^m$ reflex in all limbs almost entirely lost. There has been no true spasm in this case.	Died.	There is no peritonitis. No unusual amount of fluid in peritoneal cavity. Right heart full; left empty. Lungs and kidneys normal. Liver congested. Brain and cord not congested.
4. Triethyl-ammonium chloride.	1 grm.	In 1^h restless. Scrambling with fore feet. Movements staccato. Trembles if touched. $2^h\ 20^m$ a violent attack of scrambling movements. If allowed to run, movements are tremulous and associated with much involuntary movement. 4^h head down, rump up. No anæsthesia. Drumming movements with fore limbs. Convulsive movements appear whenever movement is attempted. The strength of the animal never seems much impaired. 7^h decided improvement in condition.	Recovered.	
5. Triethyl-ammonium iodide.	2 grms.	$1^h\ 3^m$ has slight tremor. $1^h\ 10^m$ tremor very distinct on provoking movement. Hind legs are thrown out in hopping in a tremulous manner. $2^h\ 13^m$ there is now no tremor or weakness when animal is taken up.	Recovered.	
6. Ethyl-ammonium iodide.	2 grms.	The second injection of 1 grm. was $2^h\ 12^m$ after the first. After the first injection 22^m drowsy. Sits in normal position, with an occasional slight tremor. 50^m trembles on being taken up, and when set down again. $1^h\ 53^m$ tremor has ceased. Injected 1 grm. Increasing apathy, ceases to notice other rabbits. $1^h 25^m$ after second injection appears somewhat anæsthetic. $2^h\ 5^m$ remains sitting perfectly still if placed on floor. Shakes if compelled to move.	Died.	Brain and its membranes not markedly congested. Right heart full; left empty. Intestines full of a brown fluid, with fœcal odour. No peritonitis.

ACTION of Simple and Compound Ammonias on Rabbits (continued).

Solution Injected.	Quantity.	Symptoms, with Notes of Time of their Occurrence in Brief.	Ultimate Result.	Post-Mortem Appearances.
7. Amyl-ammonium iodide . .	1 grm.	In 15^m there is distinct trembling of all body. 1^h tends to sink upon belly, but soon recovers itself. Hind legs specially affected. 1^h 30^m lies several seconds with legs fully extended. In 2^h 20^m drumming and scrambling movements. Cannot remain more than 2^s or 3^s in sitting posture. No anæsthesia. 3^h 35^m improvement commencing.	Recovered.	
Do. do. . . .	1·8 grm.	42^m cannot sit up. Hind legs almost powerless. Has a good deal of tremor in fore limbs. 1^h 50^m pupils much dilated. Looks collapsed. Head tends to fall forwards. Heart and respiration regular. Never attempts to move. 2^h tends to roll on to side, but can still regain belly position. Shortly after died.	Died.	Next day. Brain appears rather congested, but cord does not. Intestines and stomach full. Right heart dilated; left empty. No congestion of kidney. Slight congestion of liver.
8. Dimethyl-ammonium iodide .	1·6–1 grm.	50^m after injection violent shivering. Almost convulsive when movement is attempted. 70^m attempts to remain sitting, only succeeds for a few seconds ($\frac{1}{2}^m$). 76^m respiration feeble. Struggles now and then, otherwise quiet. 85^m corneal reflex persists, but otherwise almost completely paralysed. 91^m corneal reflex gone. Struggles. Death occurs within an hour from this time.	Died.	
9. Diethyl-ammonium iodide .	1 grm.	In 20^m appears weaker on legs, which are occasionally stretched out behind it. It is, however, able to sit up. 45^m the stretching out of legs seems to be spasmodic in character, but this may be due to paralysis having advanced further in flexors than extensors. The animal, however, still endeavours to restore leg to normal position. In 2^h 5^m appears stronger. Able to sit up better. There has never been any anæsthesia.	Recovered.	
10. Trimethyl-ammonium iodide .	2 grms.	First injected 1 grm. Second injection was 2^h 45^m after first. In 13^m lies on belly. Hind legs specially weak. 1^h 9^m getting stronger. 29^m after second injection is apathetic, and trembles when taken up. Does not appear anæsthetic, and runs if placed on floor, and excited. Position normal.	Recovered.	
11. Triethyl-ammonium iodide .	2 grms.	No symptoms but of weakness.		
12. Tetramethyl-ammonium iodide	1 grm.	Immediately shrieks. Salivation profuse. 2^m lies apparently paralysed. 4^m heart still beating. Cornea insensible.	Died.	Lungs not congested. Liver congested.
	·2 grm.	Recovered.	Recovered.	
	·5 grm.	15^s no shriek. Salivation profuse. 2$\frac{1}{2}^m$ head falls on one side. 4^m respiration ceased. Heart beating. Movement of limbs very violent. 4$\frac{1}{2}^m$ dead.	Died.	Liver congested. Brain and cord. Heart beat a long time after thorax opened.

ACTION of Simple and Compound Ammonias on Rabbits (continued).

Solution Injected.	Quantity.	Symptoms, with Notes of Time of their Occurrence in Brief.	Ultimate Result.	Post-Mortem Appearances.
13. Tetraethyl-ammonium iodide .	1 grm.	In 11^m animal trembles and shivers. Head tends to drop. 20^m head and forequarters much paralysed. Corneal reflex still present. Reflex still in legs. 23^m respiration ceases. Corneal reflex still good. Eye protruded. 26^m nostrils still twitch, and heart beats. 36^m dead.	Died.	No congestion of intestines. Heart still contracted.
Do. do. . . .	·5 grm.	In 21^m lies with head on table. Shaking from side to side. Head tends to fall on one side. 26^m convulsive springing, lurching, and shuffling. Dyspnœa. Clonic spasm of jaw. 30^m death.	Died.	Brain and cord; kidneys and liver congested.
14. Methyl-ammonium sulphate .	·7 grm.	No symptoms.	Recovered.	
	1·9 grm.	No symptoms.	Recovered.	
15. Ethyl-ammonium sulphate . .	2·0 grm.	Did not vary from normal, except in 25^m the hind legs appeared rather weak.	Recovered.	
	2·0 grm.	No symptoms.	Recovered.	
16. Amyl-ammonium sulphate . .	2·0 grm.	29^m respiration accelerated. Legs slip from beneath it. 63^m sinks on belly, but can rise again. 70^m lies on belly with legs extended behind it, but can still take a few springs if excited. 95^m effect of poison passing off.	Recovered.	
17. Dimethyl-ammonium sulphate	·7 grm.	No symptoms.	Recovered.	
	2·0 grm.	In 10^m sinks on belly on table, cannot rise. 62^m starts when touched. 3^h 50^m begins to run.	Recovered.	
18. Diethyl-ammonium sulphate .	·2 grm.	In 35^m lies with legs extended behind it, but can easily rise.	Recovered.	
	2·0 grm.	No symptoms.	Recovered.	
19. Trimethyl-ammonium sulphate	·7 grm.	For 2^h 16^m after completion of injection lies flat on belly.	Recovered.	
	2·0 grm.	In 12^m tends to lie with legs out behind, but can sit up. Profuse salivation. 51^m hind legs are quite paralyzed. Respiration snoring. Lies flat on side. 1^h 26^m raises head on noise. 3^h 15^m died. Corneal reflex to the last.	Died.	
20. Triethyl-ammonium sulphate .	·2 grm.	26^m lies down on belly, then rises, but soon lies again. 1^h 57^m lies with legs extended behind it.	Recovered.	
	2·0 grm.	12^m tremors in fore paws. Convulsive shudder. 17^m reflex gone. Respiration ceases.	Died.	

RABBITS, Tabulation of Results.

Fatal.

10 c.c. or less.*	Fatal in—	20 c.c. or above 10 c.c.	Fatal in—
10 c.c. Dimethyl-ammon. chloride.	Several hours.	20 c.c. Triethyl-ammon. sulphate.	17^m.
10 c.c. Trimethyl-ammon. chloride	A few hours.	20 c.c. Trimethyl-ammon. sulphate	$3^h\ 15^s$.
10 c.c. Biethyl-ammon. iodide. . .	$2^h\ 30^m$.	20 c.c. Ethyl-ammon. iodide . .	Several hours.
10 c.c. Tetraethyl-ammon. iodide.	$2^h\ 30^m$.	18 c.c. Amyl-ammon. iodide. . .	3^h.
10 c.c. Tetramethyl-ammon. iodide	$2^h\ 4^m$.		
5 c.c. Tetramethyl-ammon. iodide.	5^m.		
5 c.c. Tetraethyl-ammon. iodide .	30^m.		

Not Fatal.

With Maximal Dose recovered from—		
2 c.c.	10 c.c. and more than 2 c.c.	20 c.c. and more than 10 c.c.
Methyl-ammon. sulphate.	7 c.c. Trimethyl-ammon. sulphate.	16 c.c. Triethyl-ammon. iodide.
	10 c.c. Ethyl-ammon. iodide.	20 c.c. Amyl-ammon. sulphate.
	10 c.c. Triethyl-ammon. iodide.	20 c.c. Ethyl-ammon. sulphate.
	10 c.c. Diethyl-ammon. chloride.	20 c.c. Diethyl-ammon. sulphate.
	10 c.c. Triethyl-ammon. chloride.	20 c.c. Dimethyl-ammon. sulphate.
	10 c.c. Amyl-ammon. iodide.	19 c.c. Methyl-ammon. sulphate.
	10 c.c. Diethyl-ammon. iodide.	20 c.c. Methyl-ammon. iodide.
		20 c.c. Dimethyl-ammon. iodide.
		17 c.c. Trimethyl-ammon. iodide.
		17 c.c. Diethyl-ammon. chloride.

The order of fatality considering—

I. The salt.
1. Iodides.
2. Chlorides.
3. Sulphates.

II. The ammonia compound.
1. The tetraethyls and tetramethyls.
2. The triethyls and trimethyls.
3. The diethyls and dimethyls.
4. The amyls, ethyls, and methyls.

appears to be:—

The former (I.) is of very secondary importance to the latter, and the difference between the iodides, and chlorides, and sulphates is magnified by the fact that in the case of the iodides alone were the *tetra* compounds employed.

In regard to rapidity of action, we find (1) tetramethyl-ammonium-iodide (5 c.c. = ·5 gr.) fatal in 5^m; (2) triethyl-ammonium sulphate (20 c.c.) in 17^m, and tetraethyl-ammonium-iodide (5 c.c.) in 30^m. No symptom of pain occurred in any case after the injection, nor of physical change in animal. There appeared occasionally a slight loss of co-ordination, but this may have been, in some cases, due to paralysis. The pupil was markedly affected in the case of trimethyl-ammonium chloride and tetramethyl-ammonium-iodide.

* Each c.c. is equal to ·1 grm. of the substance named.

From these experiments it appears that amongst the drugs contained in the table, the most marked disturbance occurs in the Rat in the case of the ethyl-ammonium sulphate, amyl-ammonium sulphate, amyl-ammonium iodide, diethyl-ammonium-sulphate, diethyl-ammonium chloride, triethyl-ammonium chloride, and tetra-methylamine ammonium iodide. In all of these tremors were noticed, and in some—the diethyl-, triethyl-, and amyl-ammonium salts—a peculiar rapping of the head upon the table was noticed, which appeared to be of a convulsive character.

The two most powerful convulsants were the amyl-ammonium-sulphate, and the tetramethyl-ammonium-iodide.

We found that the iodides not enumerated amongst those causing marked nervous disturbance have little tendency to produce spasmodic movements. In them loss of reflex, first in the hind legs, and then in the anterior part of the body, is most marked.

It appears to us that as a group the salts of the compound ammonias have a complex action: they affect the spinal cord, motor nerves, and muscles. The extent to which these structures are affected by the different compounds varies with each compound.

The spinal cord appears to be first stimulated, and then paralyzed. The symptoms which lead us to suppose that it is first stimulated are the twitchings which occur in the early stage in Rabbits and Rats, when poisoned with the substances mentioned in the tables, and the convulsions which occur in Frogs poisoned by ethylamine and tetraethylamine-iodide. That the spinal cord is paralyzed at a later stage, both as a conductor of motor stimuli and as a reflex centre, we infer from the failure of reflex action both in Frogs and Mammals, and from the fact that a stimulus applied to the hind foot frequently induces motion, not of the corresponding hind leg, but of one of the fore legs.

The convulsions which occur shortly before death in mammals are, perhaps, to be regarded as due, not to the irritant action of the poison on the nerve centres, but rather possibly to its paralyzing action on the motor nerves: this motor paralysis causes enfeebled breathing, and a consequent venous condition of the blood with asphyxial convulsions. That the compound ammonias and their salts paralyze the motor nerves is shown by our direct experiments on the nerve muscle preparation, in which the nerves were almost always paralyzed before the muscle. The muscles, however, are by no means unaffected—at first their power may seem to be increased, so that they respond by a more powerful contraction to irritation; afterwards, however, they become weakened, and tend to become completely paralyzed by the continued action of the poisons. This increase of irritability is not observed in the case of some of the compounds, even as a temporary condition.

Comparison between the Action of Ammonia and the Compound Ammonias.

Ammonia itself has a convulsant action, the convulsions apparently being due to its effect upon the spinal cord, like those of strychnia. It differs, however, from strychnia in this respect, that the convulsions do not continue long, apparently because the motor nerves soon become exhausted, so that the excited spinal cord can no longer induce muscular contractions. The only one of the compound ammonias, in which one atom of hydrogen only is replaced by an alcohol radical, that we have experimented with is ethylamine; and this we find has also a convulsive action, probably the same in nature as that of ammonia. It has but a feeble paralyzing action on motor nerves. But this is only true of a single stimulus or of a few stimuli. When the nerve is subjected to rapidly repeated stimulation, it becomes very quickly exhausted. Ethylamine, therefore, while not directly paralyzing the excitability of the nerve, greatly lessens its endurance and power of work. It will thus have a similar effect to ammonia in shortening the convulsions, and thus rendering them like those of ammonia, and unlike those of strychnia. Its action on muscle itself appears to be very similar to that of ammonia. First it increases the excitability of the muscle, but afterwards diminishes it, and renders the curve both lower and longer.

Trimethylamine was found by Husemann to have a tetanising action even on Frogs, like that of ammonia. In our experiments, however, we found gradual failure of the circulation and of reflex without any spasm. This difference between his results and ours may be possibly due either to our having employed different kinds of Frogs or to our having experimented at different seasons and under different temperatures. Another possibility is, that the Frogs he employed were stronger, and that their circulation was more vigorous than ours: for we have already noted that ammonium bromide produced tetanus in Frogs, but this came on at a late period in the poisoning, and unless the Frog was strong, and the circulation vigorous, the animal died before the tetanus made its appearance.

With triethylamine we noticed a great failure of reflex, unaccompanied by spasm; with both triethylamine and trimethylamine the action appeared to be slower than that of ethylamine. In one case of poisoning by the latter, tonic spasm occurred in 70^m after injection, whilst in two hours after the injection of a larger quantity of triethylamine and trimethylamine, a faint reflex action was still present, and the circulation was maintained. The action of trimethylamine and triethylamine on motor nerves and muscle is very much the same as that of ethylamine or ammonia. From a comparison of ammonia with these compound ammonias it appears that the replacement of hydrogen by alcohol radicals tends to diminish the convulsant action of ammonia itself, and that the diminution is greater in proportion to the number of hydrogen atoms substituted.

We have not obtained any distinct evidence that the substitution of alcohol

radicals for hydrogen increases the paralyzing action of ammonia on motor nerves, or indeed alters its effect upon the muscle.

We shall presently have to notice, however, the marked change which occurs in physiological action, when we pass from an ammonia in which nitrogen is combined with three atoms of an alcohol radical to those in which we have it combined with four atoms as tetramethyl- and tetraethyl-ammonium iodides.

CHLORIDES.

Ammonium chloride has been shown by BOEHM and LANGE to produce convulsions resembling ammonia itself.

Amylamine hydrochlorate has been shown by DUJARDIN-BEAUMETZ to have a convulsant action upon Rabbits.

Our experiments on Frogs have led to the following results:—

We found that methylamine chloride caused gradual failure of reflex action generally unaccompanied by spasm, while the diminished reflex produced by ethylamine chloride was of a spasmodic nature, though there was no true tetanus. With amylamine chloride we observed no spasm. In one case there was a tendency to spasm chiefly in the hyoglossus muscle.

The dimethyl- and diethyl-ammonium chloride cause weakness, lethargy, and failure of reflex action, but no distinct spasm. A tremor is observed on movement, but this seems to be rather due to failure of motor nerves than to increased excitability of nerve centres.

Their action upon motor nerves and muscles appears to have been much the same as that of ethylamine: the nerve not being directly paralyzed, but its power of transmitting stimuli continuously being greatly diminished.

The muscle has at first its contractility increased but afterwards diminished (Plate 8, fig. 1, *a*, *b*, *c*, *d*). In these experiments on Frogs the chlorine does not appear to have altered the action of the compound ammonias with which it is combined.

From experiments on Rats we find that both diethyl- and triethyl- ammonium chlorides have a similar action. The most marked symptoms are motor weakness and tremor. The tremor is most perceptible when the animal moves, and there is a very curious spasmodic movement of the head causing the chin to rap upon the floor.

Before death, convulsions occur, but these are probably asphyxial.

In Rabbits the effect is somewhat similar. The movements become tremulous, are exaggerated and scrambling in character, suggestive of impaired co-ordination. No anæsthesia was observed. Reflex was lost gradually and disappeared, first in the hind limbs.

The most marked effect of the chlorine in altering the action of the compound ammonias appears in these experiments to be a tendency to produce tremor. It is perhaps not quite easy to say positively what the cause of this tremor is, but we are

inclined to regard it rather as an indication of failing power in motor nerves than to increased irritability in nerve centres.

Iodides.

As we have already shown in an earlier part of this paper, ammonium iodide has a powerful paralyzing action, both on nerve centres and motor nerves, producing sluggish movements and motor paralysis.

From experiments on Frogs we find that methyl- (Plate 8, fig. 2, *a*, *b*), ethyl-, and amyl- (Plate 8, fig. 3, *a*, *b*, *c*) ammonium iodides all produce torpor. In the ethyl-ammonium iodide, Goltz's "croak" experiment succeeded as it did in the case of simple ammonia iodide. With the amyl-ammonium iodide, jerking or staccato movement of the limbs was observed, apparently due to failure of motor power. The methyl-, ethyl-, and amyl-ammonium iodides in small doses increase the excitability both of nerve and muscle. In large doses they are powerful poisons to motor nerves; they have a tendency to alter the formation of the muscle curve, and produce in it a curious hump, but they do not appear to affect muscle as much as nerve.

The occurrence of the croak in the ethyl-ammonium iodide would appear to indicate rapid paralysis of the higher nerve centres; and the staccato movement in the amyl-ammonium iodide, more rapid failure of motor nerves.

The dimethyl- and diethyl-ammonium iodides produced increasing lethargy, with no spasm; with the diethyl-ammonium iodide the "croak" experiment succeeded, as it did with the ethyl-ammonium iodide.

Their action upon muscle and nerve seems to be similar to that of the methyl- and ethyl-ammonium iodides. Trimethyl- and triethyl-ammonium iodides have an action like that of the dimethyl- and diethyl-ammonium iodides, but they appear to have a greater paralyzing action on muscle and nerve (Plate 8, fig. 4, *a*, *b*, *c*), the primary increase in excitability not being marked, and paralysis of both occurring more readily. The tetramethyl- and tetraethyl- (Plate 8, fig. 5, *a*, *b*, *c*) ammonium iodides present a marked contrast to the other iodides, as Frogs poisoned by them exhibit spasmodic twitchings of the trunk and extremities. The higher reflexes cease very rapidly. The nerve is generally completely paralyzed. The muscle is only slightly affected when the poisoning is rapid, but if it be slow it is completely paralyzed also.

All the iodides render the beats of the heart slow, and tend to produce still-stand in diastole.

In the case of triethyl-ammonium iodide a vermicular movement of the heart was observed.

The tetraethyl- and tetramethyl-ammonium iodides appear to have a more powerful action than the others in producing diastolic still-stand of the heart.

Experiments on Rats.

Methyl-, ethyl-, and amyl-ammonium iodides all produce increasing weakness with a sprawling or waddling gait. The power of the cord to conduct motor impulses appears to be diminished so that the hind legs become more paralyzed than the fore legs. Its conducting power for sensory impressions is not paralyzed at this time, as stimulation of the hind legs will produce movement in the anterior part of the body. In the case of poisoning by amyl-ammonium iodide, twitching of the limbs and head were more marked than that of the methyl or ethyl compounds.

The dimethyl- and diethyl-ammonium compounds also cause progressive paralysis. In the case of the diethyl-ammonium iodide, an occasional instantaneous twitching in back and forelimbs was observed, resembling an effort at hiccough.

The tetramethyl-ammonium iodide has an action very different from the others, producing powerful convulsions. It kills also much more rapidly, and is fatal in very much smaller dose.

Experiments on Rabbits.

In Rabbits the methyl-, ethyl-, and amyl-ammonium iodides all cause increasing weakness. The conducting power of the cord appears here also to be affected, the hind legs becoming sooner paralyzed than the fore legs.

In the case of the methyl-ammonium iodide there is a distinct trembling of the body not noticed in the other two.

General Action of the Iodides.

A distinct alteration appears to be effected in the action of the compound ammonias by the combination with iodine. All the iodides, both of ammonia itself and the compound ammonias, have a powerful paralyzing action on the motor nerves. Muscular irritability is as a rule decreased; occasionally it is increased at first, as in the case of the methyl-, ethyl-, and amyl-ammonium iodides.

The muscle curve in all cases shows a tendency to become humped. This tendency is more marked in the methyl, ethyl, and amyl compounds than in the di- or trimethyl, ethyl, and amyl compounds. It is more marked when the muscle is stimulated directly than when it is stimulated through the nerve. They all render the muscle more easily exhausted, so that the tetanic curve becomes lower and is sustained for a shorter time.

SULPHATES.

Experiments on Frogs.

Ammonium sulphate soon causes the movements to be accompanied with twitchings and clonic spasm. It sometimes, though rarely, produces complete tetanus; the peripheral ends of motor nerves are paralyzed by it, and the muscular substance is also paralyzed, though later than the nerve.

The heart is considerably affected by the poison, and is frequently found arrested in diastole, and filled with dark blood. In this point it appears to agree with the iodide.

Methyl, ethyl, and amyl sulphates all cause gradually increasing lethargy and failure of reflex movement.

Methyl-ammonium sulphate paralyzes muscle and nerve very completely, the nerve being paralyzed before the muscle. The ethyl- and amyl-ammonium sulphates have much less paralyzing action upon muscle and nerve, but render them liable to rapid exhaustion.

In poisoning by them the heart was considerably affected, and beat very slowly; probably the slighter effect on the muscle of ethyl and amyl sulphates in our experiments was due to their greater effect upon the heart, so that they were carried in lesser quantity to the muscle. This is exactly what one finds with such a poison as veratrine, which has an extraordinary effect on the muscle of a Frog in small doses, but has little effect on the muscle when the dose is large, the heart being so quickly arrested that but little effect is produced upon the muscle.

Dimethyl- and diethyl-ammonium sulphate both cause weakness, with tremulous movement; but in the case of diethyl-ammonium sulphate, strong irritation causes a powerful movement in the limbs, occurring after a considerable latent period. The nerve appears to be powerfully paralyzed by the dimethyl-ammonium sulphate, while the paralyzing action is but slightly marked in the case of the diethyl-ammonium sulphate; the paralysis of the muscular tissue is also more marked in the case of the dimethyl-ammonium sulphate (Plate 8, fig. 6, *a*, *b*). Both lessen the activity of the circulation, and render the cardiac pulsations slow.

The trimethyl- and triethyl-ammonium sulphates both cause the movements to become weaker and tremulous, and sometimes staccato.

The trimethyl-ammonium sulphate (Plate 8, fig. 7, *a*, *b*, *c*, *d*) appears at first to increase the excitability of the animal, and even when the muscular power has failed, so that irritation of the foot no longer will cause it to be withdrawn, tremor occurs over the whole body from the stimulus. The nerve is either much weakened or paralyzed, so that it either soon gives way when tetanised, or does not respond to stimulus at all. The muscle is also paralyzed; the minimal irritability is much impaired in poisoning by trimethyl-ammonium sulphate, although the contractile power remains considerable.

One of the most marked points in the action of the sulphates of ammonia and

compound ammonias on the Frog appears to be their tendency to affect the circulation, and to render the beat of the heart slow, or arrest it entirely in diastole. Muscle and nerve are both paralyzed, the paralysis of the muscle being later than that of the nerve.

We have noted above a number of more or less exceptional instances, but in many of those there can be little doubt, we think, that the exceptional action was due to alteration in the circulation caused by the poison.

In their action upon the circulation the sulphates resemble the iodides. The spinal cord appears to be stimulated, so that convulsions or tetanus are produced by the ammonium sulphate. The combination with ethyl and methyl appears to lessen this stimulating action, although we notice in the case of the triethyl-ammonium sulphate a tendency to diffusion of stimuli in the cord, irritation of the foot being responded to by tremor over the body.

In the case of the Rat we find the amyl-ammonium sulphate to be one of the most poisonous of the whole series used in the case of these animals. There is violent tremor, increased on movement; a gait like that of paralysis agitans; sudden general clonic spasm, succeeded by springing from side to side.

In the case of ethyl-ammonium sulphate and diethyl-ammonium sulphate the movements are likewise tremulous; rapping of the head upon the floor is observed, and there is frequently a spasm of many of the trunk muscles, giving the impression of a hiccough movement. Respiration, at first accelerated, becomes very feeble, and a gradual loss of reflex precedes death.

The circulation was slowed by the action of these poisons, the heart tending to diastolic arrest, the right side especially being much engorged.

It was found that stimulation, both direct and indirect, elicited a powerful contraction of the poisoned muscle. The changes in circulation no doubt account for the slight effect of the poison upon the muscle. In the case of the amyl-ammonium sulphate, congestion of the membranes of the brain and of the cord itself were observed.

General action on Rabbits.

In the case of Rabbits, in which the whole series of these poisons was investigated, there was observed a gradual loss of power, the animal tending to lie on the belly, with the legs extended; the hind legs appeared to be chiefly affected.

In the case of the triethyl-ammonium sulphate, and the trimethyl-ammonium sulphate, there was a certain amount of tremulousness and starting when touched. The paralysis in the hind legs became complete before it did in the fore legs.

In the case of trimethyl-ammonium sulphate, profuse salivation was an early symptom, and corneal reflex persisted to the last.

The sulphates were less fatal to Rabbits than the corresponding chlorides or iodides, with the exception of trimethyl and triethyl sulphates, in which there was trembling and slight spasmodic movements, probably indicative of irritation of the spinal cord.

The symptoms were those of paralysis of the spinal cord and motor nerves. The conducting power of the cord for motor impressions appears to be paralyzed, as the hind legs fail before the fore legs. Death occurs in Rabbits and Rats by failure of respiration.

Difference between the Action of the Salts of the Compound Ammonias.

Our experiments appear to us to show that the salts of the compound ammonias vary in their action: (*a*) according to the acid radical with which they are combined; and (*b*) according to the number of the atoms of hydrogen which have been replaced in the ammonia by an alcohol radical. The influence of the acid, however, appears to us to be less marked than in the case of ammonia itself.

The iodides appear to have the strongest paralyzing action, both on the central nervous system and on the peripheral nerves. Next to them come the chlorides, and the sulphates have the least action.

The paralysis of the higher reflex, *e.g.*, of the cornea, was more marked in Frogs than in Mammals. In the latter, indeed, corneal reflex was observed almost at the last.

We have only examined the action of the iodides of tetramethyl- and tetraethyl-ammonium, so that we cannot compare their actions with those of the corresponding chlorides and sulphates. We have already drawn attention to the fact that their action appears to differ very greatly from the compound ammonias in which only three atoms of hydrogen have been replaced by an alcohol radical. In the tetra compounds convulsant action is very strongly marked, while in the triad compound ammonias it is much less so, or may be altogether absent.

In the case of warm-blooded animals salivation was noticed before death in poisoning by trimethyl-ammonium sulphate, tetramethyl-ammonium iodide, and tetra ethyl-ammonium iodide; it also occurred, to some extent, in amyl-ammonium iodide. In one or two others a similar action was observed to a less extent.

We have not investigated fully the action on the spinal cord and higher nerve centres of these different compounds, because the number of substances on which we have experimented was so great that we thought it better to leave this subject for a subsequent research, and to confine ourselves more especially to their action on muscle and nerve.

The results of our experiments on these tissues are shown in a condensed form in the following paragraphs:—

Differences between the Action of Salts of the Compound Ammonias on the Frog's Muscle and Nerve.

For convenience sake we will group the bodies, first, according to the acid radical; and secondly, according to the base they contain.

Variations in Action according to the Acid Radical.

Chlorides.

(*a.*) *Irritability* is, as a rule, slightly increased.

(*b.*) *Tetanus* from the muscle is often more extensive, whilst that from indirect stimulation is less extensive than on the normal side.

(*c.*) The *curve* is often exaggerated in direct stimulation.

It is frequently higher, and may be slightly shorter or longer than normal. On repeated stimulation, whether direct or indirect, the curve elongates to a greater or less extent. There is, as a rule, less elongation, less succeeding contraction, and less tendency to develop a distinct second hump than is to be seen in the iodides.

(*d.*) The nerve gives way somewhat before the muscle, but these substances (*i.e.*, chlorides) are not so fatal to nervous irritability as are the iodides. Amyl-ammonium chloride has a relatively stronger action on nerve than on muscle.

Iodides.

(*a.*) *Irritability* is, as a rule, decreased, the exception being occasionally found in ethyl-ammonium iodide, and di- and triethyl-ammonium iodides.

(*b.*) *Tetanus* is diminished in extent in almost every case.

(*c.*) The *curve* shows a strong inclination in all, but most in those lowest in the series, to become two-humped, the second horn or hump passing into a contracture, with very gradual decline.

(*d.*) In all cases the nerve becomes paralyzed much before the muscle.

Sulphates.

(*a.*) Minimal *irritability* is increased, or normal in the case of ethyl-ammonium sulphate, diethyl-ammonium sulphate, and triethyl-ammonium sulphate. It is decreased by amyl-ammonium sulphate, and by all the methyl-sulphates.

(*b.*) *Tetanus* produces more extensive contraction on direct stimulation in the case of the ethyls, and in very slight poisoning in some instances in the methyls, but in the latter it is usually diminished.

(*c.*) The *curve* is chiefly affected by the methyl compounds, on which it is usually lower and longer, and shows increased viscosity. It seldom displays the strong tendency to the double hump form which is so common amongst the iodides.

In the ethyl compounds the curve is usually somewhat exaggerated in relationship to the normal.

(*d.*) The failure of the nerve occurs somewhat sooner than that of the muscle. This is much more marked in the methyl than in the ethyl compounds.

On summing up those results, it appears that the iodides paralyze motor nerves more quickly than either chlorides or sulphates. We did not observe any marked

difference between the paralyzing action of the corresponding chlorides and sulphates. In the case of the muscle we notice that the irritability is increased, as a rule, in poisoning by the chlorides; is sometimes increased and sometimes diminished by the sulphates; and, as a rule, though with some exceptions, it is decreased by the iodides. The contractile power of the muscle, as shown by the extent and duration of tetanic contraction on direct stimulation, appears to be least affected by the chlorides; somewhat more so by the sulphates; and most of all by the iodides. The alterations in the form of the curve have already been described in detail.

Variations amongst the Ethyls and Methyls.

The least operative compounds examined were the diethyls and triethyls. Thus, in these alone, in the case of the iodides and sulphates, was minimal irritability equal to or greater than the normal.

(*a.*) In the case of the chlorides, however (in which the ethyls, methyls, di- and trimethyls only were examined), there was not a material difference between the corresponding compounds.

(*b.*) Amongst the iodides there is a strong tendency to loss of irritability of the nerve with all the compounds, but this is pre-eminently the case with the tetraethyl- and tetramethyl-ammonium iodides, which have an extremely powerful paralyzing action. The methyl compounds appear, however, to be operative in a slightly smaller dose.

(*c.*) The smaller group of the chlorides does not present such striking variations, but the corresponding methyls are slightly more active than the ethyls.

(*d.*) Amongst the sulphates we find the ethyls more often to produce an exaggerated single curve and an increased tetanus than do the methyls. There may, however, as shown in the chart of trimethyl-ammonium sulphate, be an increase in tetanic contraction as a result of stimulation in an early stage of poisoning.

(*e.*) The methyl compounds of the sulphate group are decidedly more fatal to the irritability of the nerve than are those of the ethyls.

(*f.*) *Ethylamine* showed development of *tetanic spasms* 70^{m} after injection. There was a gradual failure of reflex and circulation.

There was increased irritability to both direct and indirect stimulation; the curve was higher, longer, and showed increased viscosity.

Triethylamine—gradual failure of reflex and of circulation. Increased viscosity of the muscle was observed, without a marked lengthening of the curve.

Trimethylamine—gradual failure of reflex and of circulation. Increased irritability and increase of viscosity. The curve is equal to or shorter than the normal.

The methyls are more active than the corresponding ethyls. The methyls, amyls, and ethyls are more effective than the corresponding di- and tri- compounds. The tetra compounds are, however, most so of all.

ACTION OF SALTS OF THE ALKALINE GROUP ON MUSCLE AND NERVE, AND A COMPARISON OF THEIR ACTIONS WITH THAT OF AMMONIA.

The bodies usually included in the group of alkalies are, in addition to ammonia, lithium, sodium, potassium, rubidium, and cæsium: these are all monads. MENDELEJEFF includes in the monad group copper, silver, and gold, in addition to the substances just mentioned; but there is such a well marked difference between the general properties of the metals last mentioned and those of the alkalies that we have not included them in our research.

On comparing the general action of ammonia with these substances, the first thing that strikes us is that ammonia is the only one which has any tetanising action. Sometimes reflex action seems to be a little excited at first in poisoning by potassium and rubidium, but this excitement is slight, soon passes off, and is succeeded by torpor.

In the case of sodium, lithium, and cæsium, the symptoms in Frogs are those of gradually increasing torpor.

Sodium has no action at all in small quantities, but in concentrated solutions appears to paralyze nerve centres, nerves, and muscles, all at the same time. Lithium, rubidium, and cæsium have a tendency to affect either the upper part of the spinal cord or the higher motor centres connected with the fore limbs, as in poisoning by lithium and cæsium the reflex disappears sooner from the arms than from the legs, and stiffness was noticed in the arms in poisoning by lithium and cæsium, though no distinct spasm was observed. The motor nerves are not paralyzed by sodium or rubidium, but with these exceptions they are paralyzed to a greater or less extent by the other substances belonging to this group. Lithium and potassium are most powerful.

In considering the effect of the alkalies, and still more, perhaps, in the case of the alkaline earths, we have carefully to distinguish between the action of the poisons on the active contraction of muscle and on the residual shortening, which continues for a greater or less time after the contraction has passed.

To this shortening we have sometimes given the name of *viscosity*, at others, and more generally, we have employed the term used by German and French writers, *contracture*.

In regard to active muscular contraction also, we must note both the height of the curve, indicating the amount of contraction and its length, indicating the length or duration of contraction. The exact difference between the action of the various substances will be seen more in detail by a glance at the accompanying tables and curves.

But we may here state generally that the contractile power of the muscle, as shown by the height of the curve it describes, is increased by ammonium, potassium, and sometimes by rubidium and cæsium.

It is occasionally increased by sodium, but is otherwise unaffected, excepting in large doses, and it is diminished almost invariably by lithium.

The duration of the contraction, as shown by the length of the curve, is increased by large doses of rubidium (Plate 8, fig. 8, *a*, *b*, *c*), ammonium (Plate 8, fig. 9, *a*, *b*), sodium (Plate 8, fig. 10, *a*, *b*, *c*), and cæsium (Plate 8, fig. 11, *a*, *b*). It is shortened by ammonium (Plate 8, fig. 12, *a*, *b*), lithium (Plate 8, fig. 13, *a*, *b*), rubidium, and potassium (Plate 8, fig. 14, *a*, *b*, *c*). It will be seen from this enumeration that rubidium, ammonium, and sodium have a double action, sometimes increasing and sometimes diminishing the length of the contraction. In the case of rubidium and sodium the difference of action depends upon a difference of dose, small quantities tending to shorten the contraction, while large doses lengthen it. Prolonged contraction is accompanied, as we have already mentioned, by an increase of contractility in the case of rubidium, but by a diminution in the case of sodium, as shown by the height of the curve. The double action of ammonia does not seem to us to depend entirely on difference of dose, but rather to the ammonium having two different kinds of action.

The residual shortening, viscosity, or *contracture*, which sometimes succeeds an active contraction, is increased by large doses of rubidium, ammonium, lithium, and sodium. It is diminished by rubidium in small doses, ammonium, cæsium, and potassium. Here, again, the different action of ammonia does not appear to us to depend entirely on difference of dose.

Its double action appears to form, to a certain extent, a connecting link between the action of some members of the alkali group, such as potassium, and that of members of the group of alkaline earths.

The relations between the various members of the present group have to be considered more fully in a subsequent section, because we find that some members of it, while having a somewhat similar action on normal muscle, will yet antagonise each other's action, and although either of them given alone will lengthen the muscular curve, the lengthening will be abolished, and the curve reduced to the normal, by the administration of the two together.

Action of Substances belonging to the Group of Alkaline Earths and Earths.

The metals which we have examined belonging to the group of alkaline earths are calcium, strontium, and barium; and to that of the earths beryllium, yttrium, didymium, erbium, and lanthanum. The first three are dyads. Beryllium is also a dyad. The atomicity of the last four is not determined. Possibly they are all triads, though lanthanum has been grouped by MENDELEJEFF amongst the tetrads. The first point of difference that we notice about this large group is that it may be subdivided into

two sub-groups:—(*a*) containing beryllium, calcium, strontium, and barium; and (*b*) containing yttrium, didymium, erbium, and lanthanum.

In group (*a*) we notice a tendency to increased reflex action. In this particular it agrees with ammonium, but differs from members of the alkaline group. We have already noted that, in some members of the alkaline group, a slightly increased reflex action might be observed at the commencement of the poisoning, but this is considerably less than in the case of most of the members of group (*a*), with the exception of barium. Excitement of the spinal cord is most marked in poisoning by beryllium; next come strontium and calcium; and lastly barium, in which excitement, if present at all, is very slight.

In group (*b*) reflex action in the cord is not increased, nor does it appear to be very much diminished till the last. In this group, however, the higher centres appear to be paralyzed. We infer this from the fact that yttrium greatly diminishes co-ordinating power in the Frog, rendering the movements ataxic, and causing the animal to lie with the legs fully stretched out, although neither muscle or nerve is paralyzed. Didymium, erbium, and lanthanum all have a similar action.

In regard to their action on motor nerves, we notice the same well marked division into two groups as in their general action: beryllium, calcium, strontium, and barium all paralyzing the motor nerves to some extent. Lanthanum has also a paralyzing action, but yttrium, didymium, and erbium have none. In this respect these three bodies agree with sodium and rubidium, and differ from all the others belonging to these two groups which we have examined.

In regard to their action upon muscle, we do not find that these bodies can be so readily subdivided into two well marked sub-groups.

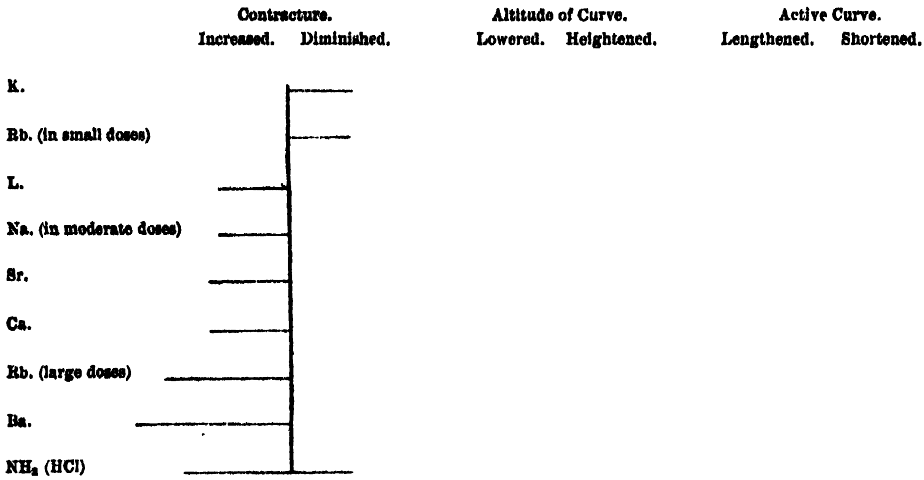
The contractility of muscle, as shown by the height of the curve, is greatly increased by barium (Plate 8, fig. 15, *a*-*d*), and occasionally, to a small extent, by erbium (Plate 8, fig. 16, *a*, *b*) and lanthanum (Plate 8, fig. 17, *a*, *b*). It is sometimes increased and sometimes diminished by yttrium (Plate 8, fig. 18, *a*, *b*) and calcium (Plate 8, fig. 19, *a*, *b*, *c*). It is diminished by didymium (Plate 9, fig. 20, *a*, *b*), strontium (Plate 9, fig. 21, *a*, *b*, *c*), and beryllium (Plate 9, fig. 22, *a*, *b*; fig. 23, *a*, *b*). We have found that the small variations occurring in the extent of contraction are best observed when the poison is applied locally in the form of solution. Where the muscles have been examined of an animal completely poisoned with the substance, the ultimate, rather than the primary result, is obtained.

The duration of the contraction, as shown by the length of the curve, is increased by barium, calcium, strontium, yttrium, and erbium. It is unaffected, or slightly diminished, by beryllium, didymium, and lanthanum (see figs. 17, 20, 22). It is obvious that the action of the rarer metals beryllium, erbium, didymium, lanthanum, and yttrium is but feeble in any direction when compared with the effect of calcium, &c.

The contracture is increased by barium, calcium, strontium, yttrium, and beryllium.

Contracture produced by barium is enormous (Plate 9, fig. 24, *a-g*). When the drug is locally applied its curve resembles greatly that produced by veratria (Plate 9, fig. 24, *b*). It appears to us to be an interesting fact that an inorganic element and an organic alkaloid should have such a similar action. Their action coincides also in the modifications which it undergoes by heat and by potash. The barium contracture, like that caused by veratria, is abolished by cooling the muscle down, or by heating it considerably above the normal. The contracture may be permanently removed by cooling down, so that it does not return when the muscle is again raised to the normal temperature. Like the veratria contracture, however, it is abolished much more certainly by heat (Plate 9, fig. 24). There is a more marked tendency for the barium contracture to relax suddenly than that caused by veratria. It is also more easily abolished by repeated stimulation.

In regard to the effect of these drugs on contracture, the same differences are to be observed between their action when injected into the circulation and when locally applied that we have already mentioned in regard to the active curve. In the accompanying diagram we have arranged some of the more important substances



belonging to the alkalies and alkaline earths so as to show their action upon muscle graphically. It will be seen that they tend to form a series, the two ends of which present some points of approximation, ammonium appearing to form a connecting link between barium and potassium.

It will be noticed that the substances here do not arrange themselves according to their atomic weight, nor yet according to their atomicities. We hope, however, to be able to consider this point more fully at a future time. We subjoin a table showing the relative position of the elements in regard to their action on motor nerves and muscles.

TABLE showing the relations of the Alkalies and Alkaline Earths as Poisons to Nerve and Muscle.

The most powerful paralyzers of motor nerves are put at the head of the column, and the others follow in the order of decreasing activity.

Those bodies which increase most the height and duration of muscular contraction and of muscular contracture are placed at the head of the corresponding columns, and at the foot are those which reduce them most.

Motor nerves.	Muscle. Height of contraction.	Duration of contraction.	Contracture.
NH_4	Ba	Ba	Ba
L	Rb	Rb	Rb
K	NH_4	NH_4	NH_4
Be	Er	Na	Na
Ca	K	Ca	Ca
Sr	Cs	Sr	Sr
Ba	La	Yt	L
Cs	Yt	Cs	Yt
La	—	Er	Be
—	Ca	—	Di
Er	Na	Be	Er
Di	—	Di	Rb
Yt	Di	La	NH_4
Rb	Sr	—	Cs
Na	Be	NH_4	La
	L	L	K
		Rb	
		Na	
		K	

TABLE showing the relations of the Alkalies and Alkaline Earths as Poisons to Nerve and Muscle—continued.

Substance.	Animal.	Dose or Application.	Symptoms in brief.	Post-Mortem Appearances.	Reactions.	Antagonisms.
Ammonium chloride	Frog	Local application— 1–1000	. . . .	. . . .	Lengthens and heightens curve, increasing also the passive shortening during the application of the solution. There is little or no increase of after-action (contracture). With weaker solutions may shorten curve.	The curve is markedly shortened by potash. And after potash has been long applied and the curve has become feeble, ammonium chloride reinstates activity of muscle. Lime shortens active curve, increases its altitude, and develops its own after-action. Restores irritability lost by barium application.
		1–500 1–200	. . . .	. . . .	Lengthens (often greatly) and usually lowers curve, increases contracture.	
Potassium chloride .	Frog . . . . (15 grms.) (30 grms.)	·02 (at once) . . . ·03 (gradually) ·1 (at once), all reflex gone.	In rapid poisoning (5m). Crouching attitude. All reflex gone, but leg drawn up occasionally spontaneously. Circulation ceased. Pigment-cells contracted. In slower poisoning, spring sudden and sharp, sometimes exaggerated, at others feeble. Tends to sink on belly. Leg moved wide of body in ataxic manner. On injection of ·1 all reflex was gone in 2m. Circulation was moderately good till last injection. The respiration was hurried.	Heart in diastole, full of dark blood, no longer irritable.	In rapid (incomplete) poisoning. Minimal irritability decreased (very slightly) on poisoned side. Tetanus firm (direct and indirect.) About 3 millims. more extension than in the normal. Curve slightly higher. Shorter with more rapid relaxation. In slower poisoning, there is a well-sustained tetanus (though not extension of muscle). A single induction shock hardly causes a visible contraction however. The nerve is completely insensible.	Counteracts veratria, barium, calcium, strontium. Lengthens some of rare metals (lanthanum, &c.). Reduces contraction of strong salt-solution muscle. Reduces contraction of lithium.
		Local application— 1–600 1–1000	. . . .	. . . .	At first and in weaker solutions shortens (hastens relaxation) and slightly heightens curve. Prolonged action of weak solution or shorter action of stronger solution (1–600) lowers curve.	
Caesium chloride . .	Frog . . . . (14 grms.) (10 grms.)	·03 grm.	Gradually increasing weakness. Reflexes at first good, but more difficult to excite. Position crouching, with extended leg. Reflexes in legs better than in arms; arms may be stiff. In 1h all reflex gone. Never any spasms. Circulation good throughout; active even when reflex ceased. Pigment-cells contracted.	Heart beating . .	Minimal irritability about equal in both muscles (*i.e.*, diminished in caesium muscle).* Poisoned muscle is less irritable to tetanising current. On stronger stimulation a good tetanus (direct stimulation) is got, but the nerve tetanus is very feeble, and there may be contraction only just on opening current. The curve is at first slightly increased in altitude, equal in other cases. It is longer than normal, but only slightly so. In more extensive poisoning the curve is rounder, but relaxation is more rapid than in the normal. Extensibility is increased.	Potash appears to shorten the elongated curve which may occur in caesium. In one case lime did not cause any recovery.
Sodium chloride . .	Frog . . . . (21 grms.) (30 grms.)	·2 grm. . . . . ·2 grm.	Movements become gradually feebler, and more difficult to provoke. Torpidity. Reflex becomes feebler, and is then completely lost. Circulation outlasts reflex. Many leucocytes are seen in web, some migrating. Red corpuscles fewer and crenated.	Auricles sensitive to stimulation; ventricle less so. In moderate diastole.	In both cases of extreme poisoning, irritability is about equal (*i.e.*, diminished in salt muscle), the tetanus both direct and indirect is equal, well sustained, but only half as extensive as the normal. The curve of both is low and very prolonged, especially so in the case of direct stimulation.	Those solutions which are strong enough to reduce another curve are themselves deleterious. Solutions of less than 2 per cent., or occasionally 1·5 per cent., increase the strontium and calcium after action. They also reduce height of curve active of calcium. Where there is veratria-like action developed potash opposes soda. There is partial opposition to Ba., Ca., Sr., and to K., only when calcium-like curve developed. Potash often cuts off the contraction of the sodium muscle without improving its contraction.
		Local application— ·6 per cent. ·7 to 1 per cent.	. . . .	. . . .	Applied locally of the strength of ·7 per cent., there appears to be no active change in the curve. (This is therefore rightly called normal salt solution.) ·8 to 1·0. There is often a shortening (slight) of the active curve usually without any increase in altitude taking place. 1· to 2· per cent. There may at first be a slight shortening of the curve, but there is from the first an increase in after action, and this soon fuses with the active curve, and produces a lower (rapidly falling curve), with considerable after action. The after-action may then become magnified into a veratria-like contracture. 2 per cent. often reduces the curve in length at once, but also much in altitude, the muscle dying in about 15m.	

* The irritability of the non-poisoned muscle is diminished by the ligature.

TABLE showing the relations of the Alkalies and Alkaline Earth[illegible] o Nerve and Muscl[illegible] continu[illegible]

Substance.	Animal.	Dose or Application.	Symptoms in brief.	Post-Mortem Appearances.	R[illegible]	
Lithium chloride.	Frog. . . . (16 grms.) (13 grms.)	·03 grm. . . . . ·04 grm.	Quietness. Spring feebler and shorter. Reflex persists longer in hind than fore legs. Reflex entirely lost in 1^h to 2^h or less. No tremor or spasm. Circulation steady at first, but becomes feebler. May be existing when reflex has ceased. Pigment-cells generally contracted.	Heart often beating feebly.	There is generally no reaction whatever on stimulating the nerve, even by tetanising current, and whilst the muscle gives a good single contraction. The curve from direct stimulation is lower, rounder, longer, and with a very gradual relaxation, or the relaxation may be more normal. The tetanus from direct stimulation is not so extensive as in the unpoisoned muscle, but it is often well-maintained and considerable.	[illegible]h heightens and c[illegible] away [illegible]ic after-action. Soda (·75 per cent.) increas[illegible] tonus witho[illegible]t heightening curve. Lime heightens curve and increa[illegible]es tonus, and is again increa[illegible]ed in tonus by lithia. Potas[illegible] reduces both.
		Local application— 1–150 1–500	. . . .	. . . .	It is doubtful whether the normal curve is really shortened at all by lithium. At first the summit may be rather reduced, and hence an apparent shortening results; but this soon gives place to a lengthening. The passage of the curve active into a tonus is most marked and invariable in this curve. The altitude is never increased above the normal.	
Rubidium chloride	Frog. . . . (10 grms.)	·015 grm.	Alternating restlessness and quiet. Leg drawn up quickly and high (reflex increased). Some stiffness of fore limbs. Movements become weaker and staccato. Reflex becomes feebler and unsustained, though rapid. At time of examination a spontaneous kick sometimes occurred. Breathing accelerated in early stage. Circulation, for some time good, gradually lessened in web, and ceased before heart stopped.	Heart in diastole, but began contracting again when blood was let out.	Irritability of both nerve and muscle to minimal stimulation diminished. The curve of the rubidium muscle, both for direct and indirect stimulation, is rather higher and rounder in the summit, with more rapid relaxation, than the normal, and therefore slightly shorter—or, at any rate, no longer—than the normal. Tetanus of both direct and indirect stimulation is more extensive.	Rubidiu[illegible] is antagonis[illegible] both by ba[illegible]ium and calciu[illegible]. The cu[illegible]re is not alt[illegible]ed by potas[illegible], except whe[illegible] vera-tria-li[illegible]e curve is pr[illegible]luced, in wh[illegible]h case potash [illegible]duces it.
	20 grms. . .	·1 grm.	Arm reflex ceased first. Leg reflex ceased 45^m after injection.	Heart in great diastole, full of dark blood.	Curve from both direct and indirect stimulation good.	
	15 grms. . .	·02 grm.	Reflex diminishes. Leg drawn up weakly and tremulously, or remains extended. Heart ceased.	Ditto ditto, not irritable.	A low, long contraction from both direct and indirect stimulation.	
		Local application— 1–400 1–100	. . . .	. . . .	With weaker dilutions the curve is usually shortened, and sometimes heightened. After the action of the solution has continued some time the curve becomes lower and flatter. 1–100. Not unfrequently there is a distinct veratria-like curve, or there may be a considerable contraction after active-curve instead.	
Calcium chloride.	Frog. . . . (14 grms.) (16 grms.)	·35 grm. . . . . ·25 grm.	Primary exaggeration of reflex. Reflex gradually lost. Occurs last in legs. No tremor. Heart beats slowly (12 per 1^m). Circulation in web gradually ceases. Pigment-cells contracted to balls. Formation of cataract in lens.	Heart ceased. Irritable.	Minimal irritability increased on poisoned side by 5 to 10 centims. The curve is usually higher for direct and sometimes for indirect stimulation. The curve is longer, and shows considerable "after action." Tetanus of muscle slightly inferior to the normal; that of nerve decidedly less, and yielding rapidly.	Is [illegible] mised by [illegible]. Re-[illegible] by very [illegible] soda [illegible] to [illegible] and 1 to 50). [illegible] l by st[illegible] time
		Local application— 1–100 1–400	. . . .	. . . .	With weaker solutions there is often an increase in extent of contraction, diminution in extensibility, slight lengthening of the arm, and increase of after-action. With stronger solutions there is a lowering (flattening) of curve, which is also lenghtened, and there is an increase in after shortening, which is very	

TABLE showing the relations of the Alkalies and Alkaline Earths as Poisons to Nerve and Muscle—continued.

Substance.	Animal.	Dose or Application.	Symptoms in brief.	Post-Mortem Appearances.	Reactions.	Antagonism
Strontium chloride	Frog . . . (12 grm (16 grm	·11 grm. . . . . ·066	Slight increase of reflex irritability; spring becomes impossible, whilst reflex remains good. Reflex of legs longest preserved. No spasm. Circulation soon slow, and only in larger vessels of the web. Ends in complete stasis. Vessels dilated. Pigment-cells sometimes dilated; at others ball-like. Some opalescence of lens.	Heart ceased. May be ghtly irrit	Minimal irritability (direct and indirect stimulation) usually increased by 5 to 10 centims. Curve of muscle lower, rounder, longer than in the normal. That of direct stimulation is the stronger. There is slight increase of after-action, but this is not so great as in case of calcium. Tetanus of both is distinctly feebler, the nerve giving way sooner than the muscle.	Is antagonised by potash. d d by very ong (1 to 75 and 1 50), resulting death muse m. Is increased by cal m. Is first decreased d then in creased by lithi
		Local 1-1 1-5			With weaker solutions, lowers, rounds, and lengthens curve, and increases after-action, as in the case of calcium. With stronger solutions the same kind of change, but more rapid and extensive. (The action of a solution of a given strength is more fatal to the preservation of muscular activity than is a solution of same strength of calcium.)	
Barium chloride .	Frog . . (21 grm	02 .	Position stiff. Fore arms under body in bow (Toad-like). After spring an occasional prolonged extension of limb is seen. Reflex fainter. Leg sometimes straightened instead of being drawn up. Examined when faint twitch in answer to stimulation remained. Circulation slowed. In 20m had practically ceased.	Auricl ting. Ven le lated and bu very slightly able. Wave of contraction travels very slowly from base to apex.	Minimal irritability of barium nerve increased. Tetanus of nerve is faint and unsustained; that of muscle inferior in altitude to the normal. The curve, on the other hand, is much higher, longer, and with a later maximum than the normal. There is but little after-contraction (see local application).	
		Local 1-1 1-3 1-3 1-8			Dilute solutions (1 to 1000 and 1 to 3000) cause, after application for 10m to 15m, a prolongation of the curve. The curve is exaggerated and the summit delayed. Stronger solutions than 1-1000 cause fibrillation and powerful (sometimes rythmic) contraction soon after application. A curve taken at this time may be exaggerated longer, higher, with late summit, or it may be a veratria-like curve. The latter results more readily from direct than from indirect stimulation. It may last for some time, and th disappear, and give way to a long curve with late mit. Note.—After neutralisation by pot, Ca., Sr., &c., the reapplication of barium solution seldom causes a return of the veratria-like curve, but of the long curve with late summit, together with passive shortening of the muscle.	Is ant gonised by- Pot sh. Sod m (imperfe r). Cal um. Str tium. Lit um, &c. Rul dium.
Beryllium chloride	Frog . . (16 grm (15 grm	Injec ,		Heart ting feebly.	There is usually complete paralysis of the nerve and very faint (if any) response of m le to te current.	K. in ases altitu of beryl lium urve. alcium does not rease extent of contracti
		ocal ap 1-100 5m to 2				

TABLE showing the relations of the Alkalies and Alkaline Earths as Poisons to Nerve and Muscle—continued.

Substance.	Animal.	Dose or Application.	Symptoms in brief.	Post-Mortem Appearances.	Reactions.	Antagonisms.
Didymium chloride	Frog . . . . (18 grms.)	Injection, ·075 grm.	Becomes difficult to rouse to movement, whilst reflex still good. No marked tremor. Is plastic, lies in unnatural positions. Reflex becomes more feeble and slow. Circulation at first good, gradually ceases.	Heart in still-stand in moderate diastole, but goes on again on stimulating.	Minimal irritability { Did. gast. { N., 40·0 centims. M., 12·5 ,, ; Lig. gast. { N., 23·5 ,, M., 15·5 ,, } Tetanus (direct and indirect) less extensive than normal { N., 40 mm. M., 42 ,, N., 45 ,, M., 45 ,, } Normal. Curve (direct and indirect) is not quite so high, but is otherwise not abnormal.	K. cl. 1-600 lengthens and heightens didymium curve.
		Local application—1-100	. . . .	. . . .	Does not materially alter length of curve. Simply reduces altitude somewhat without materially altering form. Increases contracture in resting muscle.	
Erbium chloride .	Frog . . . . (18 grms.) (15 grms.)	Injection, ·04 grm. ,, ·105 grm.	At first no motor variations, then a very gradual deterioration of reflex. Tendency to plasticity and maintenance of unusual positions. In one case quacking frequent. Circulation for long time active, and pigment cells dilated; then circulation becomes feebler.	Heart beating feebly, auricles stronger than the ventricle.	Minimal irritability very equal. Tetanus equal. Curve of poisoned muscle higher (and longer from wurf)—this due in part to the long ligature of the other limb.	
		Local application—1-100	. . . .	. . . .	There is at first a slight shortening and elevation of the normal curve. The curve soon becomes reduced in altitude. There is no increase of contracture.	
Lanthanum chlor.	Frog . . . . (16 grms.)	·06 grm. . . . . . (Frog, 12 grms.)	Lies flat. Arms extended. Arms stiff. Will then lie with leg extended. Reflex is, however, good and strong. Circulation good throughout.	Heart beating . .	Minimal stimulation: irritability of lanthanum nerve greater than normal. Effect of tetanus (direct) is equal, (indirect) lanthanum muscle somewhat impaired. Curve of lanthanum (direct and indirect) somewhat shorter and higher—shorter because of more rapid relaxation.	Potash lengthens the lanthanum curve. Calcium markedly heightens, lengthens, and causes after-action at end of contraction.
		Local application—1-100	. . . .	. . . .	Causes increasing tonus (passive contraction); a slight shortening of curve—more apparent than real, owing to the said shortening, which continually raises abscissæ.	
Yttrium chloride .	Frog . . . . (22 grms.) (16 grms.)	·02 grm. . . . . ·2 ,,	Restlessness succeeded by sitting up in stiff and unusual position. No tremor. Lethargic. Leg drawn up wide of body in an ataxic manner. Will lie long with limb extended. Breathing accelerated. Circulation good throughout.	Heart beating . .	Tetanus equal, or rather stronger in the yttrium gastrocnemius (due to the long ligature of the contrast leg, 3ʰ to 5ʰ). Curve (normal) equal to or slightly less in altitude than the yttrium. Yttrium curve has very slow relaxation in its later part, giving it quite a special appearance. This is equally the case in direct or indirect stimulation. Irritability is greater than in the normal.	
		Local application—1-100	. . . .	. . . .	Slight passive shortening of muscle. Slow relaxation after contraction. There is, however, shortening of more active part of curve in some cases.	

The effect of heat, cold, and stimulation must be postponed to another paper.

ACTION of elements upon general condition of organism as a poison acting gradually.

Substance.	Proportion to gramme of body-weight of Frog in which element acts as a poison.
Potassium chloride	·0013
Beryllium choride	·0013
Rubidium chloride	·0013 to ·0015
Barium chloride	·0013
Ammonium chloride	·0015
Cæsium chloride	·0021
Lithium cloride	·0023 to ·0032
Lanthanum chloride	·004
Didymium chloride	·0042
Erbium chloride	·006
Strontium chloride	·0055 to ·0075
Yttrium chloride	·009
Sodium	·0095
	[illegible]

Monads.		Atomic weights.
Potassium	·0013	39·10
Rubidium	·013 to 15	85·4
Cæsium	·0021	13·3
	)023 to 32	7
Sodium	·0095	23

Dyads.		Atomic weights.
Beryllium	·0013	9·4
Barium	·0013	13·7
Lanthanum	·004	93·6
Didymium	·004	95
Erbium	·006	112·6
Strontium	·0065	87·6
Yttrium	·009	61·7
Calcium	·013	40

ON THE ACTION OF ALKALI AND ACID ON MUSCLE.*

The remarkable results obtained by GASKELL upon the action of very dilute acids and alkalies on the blood-vessels, induced us to examine the action of similar solutions upon voluntary muscle. GASKELL found that alkalies cause contraction, and dilute acids relaxation, of the involuntary muscular fibres of the blood-vessels. Our observations show that this is also the case with voluntary muscular fibre, but, in addition, we note that acids beyond a certain strength cause a permanent contraction.

We tested the action of dilute acids and alkalies on muscle in two ways:—first by applying them directly to the muscle, and secondly by causing them to circulate artificially through the vessels supplying it. As water alone has a destructive action on muscular fibre, the acid and alkali was in all cases added to a 0·75 per cent. solution of sodium chloride.

The muscle-chamber designed by one of us (CASH), which was used in these and many other experiments, consists of a glass cylinder 3 centims. broad, 7 centims. long, and with

* This part of the paper was received June 15, 1881, but publication was deferred.

a capacity of about 40 cub. centims. Tubes (*a*)* for the ingress and egress of the fluids are let into the sides of the cylinder, two above and one below. The upper end of the cylinder is fitted accurately with a stopper made of cork and vulcanite. The vulcanite lid (*b*) and the cork have an opening in the centre, which can be completely closed by means of a brass sliding clamp (*c*), which is moved by a screw (*d*) provided with a milled head. This slide-clamp holds securely the femur, if the gastrocnemius of the Frog be used; the illium, if the triceps. A binding-screw (*e*) is attached to the brass arm of the clamp, and this receives one of the wires of the secondary coil for direct stimulation. The second connexion with the muscle is effected by means of a long and very fine coiled wire (*f*), which is in contact above with another binding-screw situated on the vulcanite cap, and below with a trout hook (*g*) bent into an S shape, on to which the wire is whipped. The lower end of the S is connected with the thread or gut which passes through the lower end of the cylinder to the lever. A second pair of binding-screws on the vulcanite lid are connected with platinum electrodes supported on a vulcanite back (*h*) which projects into the cylinder. These are intended for indirect stimulation of the muscle. Finally, the stopper carries a groove round the central opening, into which a metal cap (*i*) fits; application of this cap, when the groove has been filled with a drop of oil, renders the upper opening practically air-tight. The stopper is of course removed when a preparation for examination is placed in the chamber. The lower end of the cylinder is permanently closed by a stopper of wood or vulcanite, which is cemented into position. It contains two openings: the first, that of a small tube (*k*), through which a few drops of oil may be introduced when it is desired to make the chamber absolutely air-tight, as in experiments on the action of gases upon muscle; the second serves for the transmission of the thread or strand of gut which connects the lever and the tendon of the muscle. It is made from a piece of thick-walled glass tubing (*l*) of 1 centim. in length, drawn out with an hour-glass contraction in the middle. The calibre at the constriction is such that a strand of very fine silk, or the best drawn trout gut just passes through it, and no more. When the cylinder is filled with liquid the inner surface of this capillary tube becomes moistened, and it is found, whilst all friction is obviated, the escape of fluid may be reduced to such an extent that twenty or thirty drops only may flow out in the twenty-four hours. We have repeatedly used the chamber in experiments extending over twelve hours, and found it practically full at the end of the experiment.

One of the upper openings in the wall of the cylinder is connected, by means of a T-tube, with two or more funnels, which contain: (1) the poison or poisons in solution to be tested; (2) normal salt solution for washing out the cylinder. The tubes connecting these with the cylinder are controlled by clamps. In order to avoid escape of current, the fluid in the cylinder is run off before stimulation is applied. The nerve can, however, be stimulated whilst the muscle remains in the solution.

* The letters apply to both diagrams A and B, Plate 10.

Further, by regulating the height of the fluid the nerve can be exposed to the action of the solution, or kept free from it. The chamber is enclosed by a belt (*m*) connected with a rod, which fits into a nut sliding up and down on a steel upright. The lever is connected with the muscle in the usual manner, and its axis moves, together with the chamber, upon the rod of which it is clamped. By certain modifications this chamber is heated or cooled, so that the effects of variation of temperature upon the poisoned muscle may be easily studied. It is also possible to test the effect produced not only by hot and cold air, but by solutions gradually heated or cooled to any desired extent.

As already mentioned, the apparatus serves the purpose of testing the effect of gases and vapours on muscles very satisfactorily.

This mode of application was chosen on account of the obstacles to the circulation of alkalies in the muscle, and also because Von ANREP* asserts (1) that the action of a solution thus locally applied is the same as when the solution has been made to circulate through the tissues. GASKELL has privately communicated to us the same result, and numerous experiments of our own have confirmed these statements.

Von ANREP, in investigating the action of potassium upon muscle, found that it caused, either when applied locally or through the circulation, a decided shortening of the muscle, which in a few minutes reached its maximum. This shortening is independent of the action of the spinal cord, for it occurs whether the muscle remains in connexion with the cord, or whether the nerves be cut. The shortening has no relationship to the irritability of the muscle. The irritability of a muscle through which a 1 per cent. solution of potash is circulated for fifteen to twenty minutes is quite abolished, while the shortening persists; occasionally a slight elongation is seen, in place of a shortening. On the other hand, he found that sodium has not this effect on muscle.

Effects of Acid and Alkali applied externally to Muscles at rest.

Dilute solutions of potash and soda, containing from one part in 4,000 to one part in 8,000, cause shortening of the muscle. The contraction produced by soda was slightly greater in our experiments than that caused by potash, the solutions applied being of equal strength, and for an equal time.

Lactic acid, in very dilute solution of 1 to 8,000 or more, seems to tend to elongate muscle which is loaded with a slight weight.

A solution of chloride of sodium alone, however, also causes relaxation of the muscle, and the continuous application of a slight weight has a similar effect.

Less dilute solutions of lactic acid, 1 in 4,000 or stronger, causes passive shortening of the muscle, and this is occasionally accompanied with fibrillary twitchings. Dilute solutions of lactic acid cause relaxation of the muscle which has been shortened by potash or soda.

There is a fairly balanced antagonism between lactic acid 1 to 8,000, and soda

* PFLÜGER'S Archiv., vol. xxi., p. 226.

1 to 3,000. Solutions of from 1 to 10,000 to 1 to 12,000 have both a slight power of counteracting the power of soda, and of lengthening the muscle; but 1 to 8,000 is the weakest dilution which is reliable for this purpose when applied externally. Normal salt solution has a distinct power of removing the shortening produced by soda, but its action is much more limited, and less complete than that of lactic acid.

External application of dilute acids and alkalies to contracting muscle (Plate 9, figs. 25, 26, 27). Soda and potash in solutions up to 1 in 8,000, or 1 in 10,000, cause a tonic shortening of the muscle, and may, at first, increase the height of its active contraction.

Lactic acid in dilute solutions of 1 in 10,000, or weaker, may cause elongation to a muscle which has already soaked for some time in a salt solution. A solution of 1 in 10,000 may cause at first a slight increase in the excitability and increased height of contraction, but this soon disappears. In dilutions between 1 in 8,000 and 1 in 2,000 it causes eventually shortening of the muscle, with occasional fibrillation and rapid diminution of the extent of active contraction. At the same time that the contrantile power is diminishing, the muscle exhibits increasing viscosity. This is shown by a slight elevation of the basal line when the stimuli succeed each other with sufficient frequency.

The permanent shortening caused by the application of an alkali is usually diminished by the subsequent application of lactic acid. After the diminution has occurred active contraction becomes feebler.

Plate 9, fig. 25, shows the result of admitting soda solution 1 in 2,000 to the chamber containing a muscle which is being periodically stimulated through its nerve. (The solution almost entirely covers the muscle, but the nerve lying on the electrodes is above its level.) Plate 9, figs. 26 and 27, show the action of 1 to 4,000 and 1 to 5,000 soda solutions on the acting curarised muscle. Here stimulation was of course direct, and the probable escape of current is therefore to be borne in mind. In both cases the subsequent action of lactic acid is shown, viz., a reduction of the basal line, and ultimately a fall in the altitude of the contraction.

Action of Acids and Alkalies when circulated through the Muscle.

The method employed was to pith and curarise a frog. A canula was then inserted into the aorta and connected with a branching tube, through which acid, alkaline, or salt solution could be supplied from a series of funnels. By elevating or depressing the funnels the pressure by which the circulation was carried on could be increased or diminished. Excepting when otherwise stated it was always effected at as low a pressure as possible. The condition of the muscle was registered by means of MAREY'S myograph. The triceps was found to be the most convenient muscle for this series of experiments on account of its great vascularity.

Moderately dilute solutions, both of acids and alkalies 1 to 4,000, after circulating for some time, caused the muscle to shorten. Galvanic stimulation to the muscle increases this effect, both of these solutions and also of weaker ones. It frequently

happens that a muscle which exhibits little or no shortening before stimulation, becomes progressively shortened after a number of stimuli have been applied, until the basal line of the curve it describes is far above the normal.

The pressure which is sufficient for the circulation of an acid solution, as a rule, quickly becomes insufficient to maintain the free circulation of an alkaline solution. This is to be expected from the fact that an alkali causes contraction of the involuntary muscular fibres of the vessels, and is in unison with GASKELL's observation.

The first effect of an alkaline solution, as a rule, is to increase the contractility of the muscle on stimulation; the same stimulus producing a greater contraction than it would in the muscle without such circulation. A gradual shortening of the muscle, independently of any active contraction, is produced by the alkaline solution: this is shown by the rise of the basal line in the curve. After the circulation has been maintained for some time, both the contractile power and the irritability of the muscle decrease; the height of the contraction occurring on stimulation not being so great, and a stronger stimulus being required.

Plate 9, fig. 28, *a*, *b*, *c*, is introduced to show the fibrillation and temporary shortening which may occur upon the first stimulations of a muscle through which lactic acid has been some time circulated.

Plate 9, fig. 29, *a*, *b*, *c*, shows that the elevation of the basal line, caused by the circulation of soda (1-20,000), is to a large extent reduced by the subsequent circulation of lactic acid 1-10,000. The altitude of the contraction is likewise reduced.

Lactic acid, when circulated through the muscle, frequently causes fibrillation, and at first shortening of the muscle after fibrillation: there may, however, not be any shortening.

Usually the height of the contractions diminishes rapidly on repeated stimulation; sometimes, though quite exceptionally, the irritability of the muscle is increased at first, and the contractions resulting from stimulation may be at first more extensive than those of the normal muscle.

Œdema of the muscle is occasionally observed as a consequence of the circulation of acid through the vessels; this is unusual after the circulation of alkalies. The impaired contractile power eventually produced by the circulation of either alkali or acid through a muscle may be restored to a greater or less extent by the circulation of a fluid having an opposite reaction. The completeness of the restoration depends upon various circumstances, amongst which we may mention the œdematous condition of the muscle, which we have already noticed as occurring from the circulation of acids.

Our experiments on the muscles of the Frog have thus shown a very marked antagonistic power between acids and alkalies, or perhaps to speak more definitely, between solutions of potash or soda and lactic acid. It seemed advisable to make some experiments on the muscles of warm-blooded animals, in order to discover whether the same antagonism was to be found in them: for this purpose we chose the gastrocnemius of the Cat. The solution to be investigated was warmed to 40° C,

and then passed through the limb by means of a canula inserted into the femoral artery. The muscle was stimulated from the sciatic nerve, the leg being previously fixed by a clamp. The muscle was extended by a weight of 40 grammes attached by a cord working over a pulley; this was allowed to remain constantly attached in some experiments to ascertain alterations in the length of the muscle due to the fluids circulated. In several cases it was applied for two minutes before each tracing.

Plate 9, fig. 30, shows the effect of acids and alkalies.

(*a.*) The lever recorded (multiplies 4 times) contractions of 12·5 millims., an opening and closing shock every 4″.

(*b.*) After alkali 1-20,000 had circulated 10^m the basal line showed a shortening of 8 millims. The active contraction was 13 millims. Ten minutes after this tracing had been taken the flow, which had previously been free from the femoral vein, became very slow, and remained so under a considerable increase of pressure.

(*c.*) Lactic acid 1-10,000 restored the circulation and reduced the contraction. The active contraction of the value of 12 millims.

(*d*) Alkali circulated 20^m has raised the basal line 10·5 millims., but shows an active contraction of less than 10 millims.

(*e.*) After 60^m circulation the basal line is still 10·5 above the normal, but the active contraction has increased to 11·5 millims.

There is here, then, a great similarity of action in the case of acid and alkali circulated through the vessels of cold and warm-blooded animals.

General Results of Experiments on the Action of Acid and Alkali on Muscle.

The experiments just described show that dilute alkalies, potash, and soda cause shortening of muscle, which is antagonised by dilute solution of lactic acid. Since the preceding section of this paper was sent in to the Royal Society we have made some further observations on this subject, and from an examination of the curves it will be seen that, by the alternate application of alkali and acid, a muscle may be made to describe on a slowly revolving cylinder a curve very nearly resembling that described on a rapidly revolving cylinder by a normal muscle when stimulated. Other tracings show that this curve may be modified very nearly at will by altering the proportions and duration of the alkali and acid. Curves may be thus described which resemble those drawn by muscles stimulated after they have been poisoned by barium, rubidium, and other substances of the groups we have examined. In these curves we see produced by varying the application of the opposing solutions the same prolonged contraction, the tendency to an exaggerated secondary hump, and increased *contracture*.

We cannot at present draw from this a definite conclusion, but it is suggestive of the question—Does the normal contraction of muscle and its subsequent relaxation depend upon such alterations in its saline constituents as to make them play at one time the part of an alkali, and at the other the part of an acid?

Plate 9, figs. 31 and 32, show the relative effects of solutions of 1 to 3,000 caustic soda solution (Plate 9, fig. 31) and caustic potash solution (Plate 9, fig. 32) upon resting muscle. The tracings were taken upon a slowly revolving cylinder. Each centimeter of the tracing represents 5^m. The lever, which multiplies fourteen times, exercises a constant traction of 10 grms. on the muscle. Fresh solution was added where stars are placed in the course of the curve. It will be seen that the shortening effect produced by caustic soda in 50^m, during which the solution was renewed every 10^m, is slightly greater than is the case with the companion muscle treated with caustic potash of the same strength. The curves, however, show a very close similarity throughout. The commencing relaxation caused by the substitution of 1 to 1,000 lactic acid is seen in each case.

The very gradual shortening of the muscle upon the first application of potash and soda is, to some extent, due to the fact that the muscles had been previously curarised. When curara has not been previously employed the first application of dilute solutions causes a more rapid primary contraction, though the total effect of the application may not be greater, if as great as in the curarised muscle. Plate 9, fig. 33, gives the effect of a stronger solution of soda, *i.e.*, 1 to 2,500, and the subsequent relaxation it undergoes upon the application of 1 to 500 lactic acid.

Plate 9, figs. 34 and 35, give the action of soda 1 to 4,000, and potash 1 to 6,000, with partial relaxation consequent to lactic acid. That lactic acid itself causes shortening, if of a certain strength, is shown in Plate 9, fig. 36, when 1 to 1,000 solution of the acid causes in 25^m a shortening of 4 millims. in the curve, or of ·3 millim. in the muscle.

The application of potash reduces this shortening to some extent, and then, its own action being no longer balanced, causes the muscle to contract rapidly. The converse of this is seen in Plate 9, fig. 37, when the alkali is first applied, and the acid 1 to 500 causes a relaxation, and then a shortening of its own. To cause a complete relaxation a higher dilution is necessary.

Plate 9, figs. 38 and 39, give tracings of passive shortening or lengthening with an active contraction (maximal stimulation) taken at intervals superimposed.

Plate 10, fig. 40, *a*, *b*, *c*, illustrates the change of form the normal muscle curve undergoes when treated with an alkali local application. The first "hump" of the active contraction is increased in altitude; the second "hump" or elevation after the notch is reduced. Owing to this reduction the curve is shortened. A passive shortening of the muscle is seen at *c*, and is, in point of fact, less than is usually produced by solutions of these strengths.

The effect of lactic acid applied in the same manner is shown in the series *a*, *b*, *c*, Plate 10, fig. 41. Here also the second portion of the curve is reduced, and the relaxation becomes much more rapid. After 60^m in lactic acid 1 to 2,500, a slight contraction of 1·5 millim. is observable.

Plate 10, fig. 42, *a*, *b*, *c*, *d*, *e*, gives the action of potash on the normal muscle, to a

large extent counteracted by lactic acid, and the subsequent passive shortening of the muscle under the non-balanced action of a strong solution (1 to 500) of the acid.

On the relative action of Alkalies and Alkaline Earths on Muscle.

We cannot enter here into a full consideration of the antagonism which certain members of these groups show with regard to the action of other members, but we may briefly state a few of the most striking facts. Thus potassium shortens the lengthened curves of veratria, barium (Plate 10, fig. 43), calcium, strontium, of large doses of sodium and of lithium (Plate 10, fig. 44), and reduces the contracture which they have caused. Sodium, which we have shown in large doses to cause a lengthened curve with increased contraction, adds to the length of calcium and strontium when applied in strong solutions. Barium, when it has produced its lengthened veratria-like curve, is, however, counteracted by almost all the substances which tend to produce a shorter curve. Thus calcium and potassium both of them lessen its altitude, and abolish its contracture. A remarkable antagonism, however, is that existing between rubidium and barium. The veratria-like curve which the former has been shown to cause when in strong solution is completely reduced by the application of a solution of barium, of such a strength as would, if applied by itself in the first instance, have caused a similar, though more extensively varied, curve. It is to be noted that in this antagonism, as in many others, the muscle yields a reaction closely similar to the normal before it develops the characteristic curve which is associated with the substance used to antagonise.

With two substances of closely-allied action we sometimes find, as in the case of calcium and strontium, an addition of effect (Plate 10, fig. 45) without any reduction having taken place. It would appear that in some cases we get the two substances which have a similar action, at one time aiding one another, in other cases neutralising one another. It is hard to say what the cause of this curious result is, and any explanation of it must be at present entirely hypothetical. At present our data are too limited to allow us to formulate any general rule regarding antagonism. We may, however, mention some antagonisms which are at any rate curious.

(1) Calcium reduces the barium curve to the normal, or thereabouts, before it causes its own peculiar form of curve.

(2) Rubidium in strong solutions has the same effect as barium in causing a veratria-like curve.

(3) Sodium usually produces with lime, not a shortening of the curve, but an increase of the after-action (contracture) which is often seen in the lithium muscle.

(4) Potash lengthens the curves of didymium and lanthanum.

(5) Lithium increases calcium effect, and calcium increases lithium effect.

(6) Potassium opposes strontium.

(7) We have drawn attention to the antagonism of barium to rubidium (when the

latter develops in strong solution a veratria-like curve), and also that potassium is antagonistic to barium.

(8) Sodium, in strong solutions, may reduce the lithium contraction before the death of the muscle occurs.

Although we have at present considered the action of ammonia, compound ammonias, alkalies, and alkaline earths, on voluntary muscle only, we have made a number of experiments which seem to show that their action on involuntary muscular fibre is very similar, *e.g.*, barium causes a very great prolongation of systole in the Frog's heart, just as it prolongs the contraction of voluntary muscle. These results we intend to investigate more fully, and hope to publish them hereafter.

All attempts to establish a relationship between atomic weight and physiological action have hitherto failed. It may be that this failure has resulted from the lethal activity on the organism, as a whole, having been taken into consideration, whereas different substances may cause death by acting on different structures. We think that by the method here pursued of investigating their relationship to one or two structures only, and by a careful comparison of their actions, some definite connection may yet be established, and we hope that the results which have been recorded may serve as a contribution towards this end.

Perhaps they may also serve to throw some light on the curious subject of the different reactions of different organisms to the same drug, but this also we purpose to follow up in a further research.

We desire to acknowledge most gratefully the great kindness of Professor RANVIER, who placed his laboratory at our disposal, and afforded us every facility for carrying out there the experiments on warm-blooded animals, and also on unpithed Frogs, which are rendered so difficult in this country by the present state of the law.

EXPLANATION OF FIGURES.

PLATE 8.

The figures represent the curves obtained by registering the contraction of the gastrocnemius of the Frog (*Rana Temporaria*) on a revolving cylinder.

Fig. 1. Frog poisoned by 1 drop 10 per cent. solution of dimethyl-ammonium chloride.

a. Ligatured leg. 5" tetanus, direct stimulation of gastrocnemius.

b. Ditto. Indirect stimulation.

c. Poisoned leg. Direct stimulation.

d. Ditto. Indirect stimulation.

Fig. 2. Frog poisoned by tetramethyl-ammonium iodide.
- *a.* Ligatured leg. Ten stimulations* (direct) of gastrocnemius, one stimulation every 1·5ˢ.
- *b.* Poisoned leg. Ditto.

Fig. 3. Frog poisoned by large dose (·2 grm.) amyl-ammonium iodide.
- *a.* Ligatured leg. Ten stimulations (direct) of gastrocnemius, one stimulation every 1·5ˢ.
- *b.* Poisoned leg. Ditto.
- *c.* Poisoned leg. Single curve, indirect stimulation.

Fig. 4. Frog poisoned by trimethyl-ammonium iodide.
- *a.* Ligatured leg. Ten stimulations (direct) of gastrocnemius; curves of direct and indirect stimulation are equal; one stimulation every 1·5ˢ.
- *b.* Poisoned leg. Ditto. Direct stimulation.
- *c.* Poisoned leg. Ditto. Indirect stimulation.

Fig. 5. Frogs poisoned by tetraethyl-ammonium iodide.
- *a.* Ligatured leg. Single stimulation of gastrocnemius (direct).
- *b.* Poisoned leg. Ditto. The nerve is no longer irritable.
- *c.* Case of profound poisoning. Direct stimulation of gastrocnemius.

Fig. 6. Frog poisoned by dimethyl-ammonium sulphate (·25 grm.).
- *a.* Ligatured leg. Direct and indirect stimulation.
- *b.* Poisoned leg. Direct stimulation. Nerve no longer irritable.

Fig. 7. Frog slightly poisoned by trimethyl-ammonium sulphate (·1 grm.).
- *a.* Ligatured leg. Tetanus 5ˢ, direct stimulation.
- *b.* Ligatured leg. Ditto, indirect stimulation.
- *c.* Poisoned leg. Tetanus 5ˢ, direct stimulation.
- *d.* Poisoned leg. Ditto, indirect stimulation.

Fig. 8.
- *a.* Normal gastrocnemius. Direct stimulation.
- *b.* Ditto. After 5^m in 1 per cent. chloride of rubidium solution.
- *c.* Ditto. After 15^m in ·75 per cent. chloride of calcium solution.

Fig. 9.
- *a.* Normal gastrocnemius. Direct stimulation.
- *b.* Ditto. After 20^m in 1–1000 chloride of ammonium solution.

Fig. 10.
- *a.* Normal gastrocnemius. Direct stimulation.
- *b.* Ditto. After 30^m in 2 per cent. solution chloride of sodium.
- *c.* Ditto. After 45^m in ditto.

Fig. 11. Frog poisoned by ·02 grm. chloride of cæsium.
- *a.* Ligatured leg. Direct stimulation.
- *b.* Poisoned leg. Ditto.

Fig. 12.
- *a.* Normal gastrocnemius. Direct stimulation.
- *b.* Ditto. After 15^m in ·1 per cent. chloride of ammonium.

* All single stimulations are by an opening maximal induction shock.

Fig. 13. *a.* Normal gastrocnemius. Direct stimulation.
b. Ditto. After 30^m in ·33 per cent. chloride of lithium.

Fig. 14. *a.* Normal gastrocnemius. Direct stimulation.
b. Ditto. After 30^m in ·1 per cent. chloride of potassium.
c. Ditto. After 30^m in ·15 per cent. ditto.

Fig. 15. *a.* Normal gastrocnemius. Direct stimulation.
b. Ditto. After 30^m in ·25 per cent. chloride of barium.
c. Ditto. After 45^m in ditto.
d. Ditto. After 15^m in ·25 per cent. chloride of potassium.

Fig. 16. Frog poisoned by chloride of erbium (slow action of drug).
a. Ligatured leg. Direct stimulation.
b. Poisoned leg. Direct and indirect stimulation give equal contractions.

Fig. 17. Frog poisoned by chloride of lanthanum.
a. Ligatured leg. Indirect stimulation.
b. Poisoned leg. Indirect stimulation.

Fig. 18. Frog poisoned by chloride of yttrium (slow action of drug).
a. Ligatured leg. Indirect stimulation.
b. Poisoned leg. Ditto.

Fig. 19. Frog poisoned by ·35 grm. calcium chloride.
a. Ligatured leg. Indirect and Direct stimulation give equal contractions.
b. Poisoned leg. Indirect stimulation.
c. Ditto. Direct stimulation.

PLATE 9.

Fig. 20. *a.* Normal gastrocnemius. Direct stimulation.
b. Ditto. After 20^m in 1 per cent. chloride of didymium.

Fig. 21. *a.* Normal gastrocnemius. Direct stimulation.
b. Ditto. After 30^m in ·2 per cent. chloride of strontium.
c. Ditto. After 15^m in ·5 per cent. ditto.

Fig. 22. *a.* Normal gastrocnemius. Direct stimulation.
b. Ditto. After 20^m in 1 per cent. chloride of beryllium.

Fig. 23. Frog poisoned by beryllium chloride (·02 grm.).
a. Ligatured leg. Tetanus of gastrocnemius, direct stimulation.
b. Poisoned leg. Ditto. Secondary coil at 2 c.m. Indirect stimulation of the poisoned muscle did not yield any contraction.

Fig. 24. Action of heat and cold on the barium curve.
a. Normal gastrocnemius. Direct stimulation, at room temperature 13° C.
b. Ditto. After 15^m in ·25 per cent. chloride of barium solution. Temperature 13° C.

c. Ditto. Application of barium solution continued. Kept for 15^m, cooled to $8^\circ{\cdot}5$ C.

d. Ditto. Heat to 18° C. Reappearance of veratria-like curve.

e. Ditto. Heat to 20° C.

f. Ditto. Heat to 30° C. The veratria-like curve disappears.

g. Ditto. Cool to 14° C. There is no return to the veratria-like curve. A simple prolonged contraction persists.

Fig. 25. Action of soda on contracting muscle. Solution of 1–2000 admitted at ×. Stimulation every 2^s.

Fig. 26. *a.* Action of soda, 1–4000, on contracting curarised muscle. Solution admitted at ×.

b. Same muscle after exposure to lactic acid, 1–4000, for 40^m. Stimulation every 2^s, direct.

Fig. 27. *a.* Action of soda, 1–5000, on contracting curarised muscle. Solution admitted at ×.

b. Lactic acid, 1–5000, has acted 1^m on muscle.

c. Ditto, has acted 5^m on muscle. Stimulation every 2^s, direct.

Fig. 28. *a.* Normal gastrocnemius. One opening and one closing stimulation every 4^s.

b. After 60^m circulation of lactic acid through aorta, 1–8000, stimulation causes fibrillation and shortening of muscle.

c. After 30^m circulation of soda, 1–6000, the strength of contraction, which had been diminished under acid, is restored; fibrillation has ceased.

Fig. 29. *a.* Normal gastrocnemius. One opening and one closing stimulation every 4^s.

b. Taken after circulation for 10^m of 1–20,000 alkaline solution.

c. Taken after circulation for 30^m of 1–10,000 acid solution.

Fig. 30. Tracing from gastrocnemius of Cat. One opening and one closing stimulation every 4^s. The solution, heated to 38° C., was circulated under pressure through the femoral artery, and allowed to escape by the femoral vein. The rest of the limb, with the exception of the sciatic nerve, which was exposed for stimulation, was ligatured. A weight of 40 grms. was applied 2^m before each tracing was taken. Abscisse constant.

a. Normal contractions.

b. Alkali, 1–20,000, has circulated 10^m.

c. Acid, 1–10,000, has circulated 60^m.

d. Alkali, as before, 20^m.

e. Ditto, 60^m. Flow from venous canula very slow and weak.

Fig. 31. Action of alkali and acid upon resting muscle (curarised). At the first five points indicated by ×, soda solution, 1–3000, is supplied to muscle in cylinder. At the last six points indicated by a ×, lactic acid, 1–1000, is supplied. The action of the soda was for $47{\cdot}5^m$; that of the acid for 42^m. Change of alkali to acid, or *vice versâ*, in all cases shown by double-headed arrow.

Fig. 32. Action of caustic potash, 1–3000, for 43^m, succeeded by action of lactic acid, 1–1000 for 42^m.

Fig. 33. Caustic soda, 1–2500, once renewed in 25^m, succeeded by action of lactic acid, 1–500, once renewed in 25^m.

Fig. 34. Curarised gastrocnemius. Caustic soda, 1–4000, twice renewed in 33^m, succeeded by action of lactic acid, 1–1500, once renewed in 25^m.

Fig. 35. Curarised gastrocnemius. Caustic potash, 1–6000, thrice renewed in 46^m, succeeded by lactic acid, 1–1500, twice renewed in 28^m.

Fig. 36. Curarised gastrocnemius. Lactic acid, 1–1000, four times renewed in 37^m, succeeded by caustic potash, 1–2500, once renewed in 34^m.

Fig. 37. Action of caustic potash, 1–2500, twice renewed for 13^m, succeeded by action of lactic acid (1–500) for 18^m, and this by action of caustic potash for $17{\cdot}5^m$.

Fig. 38. Action of caustic potash, 1–4000, for 20^m, succeeded by lactic acid, 1–1000, 48^m. The muscle is subjected to maximal stimulation before the change of each solution.

1. Contraction of normal muscle.

2, 3. Contractions of alkali muscle.

4, 5, 6, and 7. Contractions of acid muscle.

Fig. 39. Action of potash, 1–1500, for 18^m, succeeded by lactic acid, 1–500, for 24^m.

1. Contraction of normal muscle.

2, 3, 4. Contractions of alkali muscle.

5, 6, 7. Contractions of acid muscle.

PLATE 10.

Fig. 40. *a.* Curve of normal gastrocnemius. Direct stimulation.

b. Ditto. After 10^m in soda solution, 1–3000.

c. Ditto. After 20^m in ditto.

Fig. 41. *a.* Curve of normal gastrocnemius. Direct stimulation.

b. Ditto. After 15^m in lactic acid solution, 1–2500.

c. Ditto. After 30^m in ditto.

Fig. 42. *a.* Curve of normal gastrocnemius. Direct stimulation.

b. Ditto. After 15^m in potash solution, 1–4000.

c. Ditto. After 15^m in lactic acid solution, 1–500.

d. Ditto. After 30^m in ditto.

e. Ditto. After 45^m in ditto.

Fig. 43. *a.* Curve of gastrocnemius which has been 20^m in barium chloride solution, 1–600.

b. Ditto. After 15^m in chloride of potash solution, 1–600.

c. Ditto. After 30^m in ditto.

Fig. 44. *a.* Curve of gastrocnemius which has been 80^m in chloride of lithium solution, 1–300.

b. Ditto. After 15^m in chloride of sodium solution, ·75 per cent.

c. Ditto. After 30^m in ditto.

d. Ditto. After 15^m in chloride of potassium solution, 1–800.

Fig. 45. *a.* Curve of normal gastrocnemius. Direct stimulation.

b. Ditto. After 30^m in chloride of strontium solution, 1–150.

c. Ditto. After 15^m in chloride of calcium solution, 1–150.

Diagrams of muscle chamber, A *and* B.

a, a. Influx and efflux tubes for solutions.

b. Vulcanite lid cemented into cork which closes the upper end of the chamber.

c. Sliding clamp which fixes the femur moved by milled-headed screw (*d*).

e Clamp for carrying wire for direct stimulation. The second connexion is made through the coiled wire (*f*), terminating in a hook (*g*) which passes through the tendon of the muscle.

h. Electrodes for stimulation of the nerve.

i. Metal cap closing central opening in stopper.

k. Accessory escape or oil tube.

l. Tube with hour-glass contraction, through which thread connecting tendon and lever works.

IX. *Description of Teeth of a Large Extinct (Marsupial?) genus,* Sceparnodon, RAMSAY.

By Professor OWEN, *C.B., F.R.S., &c.*

Received October 2,—Read October 15, 1883.

[PLATE 11.]

THE only known Mammals of Australia with rootless, ever-growing scalpriform incisors, in bodily size suitable for wielding those about to be described, are the *Diprotodon*, the *Nototherium*, and the *Phascolonus*, all of which have become extinct. But the incisors of the known species of the above genera differ in shape from each other and, in a still more marked degree, from those of *Sceparnodon;** nor do any such teeth from other and smaller Mammals match with the present Fossils.

My first cognizance of this form of tooth was derived from casts, which were kindly transmitted to me in October, 1881, by EDWARD P. RAMSAY, Esq., Curator of the Museum of Natural History, Sydney, New South Wales.

In the letter advising me of their transmission, Mr. RAMSAY writes:—

"The flat teeth are those for which I proposed the name *Sceparnodon*, but which name need not be retained by you, as no description has been published of them. The smaller of the flat teeth was obtained in the central part of South Australia; I believe near Lake Eyre. I found it among a collection which I was asked to determine at the Melbourne Exhibition, and I took casts of it. Those numbered A 3292, A 3295 came from Gelgoine Station, New South Wales, and were found in a deep hole in the creek which was being cleaned out for water; they are quite black, glossy, and seem to be impregnated with iron.

(Signed) "ED. P. RAMSAY."

Comparison of these casts of teeth, more or less mutilated, led me to the conclusion my valued correspondent had arrived at, and to the retention of the generic name proposed for the extinct animal to which they had belonged. But I deferred their description in the hope of receiving actual and better preserved specimens, affording also the means of adding characters of microscopic structures to those of size and shape.

A portion of such tooth (Plate 11, figs. 1, 2, 3) reached me this year, through the kindness of Mr. C. H. HARMANN, of the Range Nursery, Toowoomba, Queensland. It was found by him in the neighbouring bed of King's Creek, from which

* Σκέπαρνον, adze; ὀδούς, tooth.

formation the most instructive specimens of *Megalania* have been obtained. With this verification of the subjects of the casts, I no longer defer making known so singular an addition to the, most probably, Marsupial fossil Fauna of Australia.

The specimen from King's Creek, though mutilated at both ends, includes a portion of the tooth, 2 inches 9 lines (70 millims.) in length, with an uniform breadth of 1 inch 3 lines (32 millims.), and as uniform a thickness of 7 millims. gradually increasing to 8 millims. at both side-margins. The curvature of the tooth is uniform and moderate, and is shown in Plate 11, fig. 1.

Sufficient of the pulp-cavity was preserved at one end to indicate the tooth to have been one of uninterrupted growth; at the opposite end the cavity is reduced to a linear fissure, fig. 3 *a*. Here the body of the tooth is seen to be composed of hard dentine, with a coat of enamel on the convex side, bending for the extent of a millimetre upon each obtuse margin; the rest of the tooth having a thinner coat of cement. One margin, slightly broader and less rounded than the other, indicates that which was in contact with, or very close to, the fellow incisor of the scalpriform pair.

The enamelled surface of the tooth presents fine and close-set longitudinal striæ; its transverse convexity is less than the corresponding concavity of the opposite side. The concavity is traversed lengthwise by a pair of low, linear, risings or ridges, *r*, *r*, fig. 2, 5 millims. apart; one extending midway between the two borders, the other ridge being nearer the outer lateral border. Most of the surface of this fossil shows a deep rufous stain which extends some way into the tooth's substance, as shown by the sections for microscopical research next to be noted; the weight of the fossil indicates metallic infiltration.

Of the intimate structure of the teeth of *Marsupialia* I detected, in 1844,* but two which seemed to call for illustration: one, from the Wombat, showed a larger proportion of the cement exterior to the enamel than in the Rodents' incisors: this character, as it was exaggerated in the molars, was exemplified in a longitudinal section of one of those teeth (op. cit., plate 103, fig. 2). The other Marsupial modification was displayed by the teeth of the Kangaroo, and the illustration was afforded by a section of an incisor. The character in question is a continuation of the more wavy terminal portion of the dentinal tubes across the boundary-line into the enamel (op. cit., plate 102). In the subjoined drawing of a similar microscopic section of the fossil incisor of *Sceparnodon* the resemblance to the Wombat (ib., plate 103), in the dental character so exposed will be seen to be closer than to any other Marsupial or to any Australian genus of Rodent. The existing members of the *Rodentia*, native to Australia, are mostly of small size: the aquatic form, *Hydromys*, exemplifies the largest, but this hardly exceeds that of our Water-Rat (*Arvicola*), in which the upper incisors show a greater fore-and-aft than transverse diameter. The small relative degree of the former diameter, or thickness, of the tooth is peculiar to *Sceparnodon*.

* 'Odontography,' pp. 373–398; plates 98–103.

The dentinal tubes, as displayed under a magnifying power of 150 diameters (Plate 11, fig. 9, *d*,), show but a slight curvature in two-thirds of their course from the pulp-cavity, but become more curved in the distal third: the terminal bend, convex towards the cutting end of the tooth, is more strongly marked than in the incisors of *Phascolomys* or of *Macropus*. The diameter of the tube is $\frac{1}{8000}$ of an inch. The dichotomous divisions of the main tubes are sparing, until the greater curve is made, and the terminal branches of these open into minute cells along the line of the enamel, and occasionally into cells at some distance therefrom as shown at *d'*, fig. 9. The undulation of the fibres of the enamel, *e*, have a parallelism which renders a seeming course transverse to the section more conspicuous than the true direction toward the cement, *c*. This constituent repeats the microscopical characters of that of the Wombat's upper incisor.

The cast of one of the specimens of an incisor of *Sceparnodon Ramsayi* (Plate 11, figs. 4 and 5), from near Lake Eyre, includes the exposed or working end of the tooth, *a*; it is bevelled off to an edge from the logitudinally concave to the convex side, corresponding to the enamelled outer surface, and which forms, as in other scalpriform incisors, the trenchant margin. This edge slightly curves from the outer (lateral) to the inner (mesial) border of the tooth. Just above the worn surface of the tooth near this border the concave side shows a feeble depression, *b*, indicative, it may be, of pressure by a contiguous tooth. The transverse concavity of the back surface of the tooth is rather deeper than in Plate 11, fig. 1, and does not show the two longitudinal ridges. This character, if traceable in the actual tooth, might suggest a specific difference.

The fractured, probably implanted, end of the actual tooth may show the termination of the pulp-cavity, but the indication in the cast is obscure. The length of the specimen, in a straight line, is 4½ inches (90 millims.); following the curve it gives 95 millims. The breadth of the biting end of the tooth is 27 millims.; that of the opposite or growing end is 30 millims. This increase indicates a relation to the growth of the animal's body, and suggests that the incisor may have come from a not fully-grown individual. There is a corresponding increase of thickness at the (broken) implanted end, which gives 13 millims.

The cast of the largest of these teeth which I have received, showing also the largest proportion of the tooth, figs. 6 and 7, repeats the character of the mid-ridge along the concave surface; but a second ridge is more remote and less defined. The depression, *b*, above or root-ward of the abraded working surface is again indicated in this incisor. The longitudinal lineation of the convex side of the tooth (fig. 8) is of a coarser character than in the portion of tooth, fig. 3, from King's Creek. The length of the cast of the tooth from Lake Eyre, in a straight line, is 5 inches (130 millims.): following the curve it gives 136 millims.: the breadth is 1½ inches (35 millims.), and this dimension is the same at both ends of the tooth, indicative of its having come from a fully-grown individual.

It seems strange that the indications of an extinct species so conspicuous as must have been the living *Sceparnodon Ramsayi*, obtained from localities so remote from each other and showing a wide geographical range, should be restricted to a front incisor, seemingly of the upper jaw. Yet the first indication of the large Carnivore (*Thylacoleo*) was a solitary carnassial;* and that of the huge herbivorous *Diprotodon*, was but a fragment of a front lower incisor.† I am in hopes, therefore, of being favoured by analogous opportunities of communicating to the Royal Society a restoration, through successive contributions, of the skeleton and dentition of the present singular, most probably Marsupial, Rodent-like, extinct, Australian Mammal.

Description of the Plate.

PLATE 11.

Sceparnodon Ramsayi, Owen.

Fig. 1. Side or edge view of portion of an incisor tooth.

Fig. 2. Outer or convex side of the same.

Fig. 3. Inner, or concave, side of the same.

Fig. 3*a*. Cross-section of the same.

Fig. 4. Side or edge view of the cast of a larger portion of an incisor.

Fig. 5. Inner or concave side of the same cast; showing, *a*, the abraded surface; *b*, the indent above that surface.

Fig. 5*a*. Cross section of the same.

Fig. 6. Side or edge view of the cast of a still larger portion of an incisor.

Fig. 7. Inner, or concave, side of the same cast; showing, *a*, the abraded surface; *b*, the indent; *r*, median ridge.

Fig. 8. Portion of the outer surface, fig. 7′, showing the fine lineation of the cement-clad enamel.

(All the figures, save 9, are of the natural size.)

* Mitchell's 'Three Expeditions into the Interior of Eastern Australia, 8vo., 1838, vol. ii., Appendix, p. 359, plate 32, figs. 10, 11.

† Ib. ib., p. 362, plate 51, figs. 1 and 2.

X. *Evidence of a Large Extinct Lizard* (Notiosaurus dentatus,* OWEN) *from Pleistocene Deposits, New South Wales, Australia.*

By Professor OWEN, *C.B.*, *F.R.S.*, *&c.*

Received, January 9,—Read January 17, 1884.

[PLATE 12.]

ON the 19th November, 1883, I received from ROBT. ETHERIDGE, Jun., Esq., the subject of the present Paper, with the following memorandum which accompanied the specimen transmitted to him by CH. S. WILKINSON, Esq., F.L.S., F.G.S., of the Department of Mines, Sydney.

"Portion of jaw and teeth from Cuddie Springs. These springs are in pleistocene deposits full of bones of *Diprotodon*, *Sthenurus*, Crocodile, &c., as far down as they have been sunk into—viz., 30 feet."

The specimen was a small fragment, as will be seen by the annexed figures; and to the bone were attached the bases of the crowns of two teeth. These were of the size of the serial teeth of the Australian *Crocodilus porosus*, of similar shape, with longitudinally striated enamel.

Under the impression of Mr. WILKINSON'S note, I first compared them with the teeth in the series of Crocodilian skulls now exhibited in the Reptilian Gallery, of the Natural History Museum. But, though a longitudinally-ridged enamel is common to the teeth of other than the Australian species, in none were the ridges so strongly developed. Afterwards, submitting the fossil to a closer scrutiny, I observed that each ridge began by a pair of feebler ones rising from the root of the crown, and uniting after a course of from 2 to 5 millims.—a character not shown by any of the Crocodilian teeth; next, after close scrutiny of the broken portion of jaw to which the teeth were attached, I determined the parts of the bone which retained their natural unbroken surface.

The fossil in question was of a jet black colour, and the surface which I concluded to be the outer one of a dentary element of the mandible (Plate 12, fig. 1, *a*) shone as does a piece of polished jet.

Now such glistening exterior with perfect petrifaction characterises other fossil remains, especially of plants, from the same formations in Australia; and, as to the coal-

* Gr. νότιον, australis; σαῦρον, lacerta.

black colour shown in parts of such fossils, that also we find in some Mammalian fossils from our own tertiaries.*

The bases of both teeth in the pleistocene fossil (Plate 12, figs. 2–5, *b*, *b'*) were anchylosed to the alveolar floor continued from the outer wall *a*, and to this also was similarly anchylosed so much of the supposed part of the crown as remained of each tooth. The portion of the jaw-bone continued from the inner surface of the anchylosed teeth showed a natural surface sloping away from the teeth upon so much as remained of the inner surface, *a'*, of the dentary bone.

Here, therefore, were plain characters, not of a Crocodilian, but of a Lacertian mandible, and of a species of that division of the *Lacertilia* called "pleurodont."†

Of existing Australian Lizards *Chlamydosaurus* is "acrodont;"‡ *Hydrosaurus* is "pleurodont;" and, moreover, is the largest known existing Lacertian. The base of the tooth in this species is striated, and that character is best shown on the inner side (Plate 12, fig. 8), which is free from the bony parapet, according to the fashion exemplified in *Notiosaurus* (Plate 12, fig. 2); but with fewer and larger ridges.

I append figures, nat. size, of a portion of the jaw of *Hydrosaurus gigas* (Plate 12, figs. 7, 8), corresponding to the fossil. The proportions of the outer wall, and of the base of the teeth thereto anchylosed, are the same; such confluent part is, also, longitudinally ridged. The pleurodont character prevails in both upper and under jaws, but the teeth are mostly wider apart in the mandible, and are juxtaposed as in the fossil, only in a small proportion of the dentigerous part. At this stage of the comparison a vertical transverse section was taken of that end of the fossil to which the more fragmentary tooth was attached. This section (ib., fig. 5) demonstrated the anchylosis of tooth to bone according to the pleurodont type. A slice of the section was prepared for microscopic scrutiny. Under a magnifying power of 120 the coarse lamellate disposition of the osseous tissue of the Lacertian mandible, the elongate bone-cells, and the fine plasmatic tubules, diverging from the vascular cells, were demonstrated at *a*, fig. 9. The basally attached portion of tooth showed the Lacertain vascularity of the part and the dentinal tubes radiating from the vascular canals, also the lamellate walls of the canals (ib., *b*).

Another character was brought to light by this section. The remains of the pulp-cavity were seen, on first inspection of the fossil, in an aperture of 2 millims. diameter at the middle of the fractured surface of each tooth-crown, fig. 3, *c*, *c*. On the Crocodilian hypothesis such aperture should expose a pulp-cavity widening as it receded from the enamelled crown. In the section above described such cavity or continuation of the aperture was longitudinally traversed, and demonstrated its contracting to a termination at 6 millims. above the anchylosed base of the tooth (Plate 12, fig. 5, *c*).

In *Hydrosaurus* the outer surface of the dentigerous part of the mandible is perforated by neuro-vascular apertures almost as numerous as the teeth, and about the

* 'History of British Fossil Mammals,' 8vo., 1846, pp. 301, 414, 420.

† 'Odontography,' 8vo., 1845, p. 240, plates 67 (*Monitor*), 68 (*Iguana*). ‡ Ib., p. 241.

level of the base of the outer wall to which they are anchylosed. This character is also manifested in the mandibular fragment of the Notiosaur. The fossil has been broken away from the lower part of the ramus at the level of one of these apertures (Plate 12, fig. 1, *d*), and the fracture exposes the common canal (fig. 6, *d*, *e*), which was traversed by the mandibular vessel, and the branch leading from that canal to open upon the outer surface, in the same relative position to the free margin of the outer wall, as in *Hydrosaurus*.

And now, it may be asked, why may not the fossil here described, which has clearly come from a saurian as large as *Megalania*, be part of an individual of that extinct Australian genus ?

True it is, that as yet I have received no portion of mandible so associated with the rest of the skull of *Megalania* as to enable me to make the requisite comparison.

But so much of the skull, with the upper jaw, as has been recovered indicates that such jaw was edentulous, sheathed with horn, as in Chelonia,* and could not have been opposed to a series of large, mandibular, conical, carnivorous teeth. Such edentulous condition led to the inference that *Megalania* had been phytiphagous; and, like many herbivorous Mammals, it was proved to be provided with formidable horns as defensive weapons.†

In *Notiosaurus* we have evidence of a second form of Lacertian Reptile of ordinary Crocodilian dimensions, so far as these are indicated by the size and number of the piercing, lacerating teeth, of which the fossil in question shows samples.

I have taken the liberty to write to the Geologist of the Department of Mines, Sydney, requesting the loan of any other specimens from the Cuddie Springs which may have been regarded as Crocodilian.

Description of the Plate.

PLATE 12.

Notiosaurus dentatus.

Fig. 1. Portion of mandible, outside view.
Fig. 2. Ib. ib. inside view.
Fig. 3. Ib. ib. upper view.
Fig. 4. Ib. ib. end view.
Fig. 5. Ib. ib. vertical section of mandible and tooth-base.
Fig. 6. Ib. ib. under view.
Fig. 9. Longitudinal slice of mandible and tooth-base, magnified 120 diameters.

Hydrosaurus gigas.

Fig. 7. Portion of mandible, with two teeth; outside view.
Fig. 8. Ib. ib. inside view.

(All the figures, save fig. 9, are of the natural size.)

* Phil. Trans., 1880, p. 1045.

† Ib., p. 1048.

XI. *On the Total Solar Eclipse of May* 17, 1882.

By Captain W. DE W. ABNEY, *R.E., F.R.S., and* ARTHUR SCHUSTER, *Ph.D., F.R.S.*

Received April 9,—Read April 19, 1883.

[PLATE 13.]

PART I. (Drawn up by Dr. SCHUSTER.)

I.—*Introductory.*

THE present paper contains an account of the photographic results obtained during the last total solar eclipse. The total number of photographs taken was six: three of these represent the corona itself, while on the three others photographic records of the spectrum of the prominences and the corona were secured.

The expedition left England on the 19th of April and arrived at Suez on the evening of May 3, where they were received on behalf of the KHEDIVE by ESMATT Effendi and by the Governor of Suez. The following day was taken up with the journey to Cairo. The members of the expedition were welcomed at the station by STONE Pasha to whose foresight and energy, as well as extensive knowledge of the country, all the members of the expedition were much indebted throughout the time of their sojourn in Egypt. It was chiefly owing to the preparations which General STONE had already made that the expedition was able to leave Cairo on the following evening, arriving at Siout early on the morning of May 6. Owing to the low state of the Nile it was impossible for the expedition to reach the site which had already been chosen for the observatory the same evening, but they arrived there the next morning. The French expedition, sent out by M. BISCHOFFSHEIM, was already on the spot, and on them had fallen the burden of choosing the site; for as all the expeditions were to be the guests of the KHEDIVE, a separation would have been inconvenient to our host, and would have had no advantages as the weather was safe within the belt of totality. We cannot help expressing our admiration for the excellent way in which the site of the observatory had been selected; for not only did the result prove that the greatest length of totality had been secured, but local circumstances were well attended to, and the observatory was well protected against the dust, which formed the greatest danger to the success of the expedition.

Colonel MOKHTAR Bey had accompanied the English party to the observatory by

order of the KHEDIVE, and was in charge of the joint expeditions during their stay at Sohag. Great thanks are due to his unremitting zeal and energy; his aid in erecting the observatory and telescopes proved of great value, and his mechanical skill helped the expedition over great difficulties.

ESMATT Effendi, also a member of the KHEDIVE's household, gave most valuable assistance, and to him, as well as to many others equally able and anxious to help, our thanks are due.

II.—*Preparations for totality.*

The observatory was built on a level piece of ground close to the banks of the Nile. It was surrounded by a double wall of sugar-canes, affording an efficient protection against the dust which occasionally was carried in large quantities along the river. The ground in the immediate vicinity of the observatory was, in addition, constantly kept wet by watermen, so that the danger to the instruments from the dust and sand was reduced to a minimum.

Half of the ground covered by the English station was taken up by Mr. LOCKYER's large equatoreal telescope, while the other half was reserved for the photographic instruments. The present report only refers to the results obtained with the latter. A double layer of bricks afforded a sufficiently firm foundation for the stand on which the cameras were mounted. The stand and clockwork had originally been made for the transit of Venus expedition of 1874, and on that occasion supported one of the photoheliographs, but during this eclipse it had to carry three cameras. An achromatic lens of 4 inches clear aperture, having a focal length of 5 feet 3½ inches, was reserved for the photographs of the corona itself. The image of the moon taken with this lens during the eclipse had a radius of ·29 of an inch. The second camera, carried on the same stand, was on a similar principle to that first used during the Siamese eclipse of 1875, and then called a prismatic camera. It was an ordinary camera with a prism placed in front of the lens. The prism used on this occasion was of white and dense flint glass, and instead of an angle of 8° it had a refracting angle of 60°, and each face had a surface of 3 inches square. The corrected lens used with the prism had a focal length of 20 inches for the yellow rays. As the plates exposed in this camera were sensitive in the red as well as in the blue, it was impossible to obtain the whole range of the photographic spectrum in focus on the plate, but in order to obtain the best results the back of the camera which carried the sensitive plate could be tilted so as to bring a larger range of the spectrum into focus at the same time. Nevertheless, an inspection of the plates obtained in this camera shows that the parts extending from the green into the infra-red are the only ones which are properly in focus, and it is to these parts that our attention was chiefly directed with this instrument.

To the third camera was attached a complete spectroscope. A lens of 2 inches aperture and 11 inches focal length served to form an image of the corona on the slit.

Both collimator and camera had a focal length of about 9 inches; the prism had a refracting angle of 62°, and an aperture of 2 by 1½ inches. The sensitive plate in this camera could be tilted with the axis of the lens in the manner already explained, and the spectrum is in fair focus throughout the range of sensitiveness of the plate. In fixing the time of exposure for each plate and camera we had to consider the probable intensity of the light, but we had also to take into account the loss of time involved in changing the plates. At first it seemed, indeed, as if 12 seconds would be required to change each slide, but after a good deal of practice the time thus lost was finally reduced to 7 seconds, though even this was a considerable proportion out of the total of 70 seconds during which totality was to last. It was therefore decided that only one plate should be exposed in each of the spectroscopic cameras; but that three photographs of the corona should be attempted with the large lens. A trial was also to be made to obtain an impression of the first flash of light at the end of totality with the prismatic camera. A detailed account of the times of exposure of the different plates actually obtained will be given under the head of "Results."

It was arranged that Mr. WOODS, who was the assistant for the photographic work, should, during totality, cover and uncover the different lenses at the proper times, whilst I undertook to change the slides. Mr. J. Y. BUCHANAN was kind enough to call out the time at intervals of 10 seconds during the whole of totality. An arrangement of this kind is absolutely necessary where plates have to be changed, and where each second is of importance.

The telescope was mounted and adjusted in the usual way; the adjustment for latitude was not perfect, and a firmer foundation would have been desirable, but as a very slight motion was of little consequence in the spectroscopic cameras, while the longest exposure for the corona was only to last 22 seconds, even a rough adjustment would have been sufficient; and the results were in no way affected by this want of adjustment or by any irregularity in the motion of the clock. The final adjustment of the clock was made only a few minutes before the beginning of totality, and its behaviour during the critical time was better than could have been expected from its previous performance.

In order to fix the position of the corona on the plate from its photographs, a platinum wire was stretched across the camera close to the sensitive plate. Before and after totality the clock was stopped, and photographs of the solar cusps were taken at an interval of 2 minutes on the same plate. In this way the position of the platinum wire could be ascertained relative to the sun's path directly before and after totality. The accurate orientation of the solar corona in the sky is a very important matter, and one which has been too much neglected in some of the recent eclipses.

All the plates were gelatine plates, specially prepared by Captain ABNEY for the occasion. Those used in the prismatic camera, as before stated, were sensitive in the red as well as in the blue. From calculations made, it appears that all the plates used

had a sensitiveness for the blue end of the spectrum between fifty and sixty times greater than an ordinary wet plate. This is a matter of no slight importance when it is remembered that in comparison with photographic observations made during all other eclipses, except, perhaps, that of 1878, this exquisite sensitiveness has in effect turned seconds of time into minutes, thus enabling much more to be done in a short totality than could be accomplished in a much longer one when using the older process.

III.—*The totality.*

During the present short eclipse it was of special importance that the signal at the beginning of totality should be correctly given. If given too early, the plates would be spoiled by the presence of the last gleam of the sun, while if too late valuable time would be lost, and there would be the danger of exposing the plates beyond the end of totality, and thus again destroying their value. Experience during the last two eclipses had taught me that the corona was distinctly visible four or five seconds before totality, and I therefore determined on giving three signals: the first when the rapidly waning crescent showed that the critical moment was approaching; the second when I could see the corona against the dark limb of the moon; and finally when totality had set in. Mr. BUCHANAN undertook to measure the interval between the two last signals. In order to make sure that all the lenses were covered at the end of totality, it was decided to utilise only 65 seconds out of the 70 we hoped to have. Proceedings during totality were carried out strictly according to the programme laid down. My time and attention were so much occupied in looking after the slides and cameras that I had only a few seconds to spare for eye-observations, and no value can be attached to impressions gained in so short a time. I did not, indeed, notice any striking difference in the appearance in this and the two previous eclipses, except the presence of the prominences which I now saw for the first time, and which seemed to colour the greater part of the inner corona with a tint which appeared to me to be much more of a light orange than of a red colour. The photographs show, however, a very considerable difference between the outline and distribution of the corona in this and the three last eclipses. In the short time at my disposal I did not notice the comet, to which reference will be made, though it was very conspicuous to the majority of the party.

IV.—*Time observations.*

It was not part of the regular programme of the expedition to take any regular time observations, and the following results have therefore little value, except, perhaps, so far as the duration of totality is concerned.

The longitude and latitude of the place of observation was determined by M. TREPIED: the former by means of four lunar culminations, the latter by means of a

meridian passage of the sun observed with a small meridian circle. The results obtained were:

Latitude: 26° 33′ 21″
Longitude: 1° 57′ 40″ east of Paris, or
2° 07′ 01″ east of Greenwich.

M. TREPIED considers these values as provisional only.

I observed the first contact through the finder belonging to Mr. LOCKYER's equatoreal (aperture 3 inches). As I was, however, not accustomed to use that instrument, I forgot that there was a reflecting mirror in the eyepiece, and therefore locked for the contact at the wrong place. I first noticed the moon on the solar disc at

$7^h\ 20^m\ 14^s$ (L.M.T.).

The first contact was observed by M. TREPIED at

$7^h\ 20^m\ 09^s$ (L.M.T.).

The observation for the last contact was, of course, much better. I gave the signal at

$9^h\ 54^m\ 46^s$ (L.M.T.).

M. TREPIED gives for this contact

$9^h\ 54^m\ 57^s$ (L.M.T.).

At the beginning and end of totality the whole party was naturally so much occupied with their special work that nobody was available to mark down the time of the signals. Mr. BUCHANAN, however, by means of a stop-watch, measured the time intervening between my two signals, which meant respectively that I saw the corona and that totality had begun. That time was 4 seconds, and though with suitable instruments the corona might, perhaps, be seen sooner, I feel confident that with the naked eye my observation gives a fair estimate of the time of visibility of the corona before totality. The time agrees well with that determined by me during the eclipse of 1878 in Colorado, where I had measured it as 5 seconds. Mr. BUCHANAN also started a pendulum clock when I gave the sginal for totality, and stopped it again at the end of totality. The time of duration of the eclipse was thus determined, and I have considerable confidence in the accuracy of the results to within one or two seconds. Owing to the brilliancy of the chromosphere and the inner parts of the corona it is by no means easy to fix the beginning of totality with absolute accuracy, especially if the observer has never previously witnessed an eclipse. The experience gained during previous eclipses gives me confidence, however, as to the duration on this occasion. The calculated time of totality was 72 seconds, the interval between my signals was 74 seconds, being probably too long, though not by more than one or two seconds. As the question of duration of totality has some interest, and as different observers differed considerably in their estimate, it may be well to give the opinion of some

of the other observers within hearing of my signals as to their accuracy. Mr. BAILLIE, who had joined the expedition, and to whose excellent sketch of the corona we shall have occasion to refer, wrote directly after the eclipse as follows: "I was seated at my table just before totality, and, being unable to look upwards, kept my eye on the watch in my hand, waiting for SCHUSTER's signal. Immediately upon hearing his voice I looked up at the sun. Totality was complete . . . . ."

Mr. LAWRANCE, Mr. LOCKYER's assistant, who was looking through Mr. LOCKYER's finder, writes as follows: "The moment Dr. SCHUSTER gave his signal of commencement of totality, I still saw BAILEY's beads and a thin crescent of the sun still uncovered. I think, however, that totality commenced within a second or two. . . . . ."

It appears from Mr. LAWRANCE's statement that my signal could not have been given too late, while the photographs give conclusive proof that I could not have been too early by any appreciable amount. For as I called out I drew the slide of the spectroscopic camera, and Mr. WOODS, on hearing me give the signal, removed the cap from the prismatic camera. The plate in this camera would have been spoiled had any sunlight been allowed to fall on it. I may have been a little late at the end of totality, as Mr. BAILLIE remarks, but when a signal is given by calling out, a good fraction of a second must necessarily intervene between the time that a phenomenon is seen and the time that the signal is actually given and realised by another observer. On the whole, it may be said that the calculated time of totality was as nearly realised as possible.

PART II.—(Drawn up by Captain ABNEY and Dr. SCHUSTER.)

V.—*The photographs of the corona.*

Across all the photographs of the corona is a transparent line, which is the shadow of a platinum wire stretched immediately in front of the slide placed there to enable us to fix, with accuracy, the position of the corona. Before and after totality the clockwork of the telescope was stopped, and the solar cusps were photographed at fixed intervals. The position of the platinum wire could thus be referred to the line of the sun's motion. On a paper print of these photographs the position of the centres of the solar images was determined as accurately as possible, and it was found that a line joining these centres made an angle of about 45 minutes with the image of the platinum wire. Thanks to the courtesy of Mr. WHIPPLE, we had an opportunity of measuring our photographs with Mr. DE LA RUE's instrument, which is now deposited at Kew. The photographs taken previous to totality showed a sufficient part of the sun uncovered to enable us to draw a tangent, and thus to determine with great accuracy the direction of the sun's motion. This method gave an angle of 49 minutes between the wire and the circle of declination, a result practically identical with that previously obtained. The angle between the lines joining the cusps and the wire are slightly different, but this is due to the moon's motion over the solar disc. Owing to

the rapid change in the lunar parallax, the calculation of the angles which the line of cusps should make with the circle of declination is rather tedious, and would not repay the trouble, and without it the orientation of the photographs possesses all the necessary accuracy. In adopting 45 minutes as the angle between the shadow of the wire and the declination circle, we shall certainly not make an error greater than half a degree, and most probably not more than a quarter of a degree. The image of the wire lies in the quadrant between the east and the south.

The backs of the plates in all the cameras were coated with asphaltum, in order to prevent photographic irradiation as far as possible, but the prominences and inner parts of the corona sent out such a strong light that in spite of this precaution the photographic effect encroaches on the disc of the moon. One of the prominences appears even reversed on one of the plates which was exposed for about 22 seconds. The images of the prominences themselves appear, however, perfectly defined. The different times of exposure could not be measured exactly for want of sufficient attendance. A very fair idea of the exposures can, however, be obtained from Dr. SCHUSTER's notes made immediately after the eclipse. He wrote :—"After drawing the slide of the spectroscopic camera at the beginning of totality, I immediately did the same for the large camera, and called on Mr. WOODS to remove the screen, which he held up in front. This he did, noticing that Mr. BUCHANAN called out '60' at the time. At the time signal '50' the screen was to be replaced, thus giving an exposure of 10 seconds. Mr. WOODS was, however, a little late in answering to the signal, and I believe that 11 seconds would not be a bad estimate for the exposure of this plate. There was no record made of the time at which the second plate was actually exposed, but experience had shown that it took about 7 seconds to change a plate in this camera, and as the exposure ended at the time signal previously fixed upon, I think we may take the exposure of this plate to have been 23 seconds within one second of error. The third plate was to be exposed for 3 seconds; these were estimated by Mr. WOODS, and as he has had much practice in exposing photographs, this time was probably correct within narrow limits."

The body of the moon appears on the photographs bordered by a well-defined black line, out of which the prominences are seen to rise. It must be borne in mind that the radius of the moon during the last eclipse was only slightly larger than that of the sun, so that even in the middle of the eclipse the brightest parts of the corona, and perhaps even the upper layers of the chromosphere, were visible. The inner parts of the corona, and especially the prominences, are over-exposed in all the photographs, but in that one which had three seconds' exposure all the details up to the body of the moon are well shown.

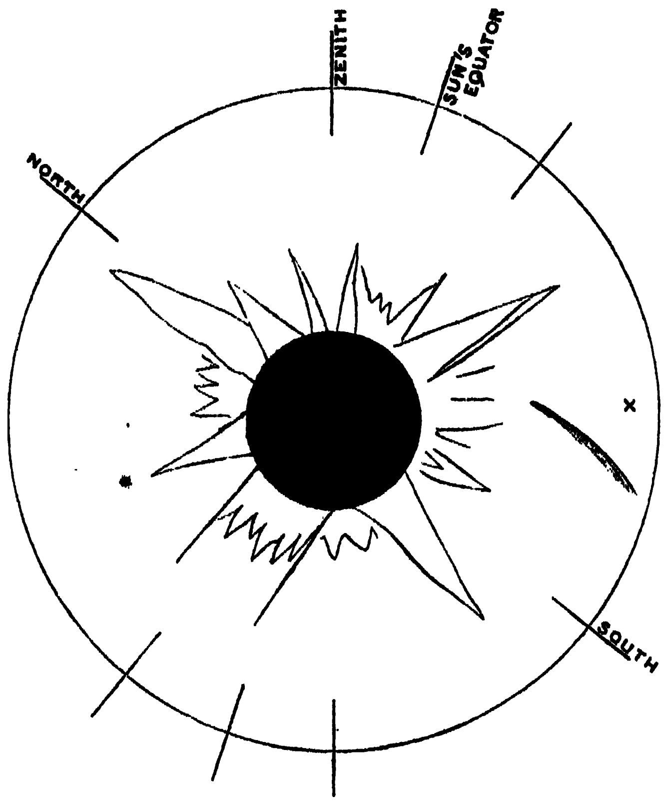
As regards the corona, we are especially struck with the irregularity of its shape. A close connexion between the outline of the corona and the state of the sun's surface is now placed beyond doubt. The corona, as seen in Colorado during the summer of

1878, may be taken as a type of what is seen at a time of few sun spots. We notice a great extension in two opposite directions. Unfortunately it has never been properly decided whether this direction agrees with the ecliptic or with the direction of the sun's equator, or does not accurately agree with either. There is, however, a probability that the direction of greatest extension varies somewhat in different eclipses. This equatoreal extension (for we may so call it, without implying any hypothesis as to its cause) has, as a rule, a form which the Siamese in 1875 not inaptly likened to a fishtail, and for this reason the longest streamers of the corona are found not near the solar equator, but symmetrically at some distance from it. Another remarkable feature of the corona of the time of sunspot-minimum are the generally short and curved polar-rifts. They are found on all good drawings and photographs of the corona, when the eclipse took place at periods of small solar activity, and have often been compared to the rays of our aurora borealis. In our photographs we look in vain for the equatoreal extension and for the polar rifts. Streamers are seen to stretch away in all directions from the moon's edge, but there is no regularity, whatever. The extent to which the corona can be traced in the photographs depends of course, on the time of exposure and the sensitiveness of the plates; and considering the great progress which has lately been made in the science of photography, it is not, perhaps, astonishing, that our photographs show a greater extent of corona than any of those previously obtained. One of the streamers reaches, indeed, to a distance of 44 minutes of arc from the sun's limb, that is about 1·4 solar diameter. The details shown on the different photographs are very interesting, but they cannot be described, and are only shown on a good drawing. Two points, however, deserve a special notice. One is the remarkable curvature of some of the coronal rays. It has long been known that these are not straight, but their general curvature shows more regularity than it did on the last occasion. The rays seem in many cases to start almost tangentially from the sun's limb; they are as a rule wider near the sun's limb, and contract as their distance from the sun increases, while others are spread out in fan-like shape. The second point to which we wish to draw attention is the transparency of the streamers: in two instances at least we can trace structural details through the luminous streamers. The distinction which has been drawn between the outer and the inner corona appears to us to be justified, the inner corona being decidedly more compact and luminous. The corona, as seen during this eclipse, does not seem to bring us any nearer to any plausible and scientific theory as to its causes. Two theories especially have been brought forward and discussed, and both of them seem to become less and less likely. The corona of last May is conclusive against any theory of meteor streams as far, at any rate, as its streamers are concerned, and the rival theory of a much disturbed solar atmosphere seems open to equally conclusive objections. It is only by means of continued observations that we may hope to solve the coronal mystery; and these we are likely to have before long, thanks to Dr. HUGGINS' important discovery.

Mr. BAILLIE's *drawing of the corona.*

Mr. BAILLIE accompanied the expedition and undertook to make a drawing of the corona during totality. Though little attention has been paid of late to pencil sketches made during eclipses, we yet venture to lay the present one before the Society, as it shows how much can really be done in so short a time by a skilled

Fig. 1.



Mr. BAILLIE's drawing of the corona.

draughtsman, who aims less at an artistic production than at the correct representation of what he sees. Mr. BAILLIE's drawing is of value as showing the relative extent of the corona as seen with the naked eye and as shown on the photographic plates. We can trace on Mr. BAILLIE's drawing all the more important streamers, and they agree in length as well as can be expected with our photographs, thus showing that there was no perceptible difference in the extent of the visible and the photographic corona. This is confirmed by the remarks made by Professor TACCHINI on the extent of the visible corona.

The comet.

Some of the observers noticed during totality a luminous streak of light presenting exactly the appearance of a comet, which our photographs prove beyond doubt

to be the case. The nucleus is exceedingly well and sharply defined, the tail is somewhat curved; it did not point towards the sun's centre, but in a direction nearly tangential to the limb. The extent of the tail was roughly two-thirds of a solar diameter.

Our photographs allow us to fix the position of the comet. It appears that the nucleus was at a distance of 45′ 50″ from the sun's centre during the middle of the eclipse, and that the line joining the comet to the sun's centre made an angle of 16′ 27″ with the circle of declination towards the west. Professor TACCHINI (C.R., xcv., p. 896, 1882; Memorie della Società degli Spettroscopisti Italiani, vol. xi., 1882) gives slightly different values. According to him the comet's position was: Dec. 18° 30′ 17″ N., R.A. 3^h 35^m 16^s. This would give a distance of 52′ 26″ from the sun's centre, and a position angle of 68′ 09″, 21′ 51″. According to our measurements the position of the comet at 18^h 24^m 36^s G.M.T. was—

Dec. 18° 34′ 59″ N.
R.A. 3^h 34^m 43^s.

An examination of our different photographs shows a slight but progressive change in the comet's position. This is in part accounted for by the moon's motion over the solar disc during the eclipse, for the position of the comet had of course to be referred to the dark lunar disc. The change in the distance of the comet from the moon's centre is however slightly larger than can be accounted for by the motion of the moon, and is probably in part due to the proper motion of the comet, which in that case must have moved away from the sun during the eclipse. The motion, if it exists, must however have been very small, and as the matter presents very little importance we have not investigated it further, especially as the comet in all probability will not be heard of any more. The different eclipse parties present at Sohag decided at a joint meeting after the eclipse to give the name of TEWFIK to the comet, in recognition of the KHEDIVE's generous hospitality.

Results of the prismatic camera.

This instrument consisted, as has already been explained, of a camera the lens of which had an aperture of 3 inches, and a focal length of 20 inches in the yellow. The prism, which was placed directly in front of the lens, had a refracting angle of 60°. The line of dispersion projected on the celestial sphere ran nearly north and south, the less refrangible side being towards the south. Only one plate was exposed, and that for 65 seconds. The first impression gained by an inspection of the photograph thus obtained would lead one to believe that sufficient care was not taken in focussing the camera. As before stated, however, it was quite impossible to have the whole of the spectrum in focus at the same time, and as special attention was directed to the less refrangible part in this instrument, it is obvious that

the violet and ultra-violet in which the strongest impressions occur could not be in proper focus. The solar line F, and everything which is below, are well defined.

It is clear that with this instrument we obtain a series of rings corresponding to the different rays sent out by the prominences. We are at once struck by the intensity of two of these rings lying close together near the boundary between the violet and ultra-violet. A comparison with the photographs obtained in the spectroscopic camera leaves no doubt that these rings are due to calcium, and are in fact coincident with the solar lines H and K. They may serve therefore as a starting-point for the determination of other wave-lengths. Measurements of the distances between the different prominence rings, and comparison with a photograph of the solar spectrum, taken with the same prism, renders it easy to identify the hydrogen lines, Hα (C), Hβ (F), Hγ (near G), and Hδ (h), and these may again stand as reference lines for the other images.

Three prominences especially are noticeable by their great intensity. We shall designate them by the numbers I., II., and III. I. and II. were close to the east point of the sun, III. was a little more towards the north. Next in intensity came a prominence (V.) near the west point of the sun. I., II., and III. show all the hydrogen lines in the visible part of the spectrum, V. shows all but C. A series of prominences towards the southern edge of the sun show F strong and Hγ very distinctly; a prominence on the northern edge (IV.) shows Hγ faintly, and only a trace of F. The prominence III. shows a number of lines in the ultra-violet. Want of focus renders them difficult to measure on this plate, but as the slit of the spectroscopic camera happened fortunately to cut the same prominence, the want is fully supplied. It is found that these lines are in part due to the same hydrogen lines which Dr. HUGGINS has photographed in several star-spectra. The prominences I. and II. show these lines also, but not so markedly. It ought to be mentioned that the prominence which we have called III. is by far the strongest in the direct photographs of the corona, and is that which we have already stated to be centrally reversed in one of the plates. The results of the prismatic camera show that the intensity of III. as compared with I. and II. was most marked in the most refrangible part of the spectrum. The same relation of relative intensity holds in the line coincident with C, but is reversed with F, I. and II. being here the strongest. This points to the conclusion that I. and II. were cooler than III., for we know that on cooling Hβ (F) becomes the strongest hydrogen line, while other lines gain in relative intensity on heating. The prominence IV. gives an anomalous result, showing Hβ (F) and Hγ (near G) but the latter with greater intensity. It was perhaps a hot, but thin and therefore black prominence.

We may turn now to the lines due to other substances. The wave-lengths were determined by measuring the distance of the image of any prominence to the corresponding image of one of the hydrogen lines. A first approximate result is obtained by employing the ordinary interpolation formula which is based on the

supposition that differences in the refractive indices of two lines are proportional to the differences of the inverse squares of the wave-length. Owing to different circumstances, and amongst others to the fact that the distances measured are not quite proportional to differences of the refractive indices, we obtain in this way approximate results only; but by taking a photograph of the solar spectrum with the same prism in the same position, we can easily determine the correction which has to be applied in order to arrive at the most satisfactory result. For instance, the distance of one image was measured from the corresponding image of C and F, both with a micrometer and directly with a finely divided scale. The interpolation formula gave 5889 by the first method of measurement and 5893 by the second, the mean being 5891. This brings us near D, and if a similar calculation is made in the reference spectrum, interpolating D between C and F, a wave-length 5907 is obtained. The true wave-length being 5882, we see that we have to apply a correction of -15 in this part of the spectrum. We thus find finally 5876 for the wave-length of the unknown line, agreeing almost exactly with the wave-length (5875) of D_3. The prominences I. and II. gave results which are practically identical.

Similarly we find for a very faint image of the prominence II. a wave-length 5315, which is evidently the well-known corona line (K 1474), which has a wave-length 5316. The image, however, is exceedingly faint. The prominence I. shows also two lines in the infra-red which are difficult to identify. One of them is very likely $\lambda=8240$; the other has a wave-length which is certainly above λ 10,000, but by how much we cannot tell. Two lines, $\lambda=4471$ and $\lambda=4394$, are seen in the blue; 4471 is the line f which is always present in prominences, but what 4394 is we do not know. YOUNG, in his catalogue of chromospheric lines, notes that he saw 4394·6 fifteen times out of 100 observations.

Besides these well-defined prominences the photograph shows two rings, which are evidently due to the lower parts of the corona, and therefore correspond to true coronal light. The wave-length of one of these rings was measured to be 5315. It is due to the green corona line (K 1474); the second is coincident with D_3. The ring in the green is particularly strong in the south-western quadrant, and hardly visible at some of the other points of the sun's limb. The yellow ring is much fainter on the whole, but more uniform all round the sun. In addition to the rings and prominences we observe on the photograph at places a certain striped appearance, which is due to continuous spectrum belonging to the prominences or to intense parts of the solar corona round the sun's limb.

A curious and rather puzzling point remains to be noticed. The prominence V. in the F ring seems to be duplicated. Two distinct impressions appear, one above the other. This cannot be due to a shift in the camera, for the other prominences, some of which are more intense, do not show the duplication. The only possible explanation which has occurred to us is, that one of the images is due to the comet, and that it is only by accident that it appears so close to an image of the prominence.

An instantaneous photograph taken in the prismatic camera about 5 seconds after the end of totality presents one or two peculiarities which are worth noticing.

In the first place, the prominences still appear, except at the places at which their light is overpowered by that of the sun. The continuous spectrum of the sun is broken up into three or four parallel bands—an appearance no doubt due to the irregularities on the moon's surface. We have here BAILEY's beads drawn out into bands. At the solar cusps traces of rings are seen which are due to the lower parts of the chromosphere extending beyond the cusps. The two brightest of these rings correspond apparently to the solar lines F and G.

We give in conclusion a table showing the lines which are seen in different prominences. The numbers indicate the order of intensity for the same line in different prominences. Thus, for instance, it is seen that in I., C is fainter than in the prominence II.; while the order is reversed for the line F.

RELATIVE intensities of lines in the prominences.

Prominence =	I.	II.	III.	IV.	V.	VI.	
Below λ 10,000	Faint						
8240 (?) ..	Distinct	..	..	..	..	..	Probably a hydrogen line.
6562 (C) ..	3	2	1				
5875 (D_3) ..	3	2	1				
5315 (1474*) ..	Very faint						
4861 (F) ..	2	1	3	Very weak	5	4	
4471 (f) ..	3	2	1				
4394	..	..	1				
4340 ($H\gamma$) ..	3	2	1	Weak, but stronger than F	4	5	
4101 (h) ..	3	2	1	..	4		
3968 (H) ..	3	2	1				
3933 (K) ..	3	2	1	..	5	4	
	Many lines in ultra-violet	..	..	..	..	..	

Results of the spectroscopic camera.

The slit of the spectroscope had a direction approximately north and south, but it did not accurately pass through the centre of the solar disc, and we have no data to fix accurately on the regions of the corona through which it passed, although its approximate intersection of the image can be guessed, as will be seen further on.

The photograph shows, in the first place, a strong continuous spectrum. This is neither equally strong nor equally extended on both sides of the moon's disc. On the northern side the spectrum can be traced to a wave-length of about 3490 towards the ultra-violet; but on the southern side it reaches further and almost as far as the solar

* KIRCHHOFF's scale.

spectrum, which had before and after the eclipse been photographed on the same plate as a reference. Towards the less refrangible side, the solar line E forms the approximate boundary, though on the southern side the continuous spectrum reaches a little further. The continuous spectrum extends from the moon's limb on both sides with great intensity up to a certain height, then there is a sudden falling off in intensity, decreasing steadily on the southern side from this point up to a height at which it cannot be traced any more; but on the southern side there seems to be a slight increase again in intensity as we go towards greater distances from the sun's limb; then there is again a sharp boundary at which a sudden decrease in intensity is noticed. From that point the continuous spectrum gradually vanishes.

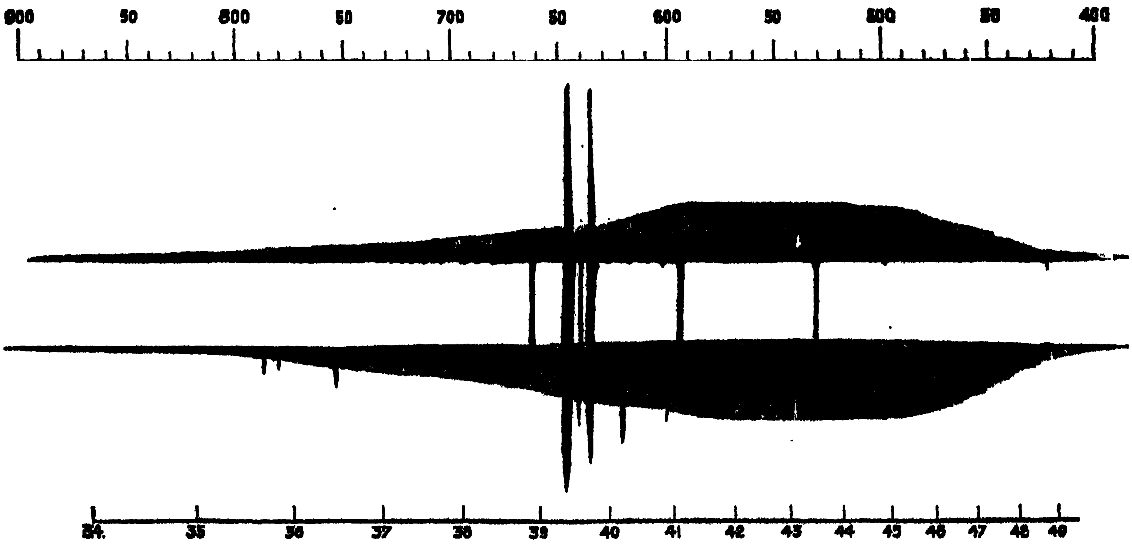
The following numbers will give an idea of the extent to which the continuous spectrum reaches at different points on the southern side:

	Height of continuous spectrum in terms of a solar radius.		
Near F	..	..	·2
For λ=4489	·37	·53	1·23
Near G	·29	·60	1·47

The first column refers to the first sharp decrease in brightness; the second column to the second falling off in intensity; and the last column gives the limit to which the continuous spectrum can be traced. On the northern side the spectrum does not reach as high, thus near G it cannot be traced further than ·9 of a solar radius.

In the lower regions the spectrum appears perfectly continuous, but in the upper regions the solar line G appears reversed.

Fig. 2.



Spectrum of the corona, from a photograph. The top scale is that of the micrometer, the bottom scale wave-lengths.

The prominence spectrum.

Close to the solar limb, on both sides of the photograph, bright lines are seen which evidently belong to the prominences. On the northern limb these are most numerous and brightest. It seems certain, in fact, from a comparison with the photograph obtained in the prismatic camera, that the slit must have crossed the prominence 3, which, as has already been mentioned, was rich in ultra-violet light. On the same plate above and below, the solar spectrum had been photographed after the eclipse, and we had, therefore, no difficulty in identifying the principal lines. We see in the first place the calcium lines H and K. These two lines are, indeed, the great feature of the photograph. Being of great intensity they do not confine themselves to the prominence regions, but appear as bright lines through the corona and over the body of the moon. The light which is due to them must, in fact, have been so strong that the scattered light was sufficiently intense in our atmosphere to give the appearance and to be conspicuous everywhere in the neighbourhood of the moon, or else that the lens forming the image on the slit was so illuminated as to give this result. It can, indeed, be traced to a distance of quite three solar radii. In addition to these lines the hydrogen lines must, of course, be expected to be present; and, indeed, their identification presents no difficulties. The full series is present, including those photographed by Dr. HUGGINS in the ultra-violet spectra of stars. Once these lines had been traced, and their characteristic distribution identified, the work of mapping was rendered comparatively easy, as they could be used as reference lines, and thus the wave-lengths of the remaining lines determined. The result is given in the following Table. The first column gives the intensity of the lines as they appear in the prominence, 1 denoting the greatest and 6 the smallest intensity. The second column gives the wave-lengths as determined by means of the hydrogen lines. In the third column the origin of the lines is identified as far as possible. In the last column we have added the numbers given by Dr. HUGGINS for the lines seen in α Aquilæ; and it will be noticed how, in the ultra-violet especially, many of the lines which we have not been able to identify are common to our photograph and to the spectrum of α Aquilæ. This correspondence was so striking that we felt justified in bringing it forward. The relative intensities of the less refrangible lines cannot be given, as their own light is so much overpowered by the continuous spectrum that even a strong line can only be traced with difficulty.

Intensity.	Wave-length of prominence lines.	Comparison.	Dr. Huggins' spectrum of α Aquilæ
	4861	Hβ (E)	
	4473	4471 (f)	
	4340	Hγ	
			4230
			4172·5
			4131
			4120
	4076	4077 (Ca)	4072
	4049		
6	4025	..	4022·5
	..	..	4000
	..	..	3997
6	3989		
1	3968	H	3967·9
4	3955	Ca	
1	3933	K	3932·8
	..	..	3915
2	3888	Hα	3862·5
6	3859±6	..	3854
3	3834	Hβ	3834
6	3816	..	3816
			3807·5
4	3795	Hγ	3795
4	3768	Hδ	3767·5
3	3757	..	3757·5
5	3746	Hε	3745·5
6	3730	Hζ	3730
6	3718	Hη	3717·5
6	3708	Hθ	3707·5
5	3699	Hι	3698
6	3693	..	3690
4	3680	..	3677·5
6	3674		
6	3667		
6	3658 }	..	3656
	* 3653 }	..	3654
	3635	Ca (?)	3637·5

The line 3859 was entered as a band, terminating sharply at 3865 and 3853; similarly the two lines 3658, 3653 appear on our photograph as joined together. The line 3955, which appears between H and K, is very likely the representative of a calcium triplet, photographed by Professors LIVEING and DEWAR, and having, according to them, a wave-length of 3972·3, 3956·0, 3947·9. The most refrangible of the lines appearing in the prominence seems also likely to be due to calcium; for there is a calcium triplet marked as very strong, diffuse at 3644, 3631, 3623·5. We are obliged to Professors LIVEING and DEWAR for their information respecting these lines.

The line spectrum of the corona.

It has been mentioned that close to the sun's limb a strong continuous spectrum hides any lines which might be present in these regions. We have reason to believe

that there are indeed a number of lines in this region. Over the fainter portions of this continuous region lines can be distinctly traced. A few of them are sufficiently strong to be seen with the naked eye, but most of them require the use of a magnifying glass. The lines are no doubt very faint, but we have succeeded in showing them even to some whose eyes were not specially trained to examine faint photographic impressions, and there cannot be the slightest doubt as to the real existence of these lines. It is more difficult to say to what extent our measurements can be considered as accurate. They have been taken with a measuring microscope, but it was only when the light was good, and when the eyes were in their best condition, that satisfactory results were obtained. We give the wave-lengths as being as exact as we can hope for with the small dispersion employed. Four or five of the lines can be easily seen under ordinary circumstances. These are one line less refrangible than G, of very peculiar appearance. At its base it is a broad band, then contracting quickly it fades away into a sharp point; close to it is a sharper and longer line. A little less refrangible than H is an easily recognised line reaching to a good height away from the sun (more than a radius). Two or three lines in the ultra-violet are also easily recognised.

The following list of lines does not pretend to be complete. There are some lines, especially in the ultra-violet, which have not been included, but they are so faint that their measurement would have been very difficult. As it is, even some of the lines given in the table can only be seen under specially favourable circumstances. On the northern side only a few of the principal lines can be traced. We have sometimes thought that a different set of lines appeared there, but the measurement and interpretation of the northern line present some difficulties. In the first place the spectrum of the corona is much fainter, and secondly the spectrum of the prominences is much stronger. Most of the prominence lines, like H and K, stretch across the moon's disc, and therefore also across the corona. Amongst the number of lines we have measured it would not be difficult to point to some coincidences with the lines of spectra of known bodies, but other equally strong lines of the same bodies seem to be wanting, and we do not feel ourselves justified in suggesting any more than accidental coincidence.

Another feature of the photograph is the fact that the FRAUNHOFER (dark) lines about G are clearly distinguishable in the coronal spectrum. About this region of the spectrum the photographic action attains its maximum, and it is in this locality that we should naturally first look for evidence of reflected solar light. That a small fraction of the coronal light is reflected light of this nature is now without doubt established, the photographic evidences being complete. We may remark that the lines are of greatest intensity a little distance from the moon's track, though at the best they are extremely faint.

LINES seen in the corona.

4526	4212
4501 double (?)	4195
4473	4179
4442	4173 short
4414	4168
4401 short	4101 h
4395	4085
4370 short and winged	4067
4340 Hγ	4057
4289	4044
4267	4015 comparatively strong
4252	3992
4241	3948
4224	

Conclusion.

In conclusion we may briefly review the principal results we have obtained.

The direct photographs of the corona are chiefly of interest in connexion with previous and future eclipses, and we believe that those obtained will be found of value, as they have been taken during a time of sunspot maximum; as they extend further than any photographs previously obtained, and as the position of the corona has been fixed by their aid to within a fraction of a degree.

The photograph taken with the prismatic camera is of importance when we come to compare spectra of different prominences, which are found to give lines with different relative intensities, caused no doubt by differences of temperature. Two prominence lines in the ultra-red have been discovered. It is also proved that the green line of the corona is a line specially belonging to the corona, forming a distinct ring round a large part of the solar disc, whilst it is only very faintly present in the prominences. A faint ring corresponding to D_3 is also seen.

The photograph of the spectrum of the corona and prominences has yielded an abundant harvest. Twenty-nine lines of one prominence have been photographed, and the great importance which the metal calcium plays in the solar eruptions has been brought to light. Other lines, well known hitherto as chromospheric lines, but not traced in the prominences, are now shown to belong to them also, and a number of unknown lines, especially in the ultra-violet, has been added to the list.

As regards the corona, we may perhaps point out that hitherto the position of only one true corona line had been fixed, though two other lines had been suspected. The corona, during the late eclipse, seems to have been especially rich in lines. THOLLON observed some in the violet without being able to fix their position, and TACCHINI could determine the position of four true corona lines in the red; from the photograph we

have been able to measure about thirty additional lines, thus increasing the number considerably.

The fact that part of the outer corona shines by reflected light has been once more proved by the presence of the dark FRAUNHOFER lines near G, and if any doubt previously existed respecting the presence of dark lines in the corona spectrum, that doubt is now completely removed.

The results have amply proved the value of the photographic method employed, and it has been shown how an eclipse of only 70 seconds' duration can be made to yield important information.

We cannot conclude this paper without a reference to the energy and zeal displayed by Mr. C. R. WOODS, the assistant told off for the photographic operations. His careful development of all the plates has largely contributed to the success of this branch of the expedition. A six months' practice in South Kensington with the particular plates employed for the observations made him thoroughly master of the manipulations required, and to his mechanical skill in the preparation of some of the pieces of apparatus we are indebted. Mr. LAWRANCE, Mr. LOCKYER's assistant, also gave valuable aid in the photographic work at the observing station.

XII. *Evidence of a large extinct Monotreme* (Echidna Ramsayi, Ow.) *from the Wellington Breccia Cave, New South Wales.*

By Professor OWEN, *C.B.*, *F.R.S.*

Received November 3,—Read November 15, 1883.

[PLATE 14.]

AMONGST the detached bones and fragmentary evidences of Mammals from the above-named locality, submitted to me by EDWARD P. RAMSEY, Esq., F.L.S., who thence obtained them, was a humerus sufficiently complete to yield the following characters. It was of great breadth in proportion to its length, and, through the unusual size and direction of the processes and ridges for muscular attachments, seemed as if the shaft of the bone had been twisted half-way round on its axis.

The head, or proximal articular surface (Plate 14, fig. 1, *a*, and fig. 3), is a transversely elongated convexity, of a narrow ovate shape, with the broader end toward the ectotuberosity, *b*—the direction of such joint being at right angles to that of the feline humerus, in which, as in *Thylacoleo*, the antero-posterior or then-anconal diameter prevails. The non-articular portions of this end of the bone extend for equal distances to the ento-*c*- and ecto-*b*-tuberosities. From the latter is continued the "deltoid" or "anterior bicipital" ridge, *f*, from which, after its course of more than one-third the length of the shaft, it is continued by a lower ridge along the thenal aspect to be lost in the bony bridge overarching the neur-arterial canal, *k*, *o*. From the ento-tuberosity, *c*, is continued the "teretial" or "posterior tricipital" ridge, along the radial border of the humeral shaft to its termination in a special process —the "tricipital," *d*. Moreover, both ento- and ecto-tuberosities are connected together by a low curved ridge or rising which bounds a small portion of the palmar surface of the shaft immediately below the head of the humerus. From the bridge, *k*, is continued a narrow ridge to the ent-epicondylar process, *i*. The distal end of the humerus is continued, ridge-like, from *i* to a process *j* midway between the epicondyles, *h* and *i*,* but bounding the ulnar trochlea, *u*. A notch below the outlet of the

* In anthropotomy the term "condyle," rightly applied to the prominent articular convexities of the "occipital," "mandibular," and "femoral" bones, is transferred from the distal articular prominence of the humerus to the processes for attachment of muscles above the joint-surfaces. I have found it convenient, in comparative osteology, to indicate the homologues of the "external condyle" and "internal condyle" of the human humerus by the terms "ectepicondyle" and "entepicondyle."

85.

neur-arterial canal, *k*, indicates the ulnar trochlea and divides the process, *j*, from the articular tuberosity or condyle, *l*, for the head of the radius. A very small proportion of this condyle is continued upon the anconal surface of the humerus (Plate 14, fig 2); the convexity there changes to a concavity, *u*, for the ulna, and from *e* is continued the ectepicondyle, *h*, as a well-marked outstanding process.

The above partial description, with the annexed figures, of this, perhaps, most modified, after the Mole's, of Mammalian humeri, suffices to show that we have the bone of a Monotreme under comparison, and that it must be referred to the terrestrial and fossorial genus represented at the present time by a much smaller species—*Echidna hystrix* (CUV.).

The subject of the foregoing description was one of several other remains of phytophagous and insectivorous Marsupials, surpassing, like *Nototherium* and *Phascolonus*, the still existing Kangaroos and Wombats in bulk, and which, from the fractured state and markings of their bones, I conclude were dragged as prey by the sole Carnivore of adequate strength and size, at the remote period antecedent, probably, to the advent of the biped population of Australia. To the hunger of these so-called aborigines for animal food I am disposed to refer the final disappearance of Beasts, of Birds (*Dromornis*, of twice the bulk of the present Emeu, for example), and of Reptiles (of which *Megalania* is an instance), conspicuous by their bulk, and as unable as the Elephants of Africa assailed by Negroes to resist the attacks of Man when impelled by the rage of hunger. Such Kangaroos as were not disabled by their weight from a rapid saltatory flight have survived with the smaller, easily-concealed kinds. A small burrowing Wombat (*Phascolomys*) still survives; its skeleton and dentition have enabled me to interpret the nature and affinities of its huge extinct ally, the Phascolone. So, likewise, the small burrowing ant-devourer, still maintaining an existence, elucidates the affinities of its larger ancient congener.

I had long hoped to receive some fossil evidences of the Monotremes peculiar to Australasia, the lowest modification of the Mammalian class, represented by the aquatic insectivore, the *Ornithorhynchus*, and by the terrestrial kind, *Echidna*, closely resembling the placental ant-eaters.

For some years after the demise of CUVIER, both genera were deemed peculiar to the Australian and Tasmanian dismemberments of the great southern continent; but, of late, a species (*Echidna Bruijnii*)* and a second kind (*Echidna Lawesii*)† have been found living in the northern tract of New Guinea. Both these species somewhat exceed in size the Australian varieties known as *Echidna hystrix* (SHAW), and *Echidna setosa* (HOME); but the fossil here described (*Echidna Ramsayi*) shows that a species

* GERVAIS, "Ostéographie des Monotrèmes vivants et fossiles," Atlas, plates vi. and vii.

† RAMSAY, E. P., "Note of a species of *Echidna* from Port Moresby, New Guinea," Proceedings of the Linnean Society of New South Wales, vol. ii., p. 3. A notice by Mr. KREFFT has appeared in the 'Annals and Magazine of Natural History' (vol. i., p. 113), of fossil remains ascribed to the genus *Echidna*.

larger than either of those from New Guinea formerly existed in Australia. I cannot conclude without referring to the humerus of a similar exceptional type, but of considerably larger size, which was so associated with vertebral, pelvic, and femoral remains of a Reptilian character as to lead me to refer these fossils to the cold-blooded air-breathing class* under the generic name *Platypodosaurus*. It is noteworthy, in relation to a geographical approach to the present limited and exclusive locality of the modification which brings Mammals nearest to Reptiles, that the Platypodosaurian remains should have been discovered at the southern extremity of the African continent.

As corresponding parts of the humeri of the existing and extinct kinds of *Echidna* are denoted by the same letters in the drawings accompanying the present paper, a "table of admeasurements" need only to be added to exemplify the size-characters, those of the humerus of *Platypodosaurus* being added.

Humerus.	Echidna hystrix.		Echidna Ramsayi.		Platypodosaurus.	
	in.	lines.	in.	lines.	in.	lines.
Length	2	0	3	4	10	6
Breadth of proximal end	1	0	1	9	5	3
" middle of shaft	0	4	0	10	2	3
" distal end	1	9	2	9	5	10
Thickness (ancono-thenal) of middle of shaft	0	$3\frac{1}{2}$	0	8	2	6

Plate 14.

Fig. 1. Anterior or palmar view of the humerus, *Echidna Ramsayi*.
Fig. 2. Posterior or anconal surface of the humerus, ib.
Fig. 3. Articular head of the humerus, ib.
Fig. 4. Anterior or palmar view of the humerus, *Echidna hystrix*.
Fig. 5. Posterior or anconal surface of the humerus, ib.
Fig. 6. Articular head of the humerus, ib.

(Parts of the tuberosities *b* and *c* are broken off in the fossil.)

All the figures are of the natural size.

* Quarterly Journal of the Geological Society, August 1880, p. 414, plate xvi., fig. 7.

INDEX TO PART I.

OF THE

PHILOSOPHICAL TRANSACTIONS

FOR THE YEAR 1884.

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C.

D.

E.

F.

H.

K.

M.

N.

O.

R.

S.

T.

V.

W.

Y.

Fig. 1A

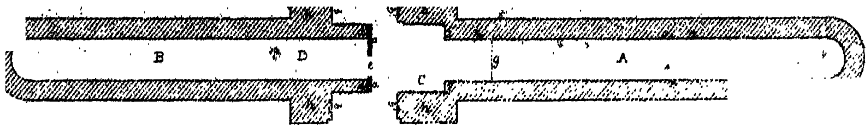
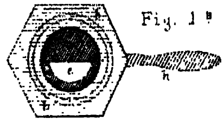
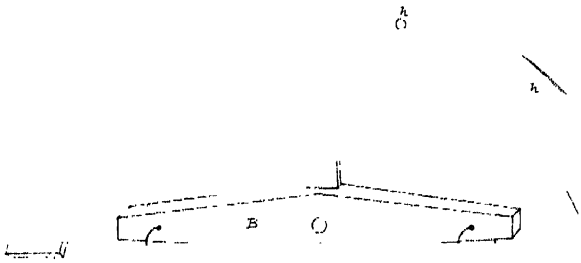
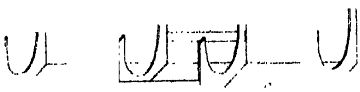


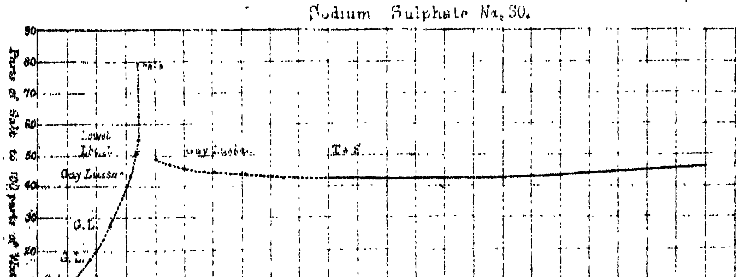
Fig. 1B



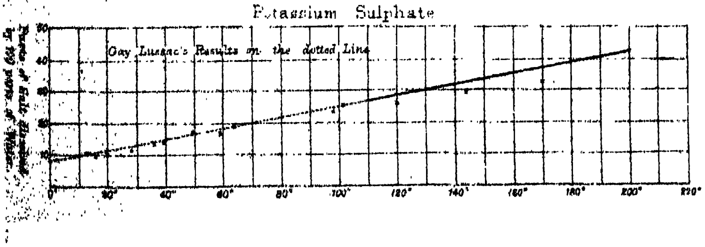




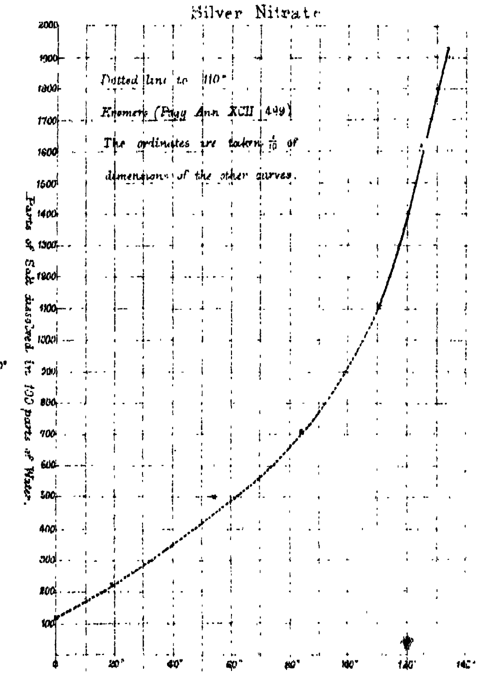
Sodium Sulphate Na2 SO4



Potassium Sulphate



Silver Nitrate



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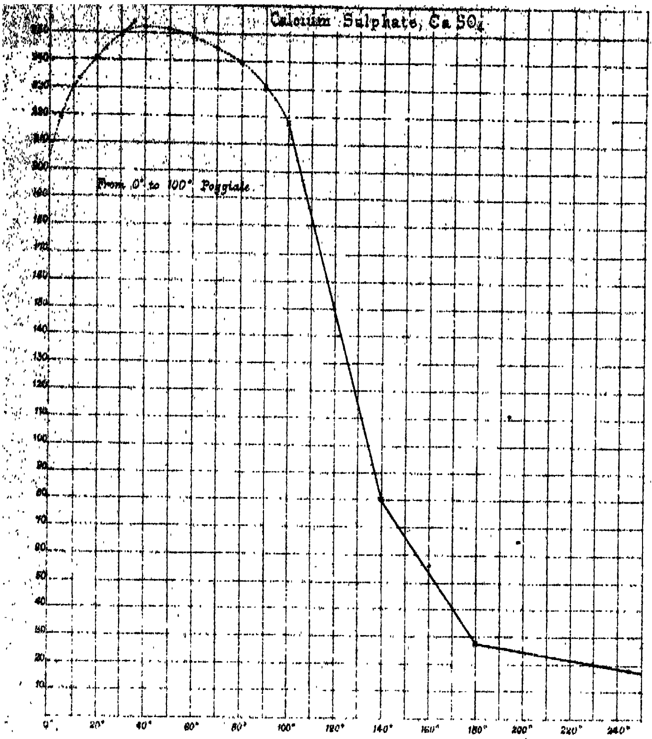
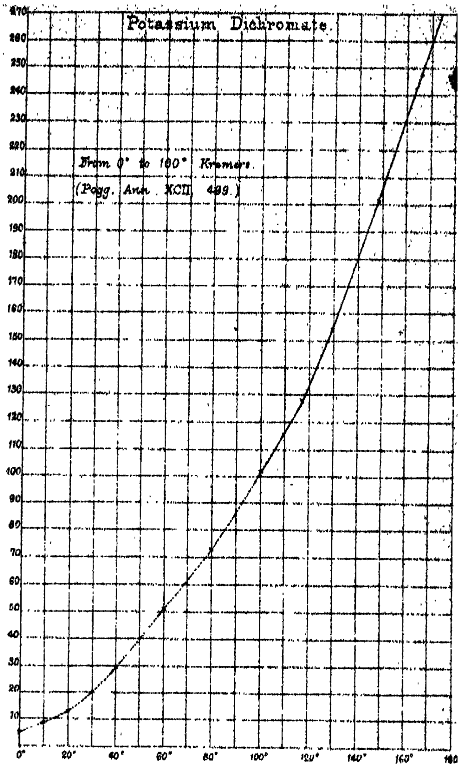
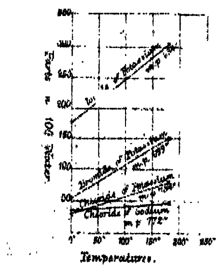
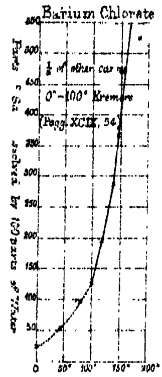
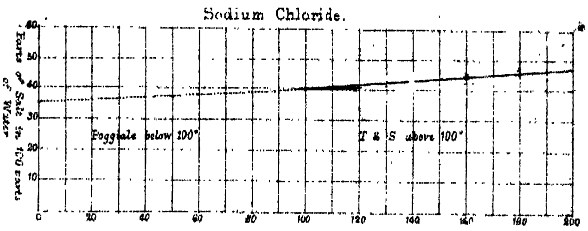
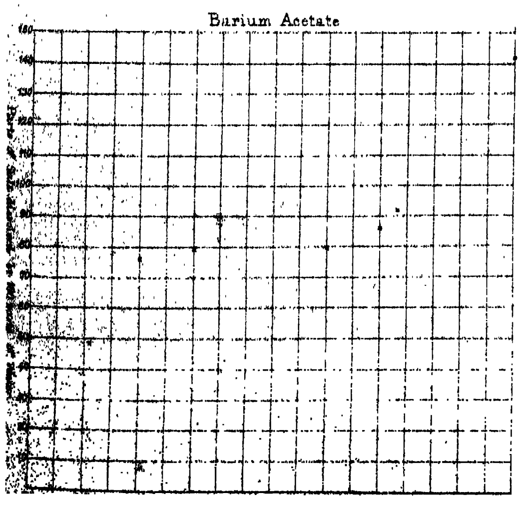

Calcium Sulphate, Ca SO4
From 0° to 100° Poggiale.

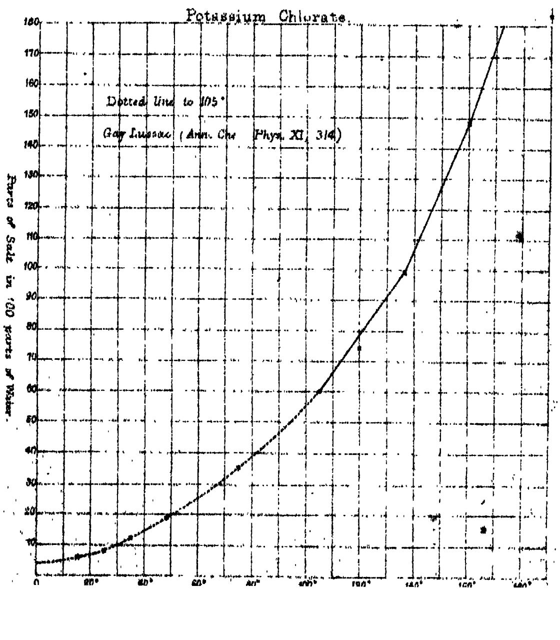

Potassium Dichromate.
From 0° to 100° Kremers.
(Pogg. Ann. XCII 499.)

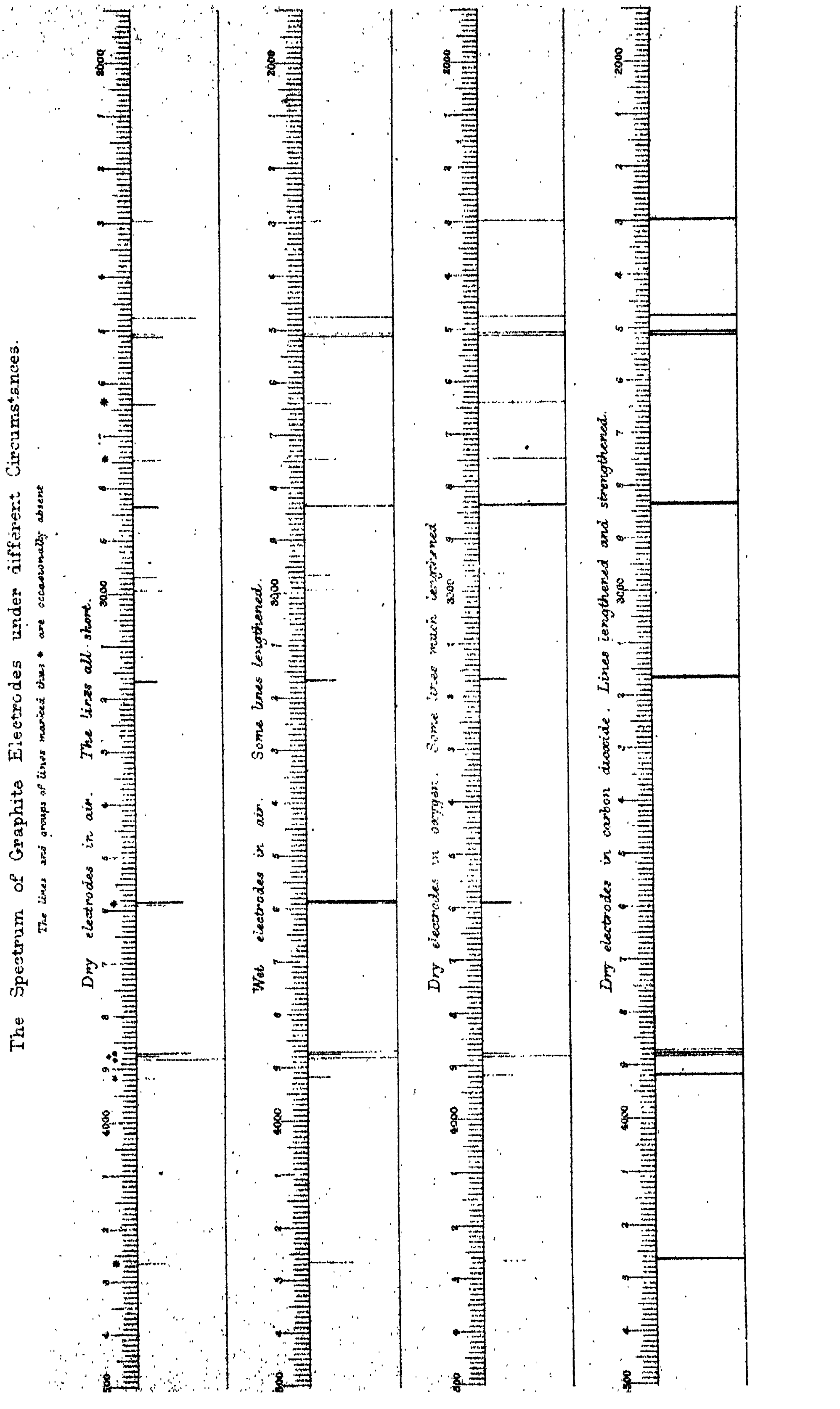

Chloride of Sodium
Temperatures.


Barium Chlorate
0°–100° Kremers
(Pogg. XCIX, 54)


Sodium Chloride.
Poggiale below 100°
T & S above 100°


Barium Acetate


Potassium Chlorate.
Dotted line to 105°
Gay Lussac (Ann. Chi. Phys. XI, 314)
Parts of Salt in 100 parts of Water.



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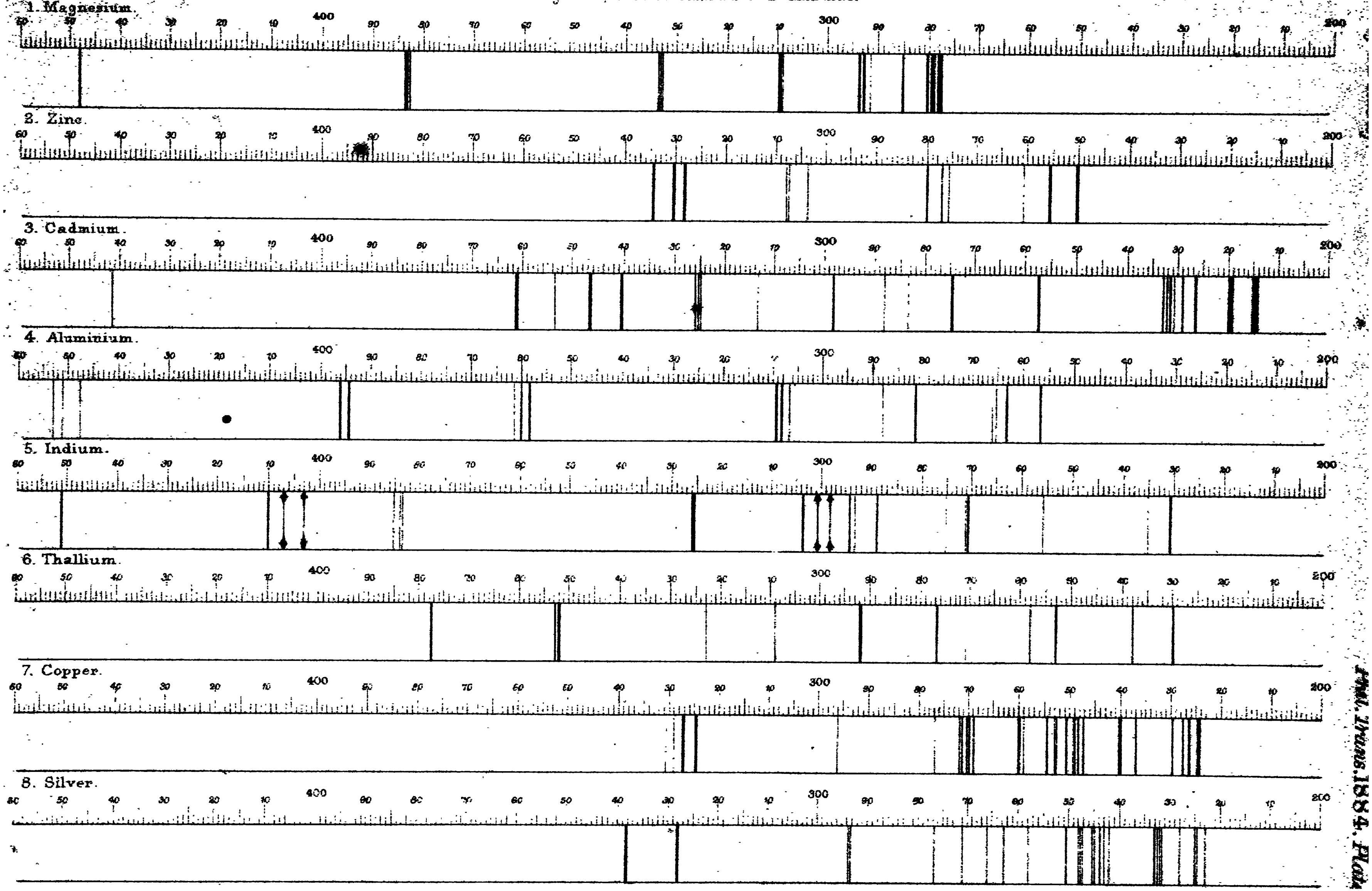
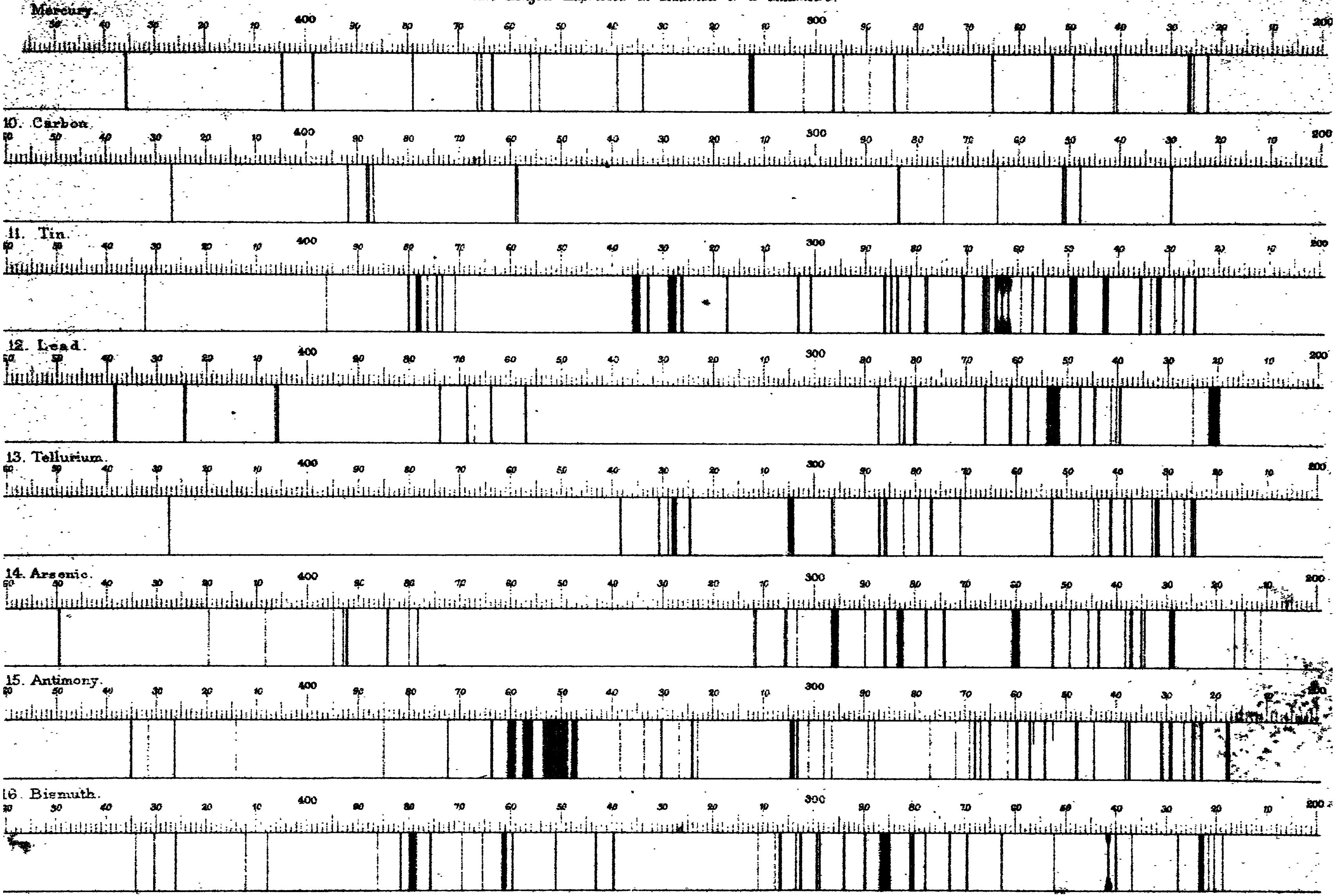
NORMAL SPECTRA.

Wave-Lengths Expressed in Millionths of a Millimetre.



NORMAL SPECTRA.

Wave-Lengths Expressed in Millionths of a Millimetre.



THE SPECTRUM OF AIR. 2ND ORDER. *Phil. Trans.* 1884. *Plate* 6.

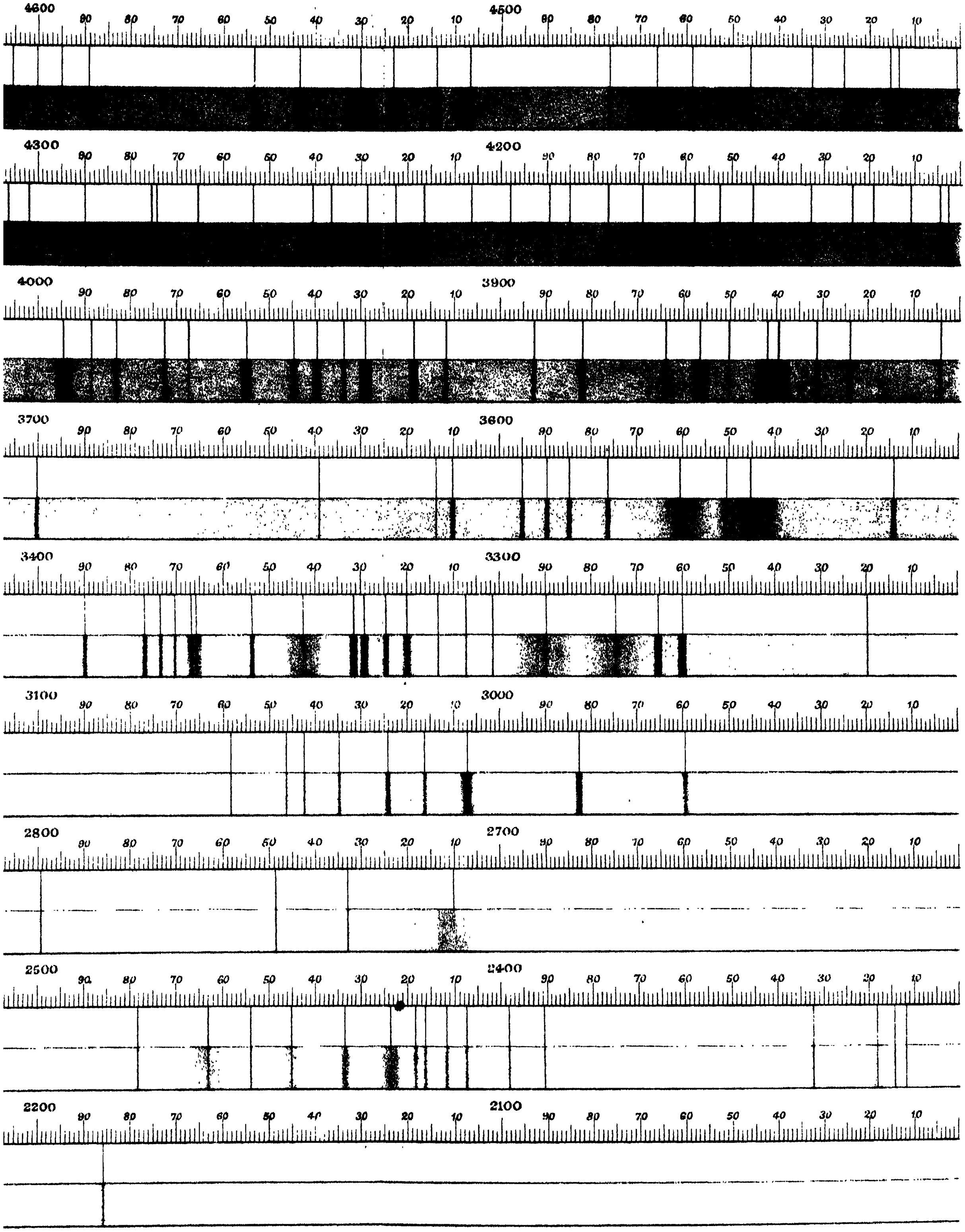


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Fig 2.

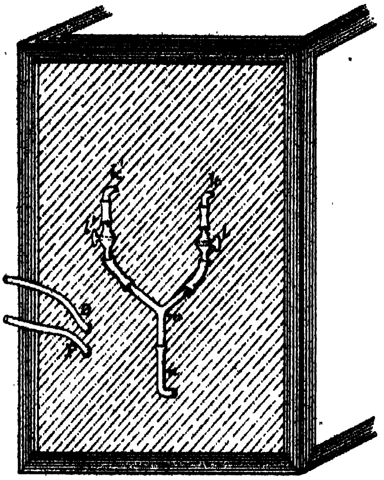


Fig. 3.

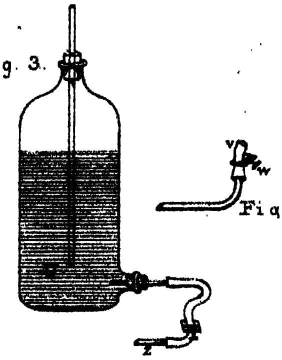
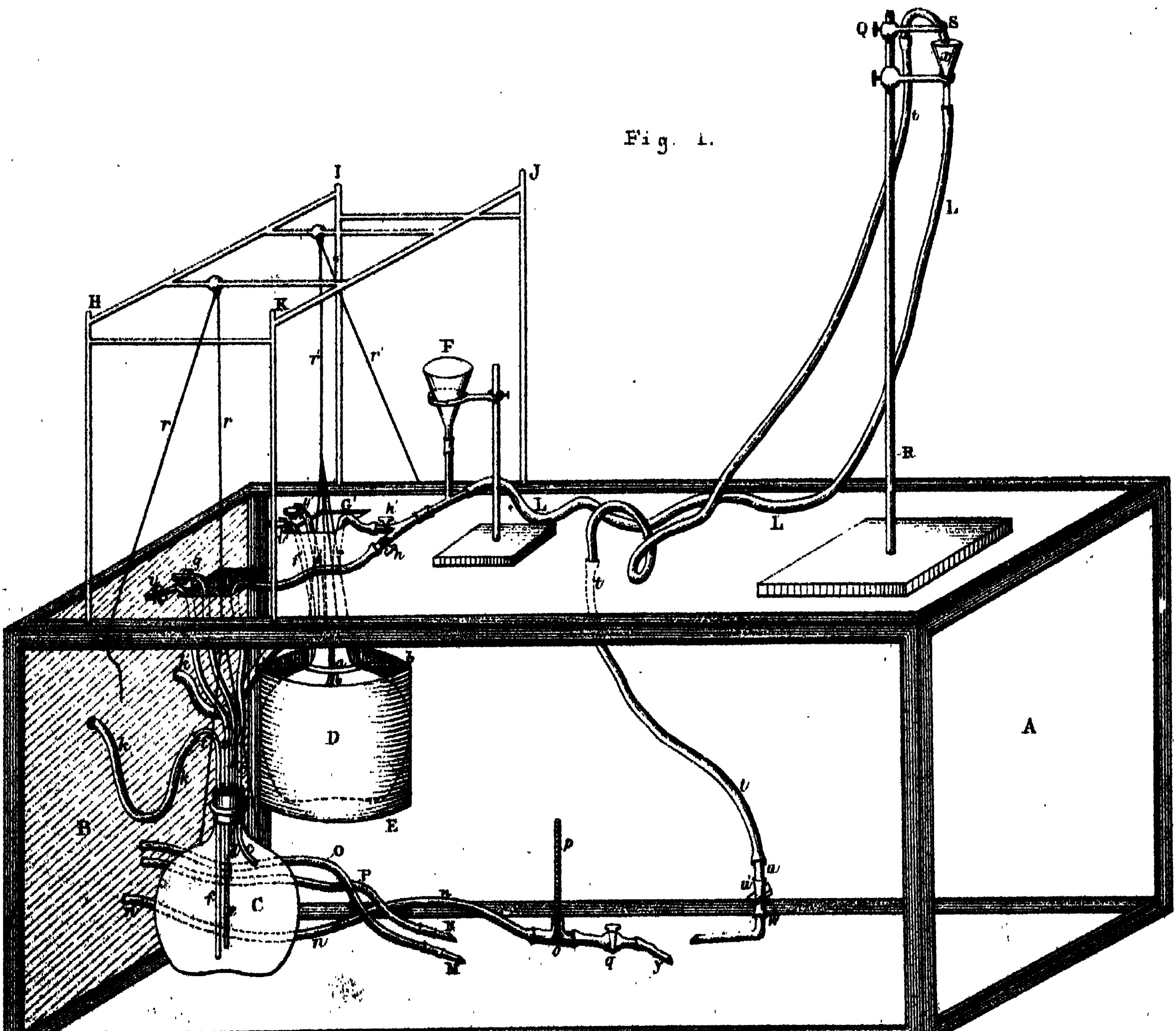


Fig. 4.

Fig. 1.



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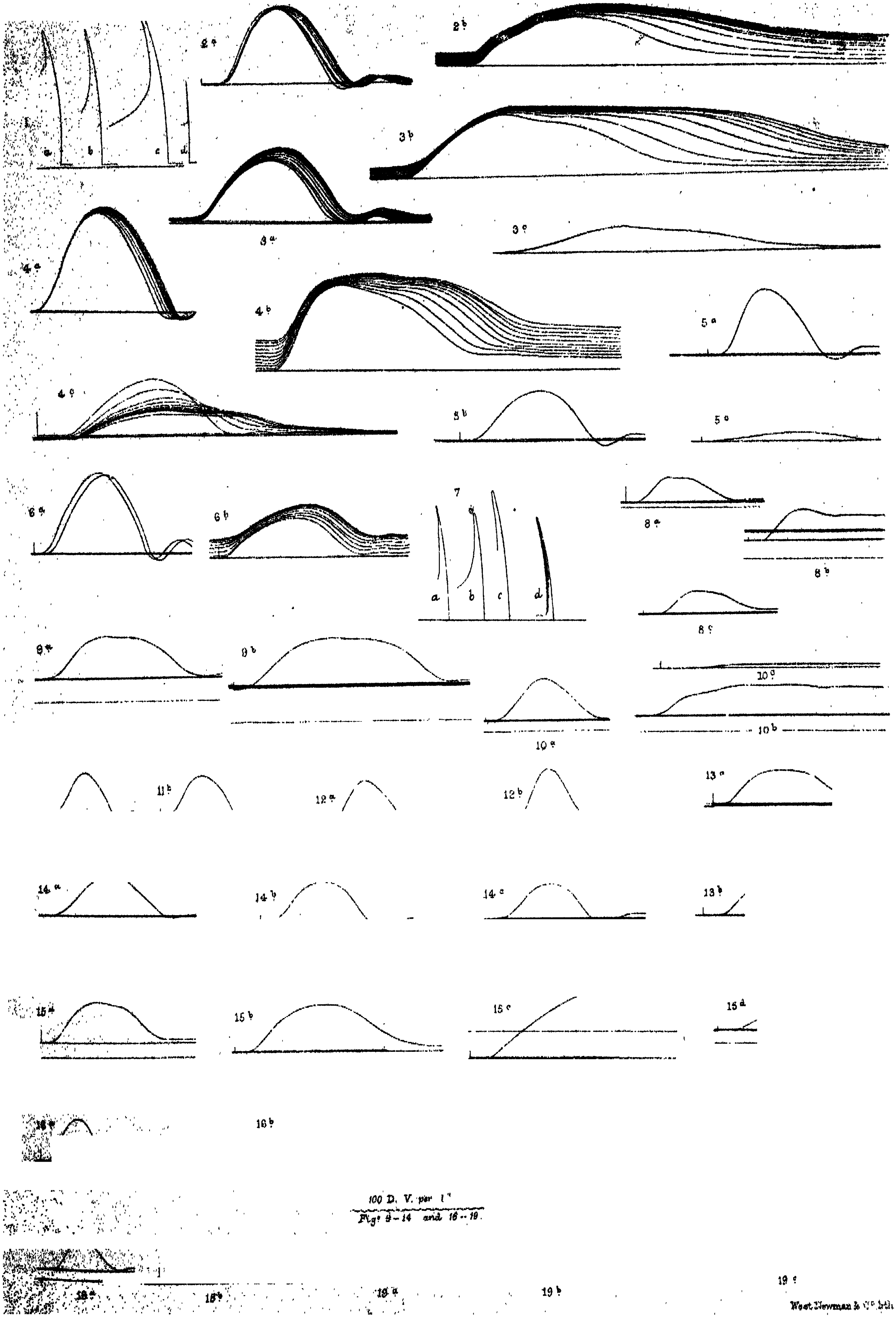

100 D. V. per 1"

Fig. 9–14 and 16–19.

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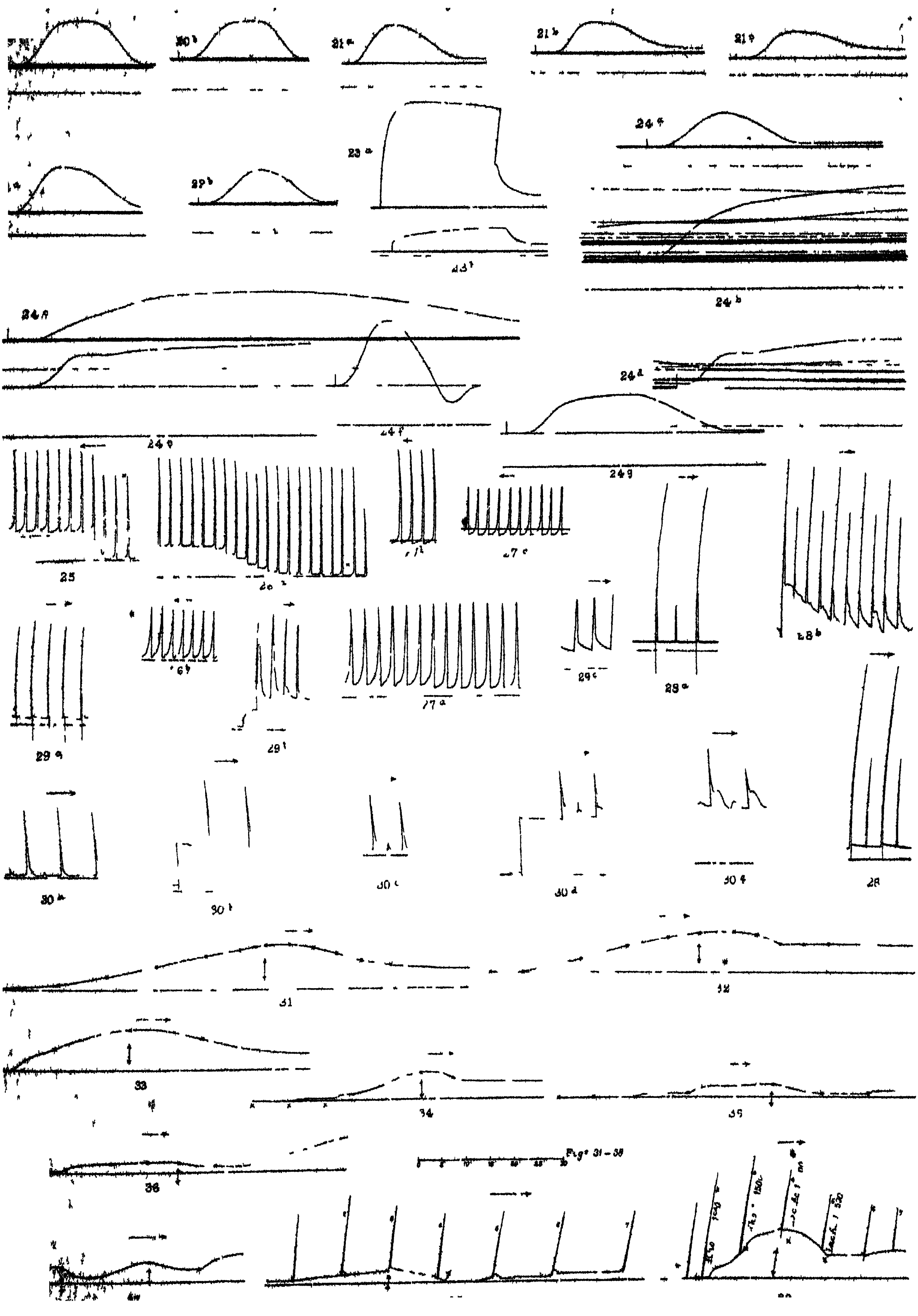
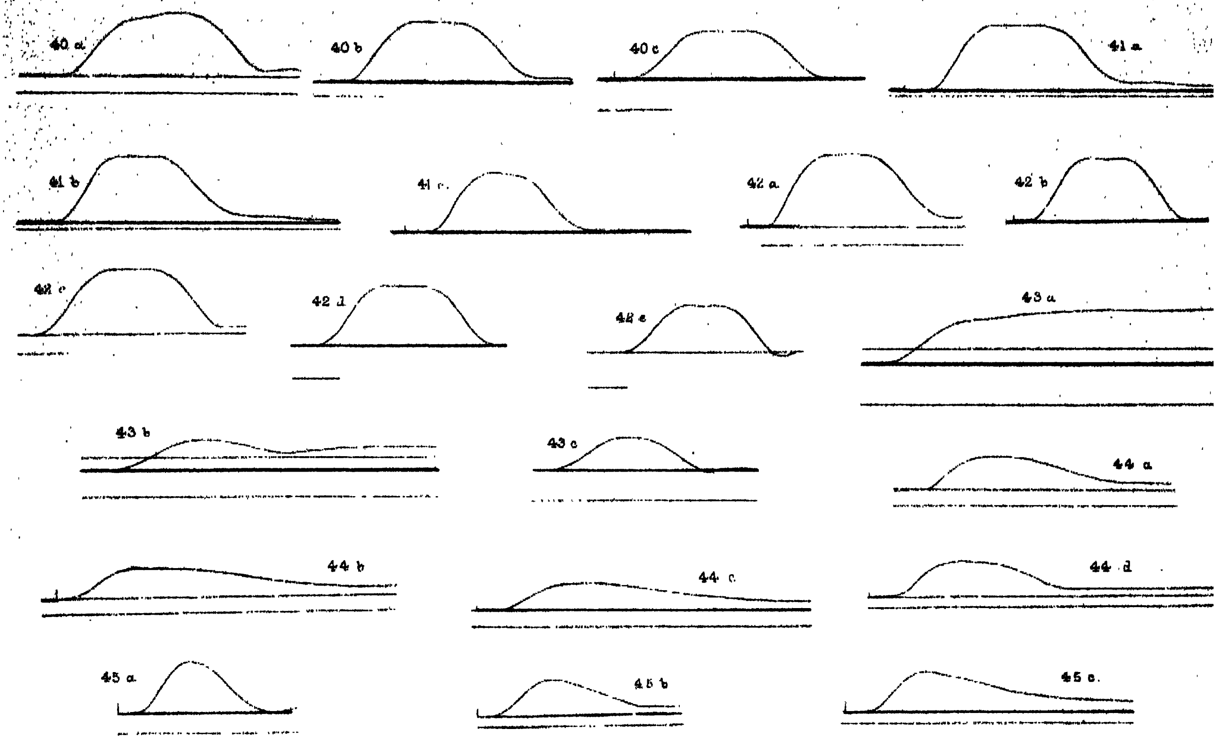
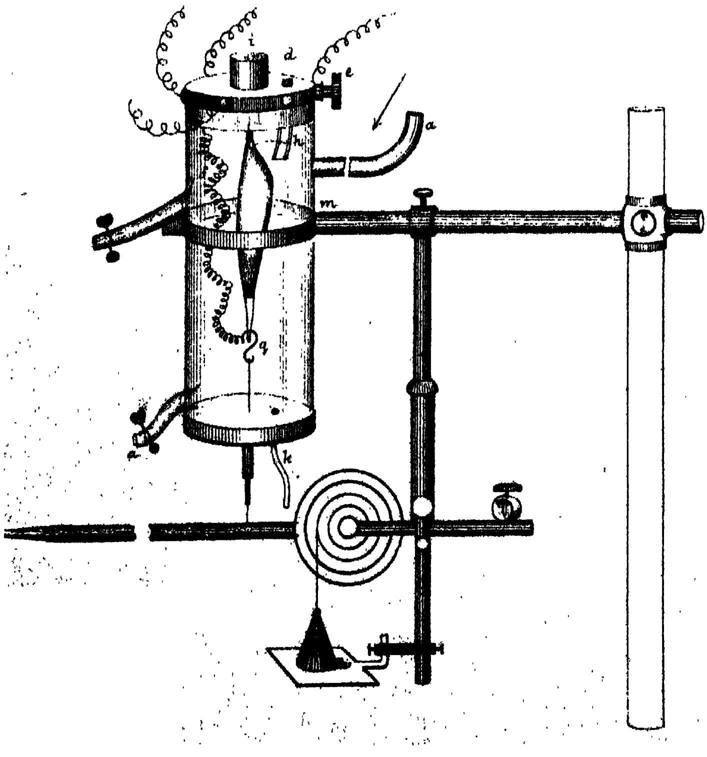


21 a
21 b
24 b
25
27 c
29 a
29 c
31
33
34
36
Fig. 31-38

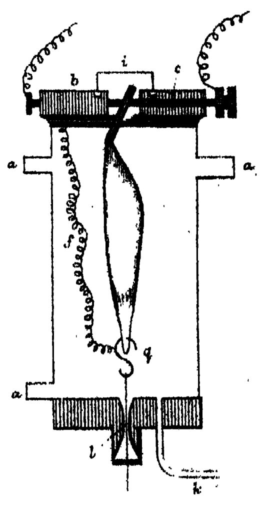
100 D. V. per 1"
Figs 40-45.



Drawing of closed Muscle Chamber with Lever in connection.
For explanation of Figures, see text



Diagramatic Section of closed Muscle Chamber (longitudinal)
For explanation of Figures see text



B

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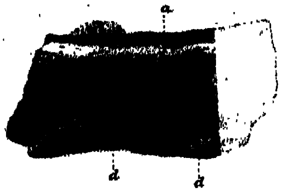


1

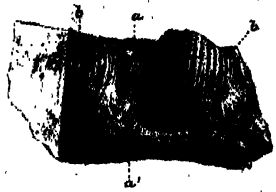


Fig. 2.

Fig. 3.

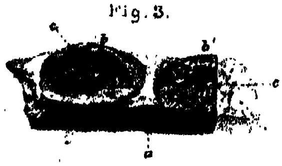


Fig. 4

Fig 5.

Fig. 6.

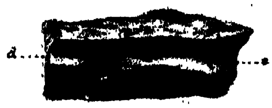


Fig. 7.

Fig. 8

Fig 9.

Fig. 1

Fig. 2

Fig. 3

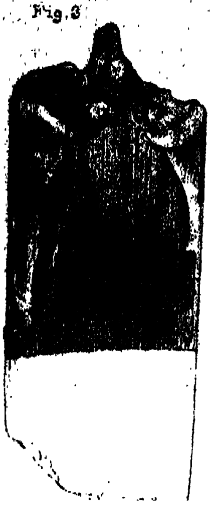


Fig. 4

Fig. 5

Fig. 3a

Fig. 6

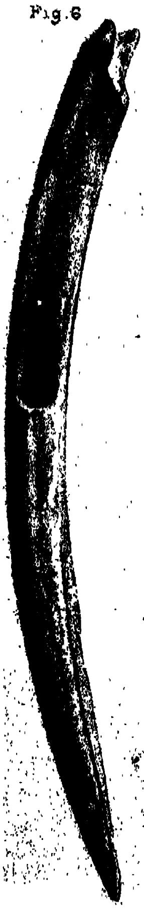


Fig. 7.

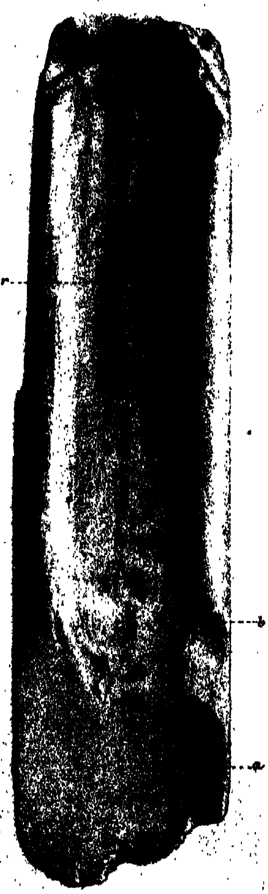


Fig. 6a

Fig. 7'

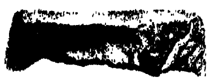
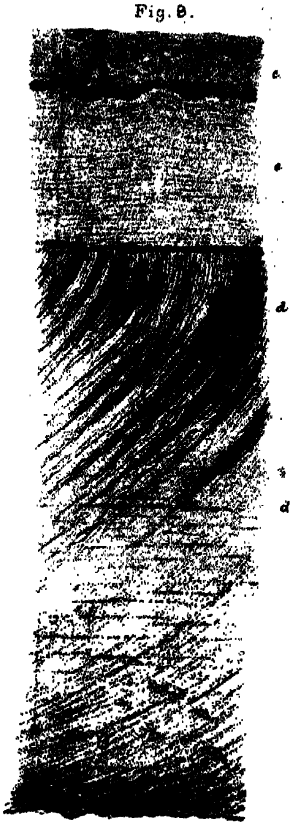
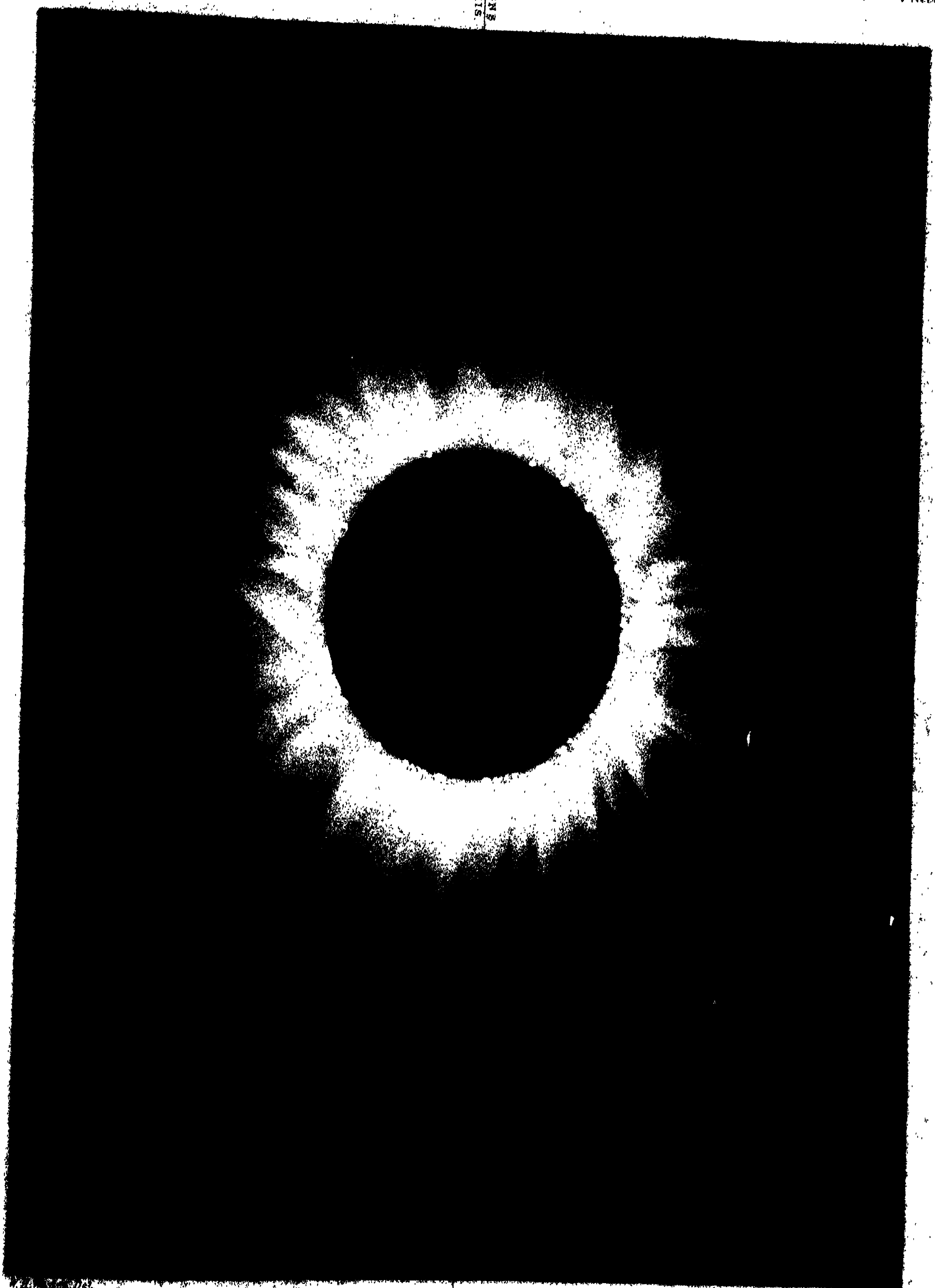


Fig. 8.

Fig. 9.



West, Newman & Co. imp.



Del.   D. J. Pound Sculp.

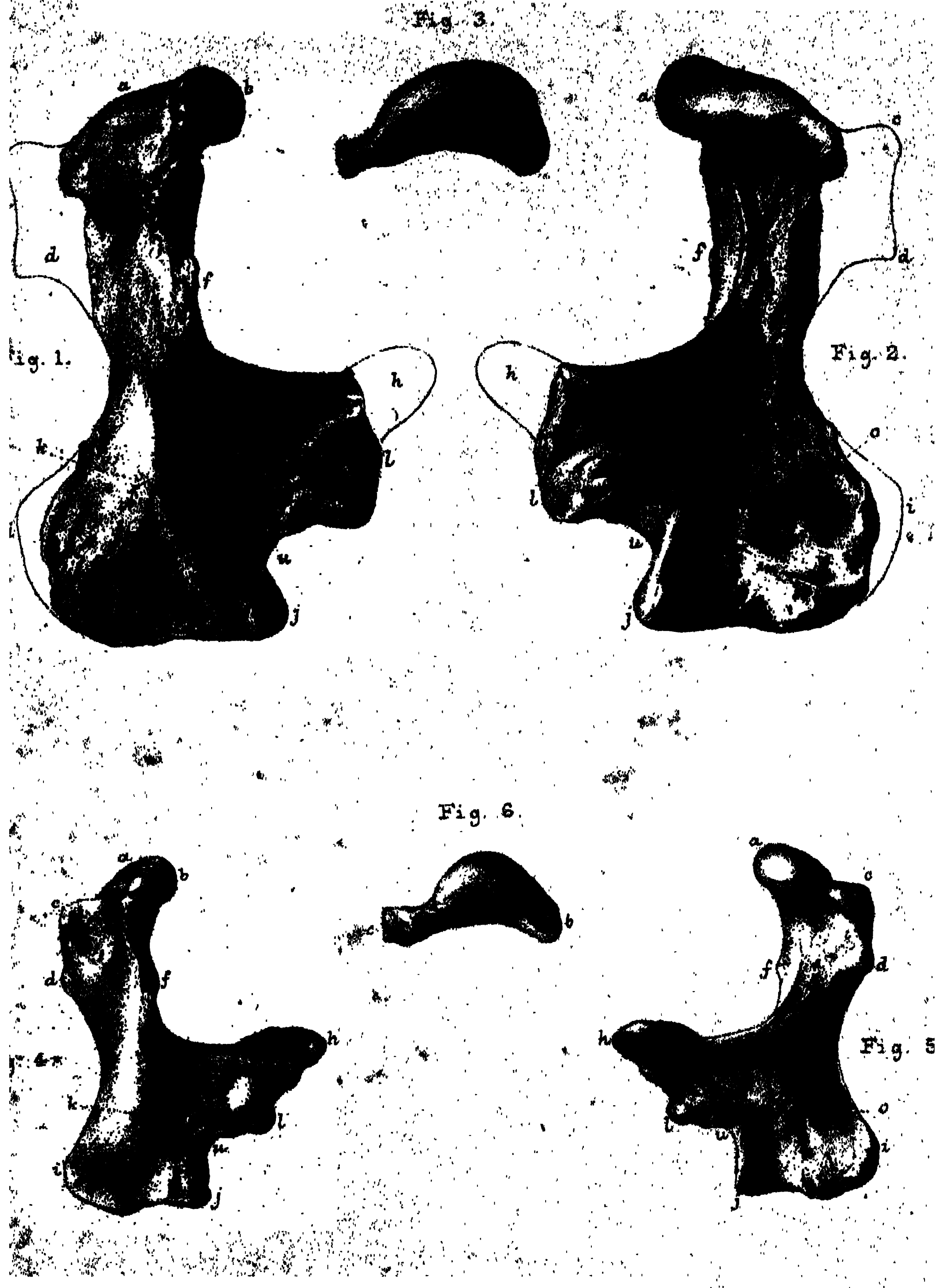


Fig. 3.
Fig. 1.
Fig. 2.
Fig. 6.
Fig. 4.
Fig. 5

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